Skills, Tasks and Technologies
Beyond the Canonical Model

Daron Acemoglu and David Autor
(Handbook of Labor Economics, 2011)

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Skills, Tasks and Technologies: Beyond the Canonical Model

- **Canonical model** — Elegantly, powerfully operationalizes supply and demand for skills
  - A formalization of Tinbergen’s “Education Race” analogy
  - Two distinct skill groups that perform two different and imperfectly substitutable tasks
  - Technology is *factor-augmenting*—Always raises productivity/wages
- Model is a theoretical and empirical success in the sense that it is widely used
But model silent on some central empirical facts of last three decades:

1. Falling real wages of low-skill workers (at least in U.S.)
2. Non-monotone shifts in inequality, despite rising ‘return to skill’
3. Widespread ‘polarization’ of employment across advanced economies
4. Skill-replacing (not augmenting) technologies

Needed: Model with richer interplay between skills, tasks, technologies

1. Distinguish between ‘skills’ and ‘tasks’
2. Endogenize assignment of skills to tasks: Comparative advantage
3. Direct competition between skills, technologies, trade in performing tasks
4. Nest canonical model as one possible case
Skills, Tasks, and Technologies: Beyond the Canonical Model

Beyond the Canonical Model of Skills and Wages

Outline

1. The canonical model: Implications and empirical successes
2. Where the canonical model falls short
3. What should an amended model offer?
4. A Ricardian model of skills, tasks and technologies patterned after Dornbusch, Fischer, Samuelson (1977, AER)
5. Some potential empirical directions
6. Conclusions
Basic assumptions

1. Two skills, high and low: $H$, $L$. Typically college v. high school
2. No distinction between skills and ‘tasks’—Skill is direct input into production
3. $H$ and $L$ are imperfect productive substitutes: $\sigma > 0$.
4. Wages are set on the demand curve

Canonical representation for aggregate output $y$:

\[ Y = \left[ (A_L L)^{\frac{\sigma-1}{\sigma}} + (A_H H)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \]

where $A_L$ and $A_H$ are factor-augmenting technology terms.
The Canonical Model

- Elasticity of substitution plays key role
  - $\sigma > 1$: $H$ and $L$ are gross substitutes. Rise in $A_H/A_L$ is SBTC
  - $\sigma < 1$: $H$ and $L$ are gross complements. Fall in $A_H/A_L$ is SBTC

\[
W_L = \frac{\partial Y}{\partial L} = A_L^{\frac{\sigma-1}{\sigma}} \left[ A_L^{\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} \left( \frac{H}{L} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}
\]

\[
W_H = \frac{\partial Y}{\partial H} = A_H^{\frac{\sigma-1}{\sigma}} \left[ A_L^{\frac{\sigma-1}{\sigma}} \left( \frac{H}{L} \right)^{\frac{\sigma-1}{\sigma}} + A_H^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}
\]
The Canonical Model

Skill premium

\[
\ln \left( \frac{W_H}{W_L} \right) = \frac{\sigma - 1}{\sigma} \ln \left( \frac{A_H}{A_L} \right) - \frac{1}{\sigma} \ln \left( \frac{H}{L} \right)
\]

Supply and demand factors represented

1. \( \ln(H/L) \) represents position of “supply curve”
2. \( \frac{\sigma - 1}{\sigma} \ln \left( \frac{A_H}{A_L} \right) \) represents position of demand curve
3. Impact of supply on wage inequality

\[
\frac{\partial \ln(W_H/W_L)}{\partial \ln(H/L)} = -\frac{1}{\sigma}
\]

4. Impact of factor technology change on wage inequality

\[
\frac{\partial \ln(W_H/W_L)}{\partial \ln(A_H/A_L)} = \frac{\sigma - 1}{\sigma} > 0 \text{ iff } \sigma > 1
\]

Consensus is that \( \sigma \in (1.4, 2.5) \), so technology that raises relative output of \( H \) also raises its relative wage
The Canonical Model

Some key testable predictions

1. Rise in supply of $H/L$ reduces skilled wage differential
   \[ \frac{\partial \ln (w_H/w_L)}{\partial \ln (H/L)} = -\frac{1}{\sigma} < 0 \]

2. Rise in supply of $H/L$ also raises real wage of $L$:
   \[ \frac{\partial w_L}{\partial (H/L)} > 0 \]
   This follows from imperfect substitutability between $H$ and $L$ and complementarity

3. Factor augmenting tech $\Delta$ always raises wages of $L$ workers:
   \[ \frac{\partial W_L}{\partial A_L} > 0 \text{ and } \frac{\partial W_L}{\partial A_H} > 0 \]
   This also follows from imperfect substitutability

4. Predictions of this model always apply to both skills
   - A bit tautological since there are only two skills/wages
   - But assume a continuum of efficiencies in each skill group: still true
   - Loosely: Wage inequality is either rising or falling in this model, not both
The Canonical Model: Implementation

- The two-factor model estimated by Katz and Murphy (1992):
  - Used data from 1963 through 1987, fit by OLS

\[
\ln \left( \frac{W_H}{W_L} \right) = \frac{\sigma - 1}{\sigma} \gamma_0 + \frac{\sigma - 1}{\sigma} \gamma_1 t - \gamma_2 \ln \left( \frac{H_t}{L_t} \right)
\]

- Replicating their approach, we get

\[
\ln \left( \frac{W_H}{W_L} \right) = 0.027 \times t - 0.612 \cdot \ln \left( \frac{H_t}{L_t} \right)
\]

(0.005) (0.128)

- This estimate implies
  1. Log relative demand for College/Non-College rising at 2.7 log points annually
  2. Elasticity of substitution \( \hat{\sigma} = 1/\hat{\gamma}_2 \approx 1.6 \)

- You can see how well this works in the next figures
- Over predicts wage growth in 2000s
The Canonical Model: Easy to See Why K-M Model Fits!
The Canonical Model: Many more Successes

2. Carneiro and Lee (2009): Fit to data for U.S. regions
   - Fit to data for three countries: U.S., U.K., Canada
   - Allow for imperfect substitutability among age cohorts
   - Explain cross-country variation in timing of rise of college premium and within-country variation in magnitude of rise in premium by age groups within countries
   - See also Fitzenberger and Kohn (2006) for German application
The Canonical Model
Explaining the College Premium by Experience Group

A. College-High School Wage Gap by Potential Experience Group

B. College-High School Relative Supply by Potential Experience Group
The Canonical Model

Explaining the College Premium by Experience Group

- The model can be extended to account for differing trends by experience group
- Estimate a regression model for the college wage premium by experience group:

\[
\ln \omega_{jt} = \beta_0 + \beta_1 \left[ \ln \left( \frac{H_{jt}}{L_{jt}} \right) - \ln \left( \frac{H_t}{L_t} \right) \right] + \beta_2 \ln \left( \frac{H_t}{L_t} \right) + \beta_3 \times t + \beta_4 \times t^2 + \delta_j + n_{jt},
\]

where \(j\) indexes experience groups, \(\delta_j\) is a set of experience group main effects. A quadratic time trend is included.

<table>
<thead>
<tr>
<th>Potential experience groups (years)</th>
<th>All</th>
<th>0-9</th>
<th>10-19</th>
<th>20-29</th>
<th>30-39</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own minus aggregate supply</td>
<td>-0.272</td>
<td>-0.441</td>
<td>-0.349</td>
<td>0.109</td>
<td>-0.085</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.136)</td>
<td>(0.095)</td>
<td>(0.079)</td>
<td>(0.099)</td>
</tr>
<tr>
<td>Aggregate supply</td>
<td>-0.553</td>
<td>-0.668</td>
<td>-0.428</td>
<td>-0.343</td>
<td>-0.407</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
<td>(0.209)</td>
<td>(0.142)</td>
<td>(0.138)</td>
<td>(0.141)</td>
</tr>
<tr>
<td>Time</td>
<td>0.027</td>
<td>0.035</td>
<td>0.016</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Time²/100</td>
<td>-0.010</td>
<td>-0.023</td>
<td>0.007</td>
<td>0.001</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.056</td>
<td>-0.118</td>
<td>0.120</td>
<td>0.138</td>
<td>0.018</td>
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<tr>
<td></td>
<td>(0.085)</td>
<td>(0.212)</td>
<td>(0.169)</td>
<td>(0.145)</td>
<td>(0.144)</td>
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<td>Observations</td>
<td>184</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
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<tr>
<td>R-squared</td>
<td>0.885</td>
<td>0.885</td>
<td>0.959</td>
<td>0.929</td>
<td>0.771</td>
</tr>
</tbody>
</table>

Source: March CPS data for earnings years 1963-2008. See notes to Figs 2 and 19.
**Overall inequality in the canonical model**

- Within group inequality is invariant to skill prices

\[
\frac{W_i}{W_{i'}} = \frac{w_L l_i}{w_L l_{i'}} = \frac{l_i}{l_{i'}} \text{ for } i, i' \in \mathcal{L}.
\]

- There can be within group wage inequality, but it will be independent of the skill premium
Overall inequality in the canonical model

- It is possible to make within group inequality responsive to the wage premium
- Assume that the two observable groups are college and non-college
- Fraction $\phi_c$ college graduates are high skill
- Fraction $\phi_n < \phi_c$ non-college graduates are high skill
- Skill premium is $\omega = \frac{w_H}{w_L}$
- College wages, $w_C$, non-college, $w_N$

$$\omega_c = \frac{w_C}{w_N} = \frac{\phi_c w_H + (1 - \phi_c) w_L}{\phi_n w_H + (1 - \phi_n) w_L} = \frac{\phi_c \omega + (1 - \phi_c)}{\phi_n \omega + (1 - \phi_n)}.$$ 

- Like Gorman-Lancaster Model
Overall inequality in the canonical model

- Because $\phi_n < \phi_c$, when the true price of skill increases, the observed college premium will also arise.
- Trivially explains wage inequality within groups as a function of skill premium.
Beyond the ‘Canonical Model’ of Skills and Wages

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1. The canonical model: Implications and empirical successes
2. Where the canonical model falls short
3. What should an amended model offer?
4. A Ricardian model of skills, tasks and technologies
5. Some potential empirical directions
6. Conclusions
Where the Canonical Model is Silent (or Mis-speaks)

1. Wage inequality (as measured by $\ln \frac{W_H}{W_L}$) rises less than predicted
2. *Real wage levels fall* for some groups
3. Wage changes non-uniform in skill
4. Polarization of employment growth across high/low-skill occupations (also non-uniform)
5. Rising importance of *occupation* as a predictor of earnings
6. Casual empiricism only
   - Directly skill-replacing technologies commonplace
   - Offshoring may function like a skill-replacing technology
Wage Inequality Rises by Much Less than Predicted

- College premium rose by 12 points between 1992 and 2008. Model predicts a *rise* of 25 log points!
- Model implies demand *decelerated* after 1992 or elasticity ($\sigma$) rose
Real wage levels fall for low-education males

![Graph showing real, composition-adjusted log weekly wages for full-time full-year workers from 1963 to 2008, with different levels for low-education males.](image)
‘Convexification’ of the Return to Education
See Lemieux (2006)
Wage changes non-monotone: Male indexed 90/50/10
Wage changes non-monotone: Female indexed 90/50/10
Non-monotone wage changes: Males full distribution
Non-monotone wage changes: Females full distribution
Polarization of Emp. Growth by Occupational Skill
Monotone in 1980s, Concentrated in Tails in 1990s and 2000s
Polarization of Emp Growth by Occupational Skill

Percent Change in Employment by Occupation, 1979-2009

- Managers
- Professionals
- Technicians
- Sales
- Office/Admin
- Production
- Operational/Laborers
- Protective Service
- Food/Cleaning Service
- Personal Care

Colors:
- Blue: 1979-1989
- Orange: 2007-2009
Polarization of Emp Growth by Occupational Skill
Harmonized European LFS Data from Goos, Manning and Salomons (2009)

See also Dustmann, Ludsteck and Schonberg (2009), *QJE*
Polarization of Emp Growth by Occupational Skill
U.S. + Eurostat Data: 10 Countries, 1992-2008. Correlation(US, EU)=0.67
Rising importance of *occupation* as a predictor of earnings
Rising importance of *job tasks* as a predictor of earnings
Where the Canonical Model is Silent (or Mis-speaks)

1. Wage inequality rises less than predicted
2. Real wage levels fall for some groups
3. Wage changes non-monotone in skill
4. Polarization of employment growth across high/low-skill occupations (also non-monotone)
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What should an amended model offer?

Objectives

1. Explicit distinction between *skills* and *tasks*
   - Tasks—Unit of work activity that produces output
   - Skill—Worker’s endowment of capabilities for performing various tasks

2. Allow for *comparative advantage* among workers in different tasks
   - Assignment of skills to tasks is *endogenous* (as in Roy, 1951)

3. Allow for multiple sources of competing task ‘supplies’
   - Workers of different skill levels
   - Machines—Task can be routinized/automated
   - Offshoring—As per Grossman and Rossi-Hansberg (2008)

4. Incorporate at least three skill groups—To study polarization

5. Goal: well-defined set of skill demands, as in canonical model

6. Ability to endogenize task-biased technological change
Related models

- Heckman and Sedlacek (1985)
- Heckman and Scheinkman (1987)
- Acemoglu and Zilibotti (2001)
- Autor and Dorn (2009)
- Goos, Manning and Salomons (2009)
- Costinot and Vogel (2010)

Our model is less general than Costinot and Vogel, but quite broadly applicable
A Ricardian Model of Skills, Tasks and Technologies

Production technology: Tasks into goods

- Static environment with a single final good, $Y$
- $Y$ produced with continuum of tasks on the unit interval, $[0, 1]$
- Cobb-Douglas technology mapping tasks to the final good:

$$\ln Y = \int_0^1 \ln y(i) di,$$

where $y(i)$ is the “service” or production level of task $i$

- Price of the final good, $Y$, is numeraire
A Ricardian Model of Skills, Tasks and Technologies

Supply of skills to tasks

Three types of labor: High, Medium and Low

- Fixed, inelastic supply of the three types. Supplies are $L$, $M$ and $H$
- Workers are homogeneous within groups
- Later introduce capital or technology (embedded in machines)

Each task $i$ defined on the on continuum has linear production function

$$y(i) = A_L \alpha_L (i) l(i) + A_M \alpha_M (i) m(i) + A_H \alpha_H (i) h(i) + A_K \alpha_K (i) k(i),$$

- Inputs are perfect substitutes
- $A$ terms are factor-augmenting technologies
- $\alpha_L (i)$, $\alpha_M (i)$ and $\alpha_H (i)$ are task productivity schedules
- For example, $A_L \alpha_L (i)$ is the productivity of low skill workers in task $i$, and $l(i)$ is the number of low skill workers allocated task $i$
Role of comparative advantage

- All tasks can be performed by low, medium or high skill workers

\[ y(i) = A_L \alpha_L(i) l(i) + A_M \alpha_M(i) m(i) + A_H \alpha_H(i) h(i) + A_K \alpha_K(i) k(i) \]

- But comparative advantage by skill differs via \( \alpha_L(i), \alpha_M(i), \alpha_H(i) \)

Comparative advantage schedule

- **Assumption:** \( \frac{\alpha_L(i)}{\alpha_M(i)} \) and \( \frac{\alpha_M(i)}{\alpha_H(i)} \) are continuously differentiable and strictly decreasing:
  \[
  \frac{\alpha_L(i)}{\alpha_M(i)} \downarrow i; \quad \frac{\alpha_M(i)}{\alpha_H(i)} \downarrow i
  \]

- Higher indices correspond to “more complex” tasks

- In all tasks, \( H \) has absolute advantage relative to \( M \), \( M \) has absolute advantage relative to \( L \)

- But **comparative advantage** determines task allocations
A Ricardian Model of Skills, Tasks and Technologies

Consider an equilibrium without capital: $\alpha_k(\cdot) = 0$

**Equilibrium objects: Task thresholds, $I_L, I_H$**

- In any equilibrium there exist $I_L$ and $I_H$ such that $0 < I_L < I_H < 1$
- and for any $i < I_L$, $m(i) = h(i) = 0$, for any $i \in (I_L, I_H)$, $l(i) = h(i) = 0$, and for any $i > I_H$, $l(i) = m(i) = 0$

**Allocation of tasks to skill groups determined by $I_H, I_L$**

- Tasks $i > I_H$ will be performed by high skill workers (Abstract)
- Tasks $i < I_L$ will be performed by low skill workers (Manual)
- Middle tasks $I_L \leq i \leq I_H$ will be performed by medium skill workers (Routine)

**Boundaries of these sets are determined by the model**

- Given skill supplies, firms (equivalently workers) decide which skills perform which tasks → *Substitution of skills across tasks*
Solving the model

- As workers are homogenous within each group:

\[ W_L(i) = p(i)A_L\alpha_L(i) = p(i')A_L\alpha_L(i') = W_L(i') = W_L \]
\[ \Rightarrow p(i)\alpha_L(i) = p(i')\alpha_L(i') = P_L \]

- Similar expressions for \( M \) and \( H \)

- From cost minimization \( \Rightarrow p(i)y(i) = p(i')y(i') \)

- Taking logs and integrating over \( i' \) get \( p(i)y(i) = P_YY = Y \), using \( P_Y = 1 \)
Solving the model

\[
\begin{align*}
\underbrace{P_L \ p(i) \ \alpha_L(i) \ l(i)}_{y(i)} &= \underbrace{P_L \ p(i') \ \alpha_L(i') \ l(i')}_{y(i')} \\
\Rightarrow l(i) &= l(i') \\
l(i) &= \frac{L}{l_L} \text{ for } i < l_L
\end{align*}
\]

and by analogous reasoning:

\[
\begin{align*}
m(i) &= \frac{M}{l_H - l_L} \text{ and} \\
h(i) &= \frac{H}{1 - l_H}
\end{align*}
\]
Notice that for task $i = I_H$ high and medium skill workers are equally productive and so are medium and low skill workers at $i = I_L$ we get:

No arbitrage between $H$ and $M$:  \[
\frac{A_H\alpha_M(I_H)M}{I_H - I_L} = \frac{A_H\alpha_H(I_H)H}{1 - I_H}
\]

No arbitrage between $M$ and $L$:  \[
\frac{A_L\alpha_L(I_L)L}{I_L} = \frac{A_M\alpha_M(I_L)M}{I_M - I_L}
\]
Equilibrium Task Thresholds: No Arbitrage Across Skill Groups

Figure 22. Determination of Equilibrium Threshold Tasks

- No arbitrage between H and M
- No arbitrage between M and L

\[ I_L^e, I_H^e \]
Relative wages in the Ricardian model

- Relative wages solely a function of labor supplies and task thresholds

\[
\frac{w_H}{w_M} = \left( \frac{1 - I_H}{I_H - I_L} \right) \left( \frac{H}{M} \right)^{-1},
\]

\[
\frac{w_M}{w_L} = \left( \frac{I_H - I_L}{I_L} \right) \left( \frac{M}{L} \right)^{-1}
\]

- So, labor supplies \( L, M, H \) plus comparative advantage schedules \( \alpha(L), \alpha(M), \alpha(L) \) determine task allocation, \( I_L \) and \( I_H \), and hence wages
Skill-biased Technical Change: A Rise in $A(H)$

- Rise in productivity of $H$ workers broadens their task set, lowers $I_H$
- Squeezes $M$ workers (excess supply of $M$) so $I_L$ also falls
Some Key Comparative Statics

Consider a rise in $A_H$ (SBTC):

- Increase share of tasks done by $H$
- Raises $W_H/W_M$ and $W_H/W_L$
- Lowers $W_M/W_L$! Why? Because $H$ and $M$ are closer substitutes than $H$ and $L$

Consider a rise in high-skilled labor supply $H$:

- Increase share of tasks done by $H$
- Lowers $W_H/W_M$ and $W_H/W_L$
- Lowers $W_M/W_L$ (Rise in $A_H$ is isomorphic to rise in $H$)

Identical comparative statics for rise in $A_L$ or $L$
Change in productivity or supply of middle-skill workers

Subtle effects

What happens when either $M$ or $A_M$ rises?

- Depends critically on this term:
  \[ |\beta'_L (I_L) I_L| > |\beta'_H (I_H) (1 - I_H)| \]

- $\beta_H(I) \equiv \ln \alpha_M(I) - \ln \alpha_H(I)$  
  $\beta_L(I) \equiv \ln \alpha_L(I) - \ln \alpha_M(I)$

- $\beta_H$ and $\beta_L$ measure the comparative advantage of $L$ versus $H$ workers in $M$ tasks

- If $\beta'_L (I_L)$ is low relative to $\beta'_H (I_H)$), high skill workers have strong comparative advantage for tasks above $I_H$

Hence, rise in $M$ displaces $L$ workers more than $H$ iff:

\[ \frac{d \ln (w_H/w_L)}{d \ln M} > 0 \iff |\beta'_L (I_L) I_L| < |\beta'_H (I_H) (1 - I_H)| \]

- Implicitly $I_L$ falls more than $I_H$ rises
How Technology Enters

Easy to model a ‘task replacing technology’

- Both $K$ and Labor can supply tasks (all are perfect substitutes)
- $K$ will supply task if can accomplish more cheaply than $L$, $M$, or $H$

Example: Routine Task Replacing technology

- Capital that out-competes $M$ in a subset of tasks $i'$ in the interval $I_L < i' < I_H$

Own wage effects

- Immediately lowers wage of $M$ by narrowing set of $M$ tasks

Cross-price effects on $W_L$ and $W_H$?

- Again depend on $|\beta'_L (I_L) I_L| \geq |\beta'_H (I_H) (1 - I_H)|$
- If $M$ workers better suited to $L$ than $H$ tasks, then $W_H / W_L$ rises
Routine Task Replacing Technology

Focal case

- Task replacing technology concentrated in middle-skill/routine tasks
- Strong comparative advantage of $H$ relative to $L$ at respective margins with $M$

Leads to wage and employment ‘polarization’

1. Wages:
   - Middle wages fall relative to top and bottom.
   - Top rises relative to bottom

2. Employment:
   - Middle-skill/routine tasks mechanized
   - Declining labor input in routine tasks
   - Given comparative advantage, middle-skill workers move disproportionately downward in task distribution
Offshoring works identically to capital that competes for tasks

- In this sense, model is like that of Grossman and Rossi-Hansberg (2008)
- But the comparative advantage setup here is more general (plausible)
First extension

Endogenous choice of skills

- Factor augmenting technical change (or introduction of skill substituting capital) will affect wages inducing a response in the supplies of skills (e.g. medium skill workers may start supply low skills)
- Workers can have a bundle of $l$, $m$, and $h$ skills
- When comparative advantage of one skill sufficiently eroded, may switch skills
- Example: Former manager, now driving delivery truck
Endogenous choice of skills

- Assume that each worker $j$ is endowed with some amount of “low skill,” “medium skill,” and “high skill,” respectively $l_j$, $m_j$ and $h_j$

- Workers have one unit of time, which is subject to a skill allocation constraint

$$t^j_l + t^j_m + t^j_h \leq 1$$

- The worker's income is

$$w_L t^j_l l^j + w_M t^j_m m^j + w_H t^j_h h^j,$$

- The worker with skill vector $(l_j, m_j, h_j)$ will have to allocate his time between jobs requiring different types of skills
Endogenous choice of skills

- Aggregate amount of skills of different types:

\[
L = \int_{j \in E_l} l^j \, dj, \quad M = \int_{j \in E_m} m^j \, dj, \quad H = \int_{j \in E_h} h^j \, dj,
\]

- \( E_l, E_m \) and \( E_h \) are the sets of workers choosing to supply their low, medium and high skills respectively.

- The worker will choose to be in the set \( E_h \) only if:

\[
\frac{l^j}{h^j} \leq \frac{w_H}{w_L} \quad \text{and} \quad \frac{m^j}{h^j} \leq \frac{w_H}{w_M}
\]
Endogenous choice of skills

- We impose a type of *single-crossing* assumptions in supplies: \( h^j / m^j \) and \( m^j / l^j \) are both strictly decreasing in \( j \) and \( \lim_{j \to 0} h^j / m^j \) and \( \lim_{j \to 1} m^j / l^j = 1 \)

- This assumption implies that lower index workers have a comparative advantage in high skill tasks and higher index workers have a comparative advantage in low skill tasks
Endogenous choice of skills

For any ratios of wages $w_H / w_M$ and $w_M / w_L$:

- there exist $J^*(w_H / w_M)$ and $J^{**}(w_M / w_L)$ such that:
  
  1. $t_h^j = 1$ for all $j < J^*(w_H / w_M)$;
  2. $t_m^j = 1$ for all $j \in (J^*(w_H / w_M) J^{**}(w_M / w_L))$;
  3. $t_l^j = 1$ for all $j > J^{**}(w_M / w_L)$

- $J^*(w_H / w_M)$ and $J^{**}(w_M / w_L)$ are both strictly increasing in their arguments

- $J^*(w_H / w_M)$ and $J^{**}(w_M / w_L)$ are defined such that

$$\frac{m^{J^*(w_H / w_M)}}{h^{J^*(w_H / w_M)}} = \frac{w_H}{w_M} \quad \text{and} \quad \frac{m^{J^{**}(w_M / w_L)}}{m^{J^{**}(w_M / w_L)}} = \frac{w_M}{w_L}$$
Endogenous choice of skills

Therefore:

\[ H = \int_0^{J^*} \left( \frac{w_H}{w_M} \right) h^j d^j, \quad M = \int_{J^*}^{J^{**}} \left( \frac{w_M}{w_L} \right) m^j d^j \]

and

\[ L = \int_{J^{**}}^{l} \left( \frac{w_M}{w_L} \right) l^j d^j \]
**Endogenous choice of skills**

- $J^*(w_H/w_M)$ and $J^{**}(w_M/w_L)$ are both strictly increasing in their arguments.

\[
\begin{align*}
\frac{H}{M} &= \frac{\int J^*(w_H/w_M) h^j \, dj}{\int J^{**}(w_M/w_L) m^j \, dj} \quad \text{and} \quad \frac{M}{L} = \frac{\int J^{**}(w_M/w_L) m^j \, dj}{\int J^*(w_H/w_M) j^j \, dj}
\end{align*}
\]

(1)

- Therefore holding $w_M/w_L$ constant, an increase in $w_H/w_M$ increases $H/L$ and holding $w_H/w_M$ constant, an increase in $w_M/w_L$ increases $M/L$.
Second extension

Endogenous technical change

- Endogenous technical change favoring *skills* is well understood from Acemoglu (1998, 2007)
- We can also consider endogenous technical change *favoring tasks* in this model
Ricardian Model: Summary

Model’s inputs

1. Explicit distinction between skills and tasks
2. Allow for comparative advantage among workers in different tasks
3. Allow for multiple sources of competing task ‘supplies’

What the model delivers

- A natural concept of occupations (bundles of tasks)
- An endogenous mapping from skill to tasks via comparative advantage
- Technical change (offshoring) that can raise and lower wages
- Migration of skills across tasks as technology changes
- Polarization of wages and employment as one possible outcome
Where the Canonical Model is Silent (or Mis-speaks)
Can the Ricardian model rationalize these facts?

1. Wage inequality rises less than predicted
2. Real wage levels fall for some groups
3. Wage changes non-uniform in skill
4. Polarization of employment growth across high/low-skill occupations (also non-monotone)
5. Rising importance of occupation as a predictor of earnings
6. Casual empiricism only
   - Directly skill-replacing technologies commonplace
   - Offshoring may function like a skill-replacing technology
Beyond the ‘Canonical Model’ of Skills and Wages

Outline

1. The canonical model: Implications and empirical successes
2. Where the canonical models fall short
3. What should an amended model offer?
4. A Ricardian model of skills, tasks and technologies
5. Some potential empirical directions
6. Conclusions
Some potential empirical directions

Some loose observations only

- Model suggests that we want to relate technical change to prices of skills via *changes in comparative advantage*
  - Measuring comparative advantage is difficult, but not impossible
  - One idea is to look at patterns of occupational specialization from ‘pre-period’ as a measure
- More generally, model makes conceptual link btwn skills, tasks and occupations
  - Occupations *do not really exist* in standard competitive wage models
  - Here, they do exist. But there is still a ‘law of one price’ for skill
Conclusions

Canonical model has been a major conceptual and empirical success

- But does not shed light on some key phenomena of interest:
  - Falling real wages for some groups
  - Non-monotone wage changes
  - Polarization of employment
  - Reallocation of skill groups across occupations
  - Rising power of occupation as predictor of wages

Possible additional insights gained by

1. Distinguishing between skills and tasks
2. Allowing for comparative advantage among workers in different tasks
3. Allowing for multiple sources of competing task ‘supplies’