Rising Wage Inequality and the Effectiveness of Tuition Subsidy Policies:

Explorations with a Dynamic General Equilibrium Model of Labor Earnings

based on Heckman, Lochner and Taber, *Review of Economic Dynamics* **1** (1998)

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The Problem of the Agent: Demographics

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Agent Solves:

$$V_a\left(H_{s,a}, K_a, s, \theta, r_t, R_{s,t}\right) = \max\left\{\frac{{C_a}^{\gamma} - 1}{\gamma} + \delta V_{a+1}\left(H_{a+1}, K_{a+1}, s, \theta, r_{t+1}, R_{s,t+1}\right)\right\}$$

subject to:

$$C_a + K_{a+1} = (1-\tau) R_{s,t} H_{s,a} (1 - I_a) + (1 + (1-\tau) r_t) K_a$$

$$H_{s,a+1} = A_s(\theta) (I_a)^{\alpha_s} (H_{s,a})^{\beta_s} + (1-\sigma) H_{s,a}$$

$$0 < \alpha_s, \beta_s < 1$$

The Euler equations are:

$$\left(C_a\right)^{\gamma-1} = \delta\left(1+\left(1-\tau\right)r_{t+1}\right)\left(C_{a+1}\right)^{\gamma-1}$$

$$\begin{aligned} &\left(C_a\right)^{\gamma-1} R_{s,t} H_{s,a} \\ &= \delta\left(C_{a+1}\right)^{\gamma-1} R_{s,t+1}\left(1-I_{a+1}\right) + \alpha_s A_s\left(\theta\right)\left(I_a\right)^{\alpha_s-1}\left(H_{s,a}\right)^{\beta_s} \end{aligned}$$

The initial conditions and Euler equations define solutions to the problem:

$$C_a^* = g_a^C (H_{s,a}, K_a, S, \theta, r_t, R_{s,t})$$

$$I_a^* = g_a^I (H_{s,a}, K_a, S, \theta, r_t, R_{s,t})$$

$$K_a^* = g_a^K (H_{s,a}, K_a, S, \theta, r_t, R_{s,t})$$

Comments:
1) The policy functions are age-specific because agents have a finite lifetime (OLG model).
2) The policy functions depend on price $R_{s,t}$ and r_t because they vary over time (perfect foresight).
Now, use the policy functions to obtain:

$$V_1\left(H_{s,1}, K_1, s, \theta, t\right) = \frac{C_a^{*\gamma} - 1}{\gamma} + \delta V_2\left(A_s\left(\theta\right)\left(I_a^*\right)^{\alpha_s}\left(H_{s,a}\right)^{\beta_s} + H_{s,a}, K_a^*, s, \theta, t+1\right)$$

The agent then decides schooling by solving:

$$s^* = \arg\max_s \left[V_1\left(H_{s,1}, K_1, s, \theta, t\right) - D_s - \varepsilon_s\right]$$

The Problem of the Firm: Notation

- $\bar{H}_{s,t}$ is the total amount of type-s human capital demanded by the firm at period t.
- K_t is the total amount of physical capital demanded by the firm at period t.
- σ_k is the depreciation of physical capital.

The problem of the firm is:

$$\pi\left(R_{1,t}, R_{2,t}, r_t\right)$$

$$= \max\left\{F\left(\bar{H}_{1,t}, \bar{H}_{2,t}, K_t\right) - R_{1,t}\bar{H}_{1,t} - R_{2,t}\bar{H}_{2,t} - \left(r_t + \sigma_k\right)\bar{K}_t\right\}$$

The first-order conditions are:

$$R_{s,t} = \frac{\partial F\left(\bar{H}_{1,t}, \bar{H}_{2,t}, \bar{K}_t\right)}{\partial \bar{H}_{s,t}}, \ s = 1, 2$$

$$\left(r_t + \sigma_k\right) = \frac{\partial F\left(\bar{H}_{1,t}, \bar{H}_{2,t}, \bar{K}_t\right)}{\partial \bar{K}_t}$$

The production function F is assumed to be:

$$F\left(\bar{H}_{1,t}, \bar{H}_{2,t}, \bar{K}_t\right)$$

$$= \left\{a_2\left(\bar{K}_t\right)^{\rho_2} + (1-a_2)\left[a_1\left(\bar{H}_{1,t}\right)^{\rho_1} + (1-a_1)\left(\bar{H}_{2,t}\right)^{\rho_1}\right]^{\frac{\rho_2}{\rho_1}}\right\}^{\frac{1}{\rho_2}}$$

Aggregation

- t_c is the year of birth of cohort c.
- $a = t - t_c$ is the age of cohort c at year t.
- $P_{t_c} = \{r_i, R_{1,i}, R_{2,i}\}_{i=t_c}^{t_c+a_R}$ is the sequence of prices cohort c will face during working life.
- $N_s(\theta, t_c)$ is the number of agents of type.
- θ in cohort c and schooling level s.

- $H_{s,a}(\theta, P_{t_c})$ is the stock of type-s human capital at age a of an agent of cohort c.
- $K_{s,a}(\theta, P_{t_c})$ is the stock of type-s human capital at age a of an agent of cohort c.

Therefore:

$$\hat{H}_{s,t} = \sum_{t_c=t-a_R}^{t-1} \int N_s(\theta, t_c) H_{s,t-t_c}(\theta, P_{t_c}) (1 - I_{t-t_c}(s, \theta, P_{t_c})) d\Psi(\theta)$$

$$\hat{K}_t = \sum_{t_c=t-a_R}^{t-1} \sum_{s=1}^{2} \int N_s(\theta, t_c) K_{t-t_c}(s, \theta, P_{t_c}) d\Psi(\theta)$$

- Earnings of a person age a at time t of cohort c :

$$W\left(a, t, H_{a,s}\left(\theta, P_{t_c}\right)\right) = R_{s,t} H_{a,s}\left(\theta, P_{t_c}\right)\left(1 - I_a\left(s, \theta, P_{t_c}\right)\right)$$

- Suppose that for two consecutive ages a and $a+1$, $I_a\left(s, \theta, P_{t_c}\right) = I_{a+1}\left(s, \theta, P_{t_c}\right) = 0.$

$$\frac{W\left(a+1, t+1, H_{a+1,s}\left(\theta, P_{t_c}\right)\right)}{W\left(a, t, H_{a,s}\left(\theta, P_{t_c}\right)\right)} = \frac{R_{s,t+1} H_{a+1,s}\left(\theta, P_{t_c}\right)}{R_{s,t} H_{a,s}\left(\theta, P_{t_c}\right)} =$$

$$= \frac{R_{s,t+1}\left(1 - \sigma_s\right) H_{a,s}\left(\theta, P_{t_c}\right)}{R_{s,t} H_{a,s}\left(\theta, P_{t_c}\right)} = \frac{R_{s,t+1}\left(1 - \sigma\right)}{R_{s,t}}$$

- We can get the ratio of $\frac{R_{s,t+1}}{R_{s,t}}$ up to a constant. Next step, we show how to get σ.
- We can get σ from microestimates of human capital production function.

- Consider the firm's wage bill of schooling level s at period t:

$$WB_{s,t} = R_{s,t}\bar{H}_{s,t}$$

- Rearranging terms:

$$\frac{WB_{s,t}}{(1-\sigma)^t R_{s,t}} = \frac{\bar{H}_{s,t}}{(1-\sigma)^t}$$

- Thus:

$$\tilde{R}_{s,t} = (1-\sigma)^t R_{s,t}$$

$$\tilde{H}_{s,t} = \frac{\bar{H}_{s,t}}{(1-\sigma)^t}$$

Digression:
Identifying the Parameters of Interest: Identifying σ

- Let
$$Q_t = \left(a_1 \left(H_{t,1}\right)^{\rho_1} + \left(1 - a_1\right)\left(H_{t,2}\right)^{\rho_1}\right)^{\frac{1}{\rho_1}} . \tag{1}$$
- $\frac{1}{1-\rho_1}$ = elasticity of substitution
- The price R_t^Q of one unit of the basket Q_t is the solution to
$$R_t^Q = \min_{H_{t,1},H_{t,2}} R_{t,1}H_{t,1} + R_{t,2}H_{t,2}$$
subject to
$$\left(a_1 \left(H_{t,1}\right)^{\rho_1} + \left(1 - a_1\right)\left(H_{t,2}\right)^{\rho_1}\right)^{\frac{1}{\rho_1}} = 1 .$$

- The first-order conditions are:

$$R_{t,1} = \lambda \left(a_1 \left(H_{t,1}\right)^{\rho_1} + (1-a_1)\left(H_{t,2}\right)^{\rho_1}\right)^{\frac{1-\rho_1}{\rho_1}} a_1 \left(H_{t,1}\right)^{\rho_1 - 1} \quad (2)$$

$$R_{t,2} = \lambda \left(a_1 \left(H_{t,1}\right)^{\rho_1} + (1-a_1)\left(H_{t,2}\right)^{\rho_1}\right)^{\frac{1-\rho_1}{\rho_1}} (1-a_1)\left(H_{t,2}\right)^{\rho_1 - 1} \quad (3)$$

- The solution to this problem is well-known:

$$R_t^Q = \left[(a_1)^{\frac{1}{1-\rho_1}} \left(R_{t,1}\right)^{\frac{\rho_1}{\rho_1 - 1}} + (1-a_1)^{\frac{1}{1-\rho_1}} \left(R_{t,2}\right)^{\frac{\rho_1}{\rho_1-1}}\right]^{\frac{\rho_1 - 1}{\rho_1}} \quad (4)$$

- The problem of the firm can be recast as:

$$\pi\left(R_t^Q, r_t\right) = \max\left\{\left[a_2 Q_t^{\rho_2} + (1-a_2) K_t^{\rho_2}\right]^{\frac{1}{\rho_2}} - R_t^Q Q_t - r_t K_t\right\}$$

- The first-order conditions are

$$\left[a_2 \left(Q_t\right)^{\rho_2} + \left(1 - a_2\right)\left(K_t\right)^{\rho_2}\right]^{\frac{1-\rho_2}{\rho_2}} a_2 \left(Q_t\right)^{\rho_2 - 1} = R_t^Q \quad (5)$$

$$\left[a_2 Q_t^{\rho_2} + \left(1 - a_2\right) K_t^{\rho_2}\right]^{\frac{1-\rho_2}{\rho_2}} \left(1 - a_2\right)\left(K_t\right)^{\rho_2 - 1} = r_t \quad (6)$$

- Taking ratios of (5) and (6) and applying logs it follows that:

$$\log \frac{R_t^Q}{r_t} = \log\left(\frac{a_2}{1 - a_2}\right) + \left(\rho_2 - 1\right)\log\left(\frac{Q_t}{K_t}\right) \quad (7)$$

- But note that from (1) and (4) are defined in terms of $H_{s,t}$ and $R_{s,t}$.

- However, we only observe $\tilde{H}_{s,t}$ and $\tilde{R}_{s,t}$. Let $\tilde{Q}_t$ and $\tilde{R}_t^Q$ be defined as in (1) and (4) but based on observables $\tilde{H}_{s,t}$ and $\tilde{R}_{s,t}$:

$$\tilde{Q}_t = \left(a_1 \left(\tilde{H}_{t,1}\right)^{\rho_1} + (1 - a_1) \left(\tilde{H}_{t,2}\right)^{\rho_1}\right)^{\frac{1}{\rho_1}}$$

$$\tilde{R}_t^Q = \left[(a_1)^{\frac{1}{1-\rho_1}} \left(\tilde{R}_{t,1}\right)^{\frac{\rho_1}{\rho_1 - 1}} + (1 - a_1)^{\frac{1}{1-\rho_1}} \left(\tilde{R}_{t,2}\right)^{\frac{\rho_1}{\rho_1 - 1}}\right]^{\frac{\rho_1 - 1}{\rho_1}}$$

- It is easy to show that:

$$Q_t = (1 - \sigma)^t \, \tilde{Q}_t \tag{8}$$

$$R_t^Q = (1 - \sigma)^{-t} \, \tilde{R}_t^Q \tag{9}$$

- Plugging (8) and (9) into (7) it follows that:

$$\log \frac{\tilde{R}_t^Q}{r_t} = \log\left(\frac{a_2}{1-a_2}\right) + \rho_2 \log(1-\sigma)\, t + (\rho_2 - 1)\log\left(\frac{\tilde{Q}_t}{K_t}\right) \tag{10}$$

- Suppose we run a regression:

$$\log \frac{\tilde{R}_t^Q}{r_t} = \beta_0 + \beta_1 t + \beta_2 \log\left(\frac{\tilde{Q}_t}{K_t}\right) + \varepsilon_t$$

- Then we can identify:

$$a_2 = \frac{e^{\beta_0}}{1+e^{\beta_0}} \quad \rho_2 = 1 + \beta_2 \quad \sigma = 1 - e^{\frac{\beta_1}{1+\beta_2}}$$

- This assumes no technical progress and is a bad assumption.

End of Digression

- To identify the other parameters of interest, consider the log of the ratio of (3) to (2):

$$\log \frac{R_{t,2}}{R_{t,1}} = \log\left(\frac{1-a_1}{a_1}\right) + (\rho_1 - 1)\log\left(\frac{H_{t,2}}{H_{t,1}}\right) \quad (11)$$

- Again, note that we do not observe either $R_{t,s}$ or $H_{t,s}$, but only $\tilde{R}_{t,s}$ and $\tilde{H}_{t,s}$.

- Therefore

$$\log \frac{\frac{\tilde{R}_{t,2}}{(1-\sigma)^t}}{\frac{\tilde{R}_{t,1}}{(1-\sigma)^t}} = \log\left(\frac{1-a_1}{a_1}\right) + (\rho_1 - 1)\log\left(\frac{(1-\sigma)^t \tilde{H}_{t,2}}{(1-\sigma)^t \tilde{H}_{t,1}}\right) \Rightarrow$$

$$\Rightarrow \log \frac{\tilde{R}_{t,2}}{\tilde{R}_{t,1}} = \log\left(\frac{1-a_1}{a_1}\right) + (\rho_1 - 1)\log\left(\frac{\tilde{H}_{t,2}}{\tilde{H}_{t,1}}\right). \quad (12)$$

- This assumes that a_1 is not time varying or, if it is, $\ln\left(\frac{1-a_1}{a_1}\right)$ is not collinear with $\widetilde{H}_{t,2}/\widetilde{H}_{t,1}$.
- But we can get σ from the production function of human capital.
- We bring this to the macro data.

Estimating the Human Capital Production Function

- We use wage and schooling data on white males from the NLSY.
- We assume that there are four observable θ types which we define according to AFQT quartile.
- We assume that the interest rate is fixed at $r = 0.05$ and that rental rates are fixed and normalized to one.

- For any given (a, θ, S) and any set of parameters π we can calculate the optimal wage

$$w(a, \theta, S; \pi).$$

- We assume that these wages are measured with error and we estimate the parameters, π, using nonlinear least squares, minimizing

$$\sum_{i=1}^{N} \sum_{a} \left(w_{i,a}^{*} - w(a, \theta, S; \pi) \right)^2 ,$$

where $w_{i,a}^{*}$ is the observed wage.

- Given these estimated parameters, we can obtain the present value of earnings for each type as college graduates or high school graduates, $\widehat{V_\theta^S}$.
- We assume that the nonpecuniary tastes for college are normally distributed, so

$$\Pr\left(\text{Coll} \mid D^S, \theta\right) = \Phi\left(\frac{(1-\tau)\left(V_\theta^2 - V_\theta^1\right) - D^S + \mu_\theta}{\sigma_\varepsilon}\right)$$

- Using data on state tuition we estimate this model as a probit.

- We take

$$\tau = 0.15 \qquad \delta = 0.96 \qquad \gamma = 0.10$$

- We calibrate the model to "look like" the NLSY in the original steady state:

$$(1-\tau)\,r = 0.05 \qquad R^1 = 2.00 \qquad R^2 = 2.00$$

- In order to match the capital-output ratio, we need a transfer from old cohorts to young.

- We take an exogenous transfer from a cohort as it retires and give it to a new cohort as it is born.

- This transfer is approximately $30,000.

- We estimate a nested CES production function allowing for a linear time trend

$$a_3 \left(a_2 \left(a_1 (\bar{H}_t^1)^{\rho_1} + (1 - a_1)(\bar{H}_t^2)^{\rho_1} \right)^{\rho_2/\rho_1} + (1 - a_2)\bar{K}_t^{\rho_2} \right)^{1/\rho_2}$$

- We estimate $\rho_1 = 1 - \frac{1}{\sigma} \doteq \frac{1}{3}$ and $\rho_2 = 0$ based on those estimates.

- We calibrate (a_1, a_2, a_3) and the transfer to yield prices (r, R^1, R^2) and a capital-output ratio of 4 in the initial steady state.

Skill-Biased Technical Change

- Unexpected shock resulting in a constant decline in a_1 for 30 years.
- The total decline in the share of low skilled labor is 30% (matching the rate of decline in the data).
- Perfect foresight.
- Transition period of 200 years.

Skill-Biased Technical Change: The Effects of Skill-Biased Technology Change

- Movements in measured wages are different from movements in skill prices, especially for young workers.
- Without intervention, economy converges to a new steady state with lower wage inequality than before the technology change.
- In the long run, society is richer and all types are better off. In the short run, low ability/low skilled workers caught in the transition are worse off.

- In the new steady state, there are more high skilled workers, but human capital per skilled worker is lower.

- During transition periods, cross-section estimates of “returns” to skill are substantially different from the actual returns faced by cohorts making educational decisions.

Tuition Subsidy

- Partial equilibrium analysis ignores the effects of changes in skill quantities on the price of skill.
- As individuals acquire more skill in response to policy change, the returns to skill decline.
- This lowers the proportion of individuals taking advantage of the policy.
- The increase in aggregate skill also affects the earnings of individuals who do not take advantage of the new policy.

- Partial equilibrium analysis fails for two reasons:

  (1) Overstates the effect of the program on participants.

  (2) Misses the effect of the program on non-participants

- Accounting for these effects in evaluating policy requires a general equilibrium, structural model of skill formation.

Tuition Subsidy: Example

- \$500 tuition subsidy
- Balance the budget in the steady states.
- Perfect foresight.
- Transition period of 200 years.

Main Findings

- Estimates of college enrollment responses based on cross-section variations in tuition are substantially overstated.
- Individuals who do not change their schooling decision are affected.

Summary

- We develop an empirically-grounded dynamic overlapping generations general-equilibrium model of skill formation with heterogeneous human capital.
- Model roughly consistent with changing wage structure.
- Partial equilibrium program evaluation can be very misleading.
- We distinguish between effects measured in a cross-section and the effects on different cohorts.

Extensions

- Additional tax and subsidy policies.
- Closer link between macro and micro models.
- Relax perfect foresight assumption.
- Incorporate a separate sector for schooling–education requires high skilled labor inputs.

Table 1: Estimated parameters for human capital production function and schooling decision (standard errors in parentheses).

Human Capital Production

$$H^S_{a+1} = A^S(\theta) I_a^{\alpha_S} H_a^{\beta_S} + (1-\sigma) H_a^S$$

$$S = 1, 2$$

	High School ($S = 1$)	College ($S = 2$)
α	0.945(0.017)	0.939(0.026)
β	0.832(0.253)	0.871(0.343)
$A(1)$	0.081(0.045)	0.081(0.072)
$H_{a_R}(1)$	9.530(0.309)	13.622(0.977)
$A(2)$	0.085(0.053)	0.082(0.074)
$H_{a_R}(2)$	12.074(0.403)	14.759(0.931)
$A(3)$	0.087(0.056)	0.082(0.077)
$H_{a_R}(2)$	13.525(0.477)	15.614(0.909)
$A(4)$	0.086(0.054)	0.084(0.083)
$H_{a_R}(4)$	12.650(0.534)	18.429(1.095)

Table 1 (continued)

College Choice Equation

$P(\delta^2 = 1) = \Lambda(-\lambda D^2 + \alpha(\theta))$

	Probit Parameters	Average Derivatives
λ	0.166(0.062)	-0.0655(0.025)
$\alpha(1)$	-1.058(0.097)	-
$\alpha(2)$	-0.423(0.087)	0.249(0.037)
$\alpha(3)$	0.282(0.089)	0.490(0.029)
$\alpha(4)$	1.272(0.101)	0.715(0.018)
Sample Size:		
Persons	869	1069
Person Years	7996	11626

(1) D^2 is the discounted tuition cost of attending college.

(2) $\alpha(\theta)$ is the nonparametric estimate of $(1-\tau)[V^2(\theta) - V^1(\theta)]$, the monetary value of the gross discounted returns to attending college.

(3) $\delta^2 = 1$ if attend college; $\delta^2 = 0$ otherwise. Λ is the unit normal cdf.

Table 2: Derived parameters for human capital production function and schooling decision (units are thousands of dollars).

	Human Capital Production	
	High School ($S=1$)	College ($S=2$)
$H^S(1)$	8.042(0.0.094)	11.117(0.424)
$H^S(2)$	10.0634(0.118)	12.271(0.325)
$H^S(3)$	11.1273(0.155)	12.960(0.272)
$H^S(4)$	10.361(0.234)	15.095(0.323)
Present Value Earnings 1	260.304(3.939)	289.618(12.539)
Present Value Earnings 2	325.966(5.075)	319.302(10.510)
Present Value Earnings 3	360.717(6.352)	337.260(9.510)
Present Value Earnings 4	335.977(8.453)	393.138(11.442)

Table 2 (continued)

College Decision: Attend College if $(1-\tau)V^2(\theta) - D^2 + \varepsilon_i \geq (1-\tau)V^1(\theta)$ $\varepsilon_\theta \sim N(\mu_\theta, \sigma_\varepsilon)$	
σ_ε (Std. deviation of ε)	22.407(8.425)
Nonpecuniary costs by ability level	
μ_1 (Lowest Ability Quartile)	-53.0190(16.770)
μ_2 (Second Ability Quartile)	-2.8173(12.760)
μ_3 (Third Ability Quartile)	29.7712(11.540)
μ_4 (Highest Ability Quartile)	-28.6494(16.966)

(1) $V^i(\theta)$ is the monetary value of going to schooling level i for a person of AFQT quartile θ. $i = 1$ for high school; $i = 2$ for college. We assume $\tau_r = \tau_h = \tau$.

(2) ε_θ is the nonpecuniary benefit of attending college for a person of ability quartile θ.

(3) D^2 is the discounted tuition cost of attending college

Table 3: Estimates of aggregate production function estimated from factor demand equations (III-1) and (III-2), 1965–1990, allowing for technical progress through a linear trend (standard errors in parentheses).

Instruments	ρ_1	Implied Elasticity of Substitution (σ_1)	Time Trend	ρ_2	Implied Elasticity of Substitution (σ_2)	Time Trend
OLS (Base Model)	0.306	1.441	0.036	-0.034	0.967	-0.004
	(0.089)	(0.185)	(0.004)	(0.200)	(0.187)	(0.007)
Percent Working Pop. < 30	0.209	1.264	0.039	-0.036	0.965	-0.004
& Defense Percent of GNP	(0.134)	(0.215)	(0.005)	(0.200)	(0.187)	(0.007)
Defense Percent of GNP	0.157	1.186	0.041	-0.171	0.854	-0.008
	(0.125)	(0.175)	(0.004)	(0.815)	(0.594)	(0.024)
Percent Working Pop. < 30	0.326	1.484	0.036	0.364	1.572	0.007
	(0.182)	(0.400)	(0.006)	(1.150)	(2.842)	(0.034)

Table 4: Simulated changes in wages and wage inequality from 1960–1990. Includes the estimated trend in technology and entrance of baby boom cohorts from 1965–80 (multiplied by 100).

Years	Coll. - HS Log Wage Diff.	Mean HS Log Wage		Mean Coll. Log Wage		Std. Deviation of Log Wages		
		Age 25	Age 50	Age 25	Age 50	HS	College	All
1960-70	6.66	-26.98	-9.17	19.41	-2.2	0.06	0.67	2.49
1970-80	-5.33	3.51	-2.32	-8.72	-5.11	2.06	-0.84	0.14
1980-90	11.74	-4.94	-1.74	11.22	-2.72	10.68	-7.87	8.12
1960-90	13.07	-28.4	-13.22	21.91	-10.03	12.8	-8.03	10.75

Figure 1: Predicted vs. actual hourly wages (in 1992 dollars) by AFQT quartile (high school category).

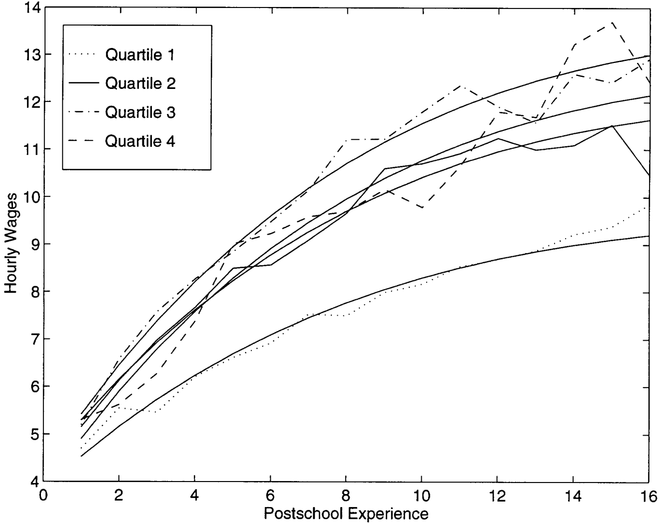


Figure 2: Predicted vs. actual hourly wages (in 1992 dollars) by AFQT quartile (college category).

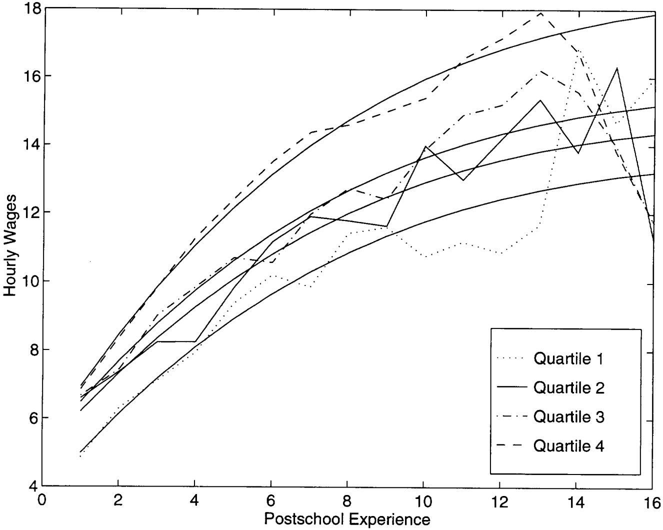


Figure 3A: Comparison of Mincer vs. estimated investment profiles (high school).

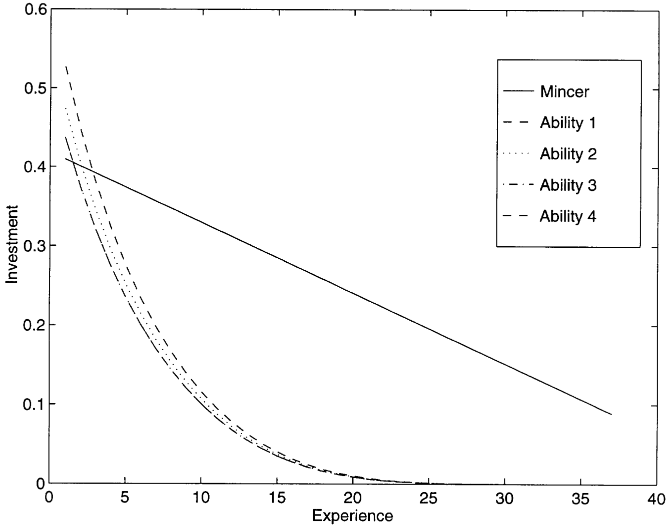


Figure 3B: Comparison of Mincer vs. estimated investment profiles (college).

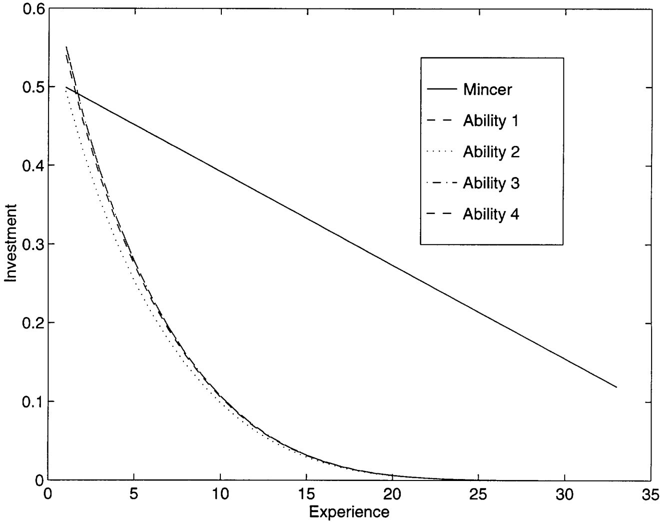
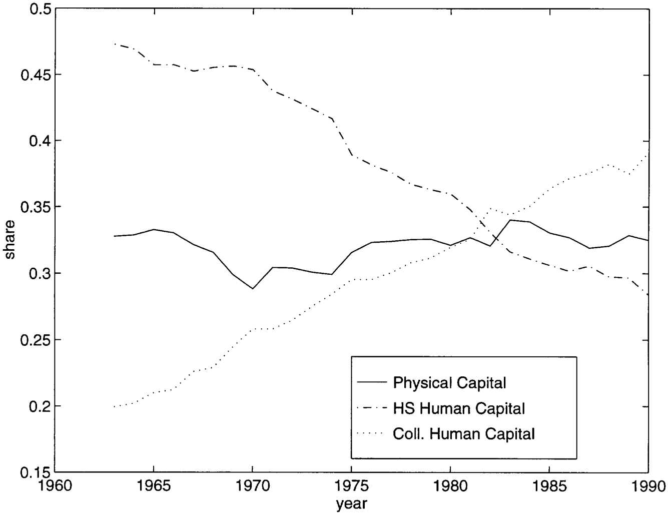


Figure 4: Labor and capital shares over time.



Note: The breakdown of labor's share is based on wages and excludes other forms of compensation.

Figure 5: Estimated trend in α_1 for 30 years.

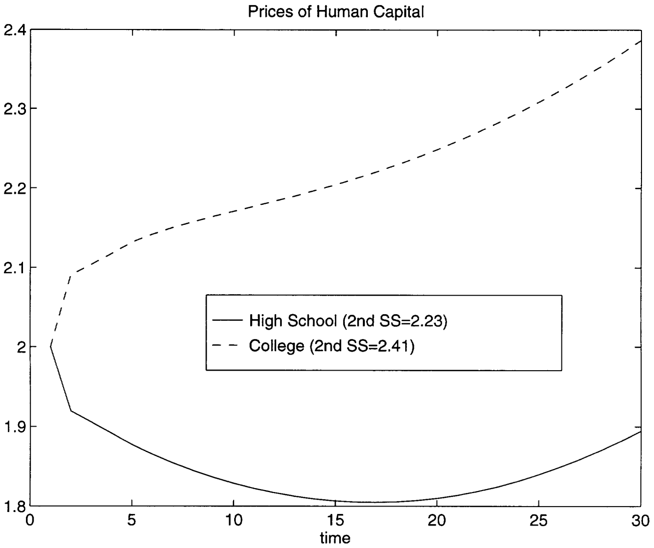


Figure 6: Estimated trend in α_1 for 30 years.

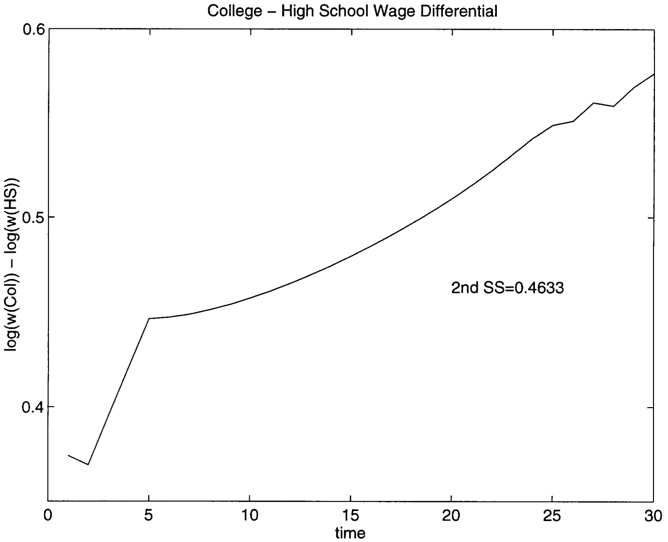


Figure 7: Estimated trend in α_1 for 30 years.

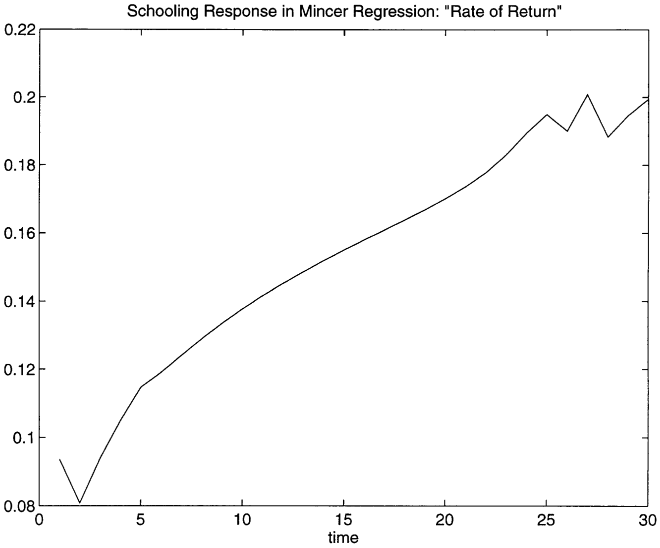


Figure 8: Estimated trend in α_1 for 30 years.

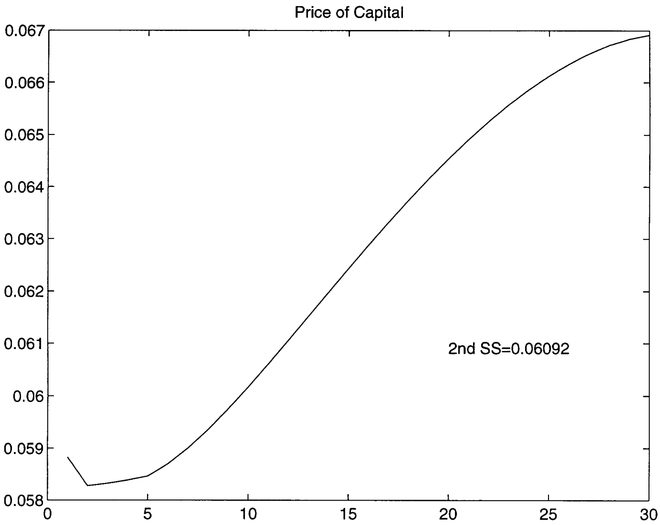


Figure 9: Estimated trend in α_1 for 30 years.

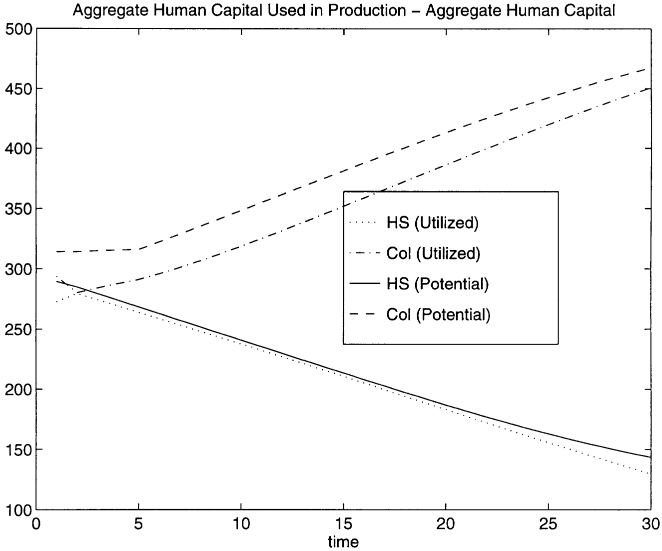


Figure 10: Estimated trend in α_1 for 30 years.

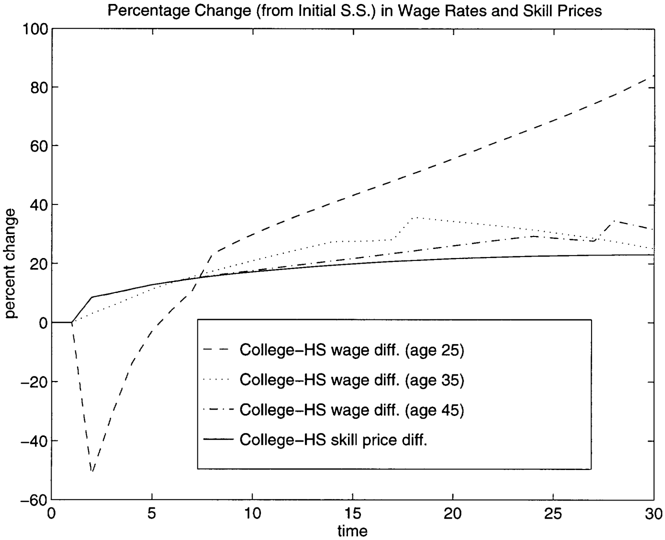


Figure 11A: Estimated trend in α_1 for 30 years.

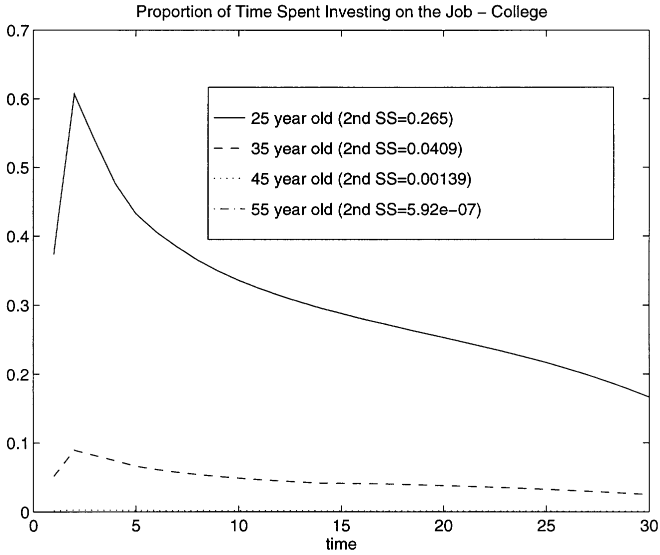


Figure 11B: Estimated trend in α_1 for 30 years.

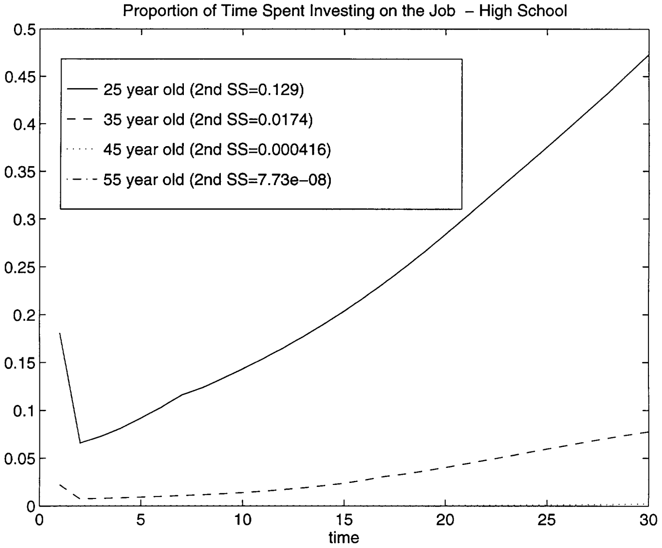


Figure 12A: Estimated trend in α_1 for 30 years.

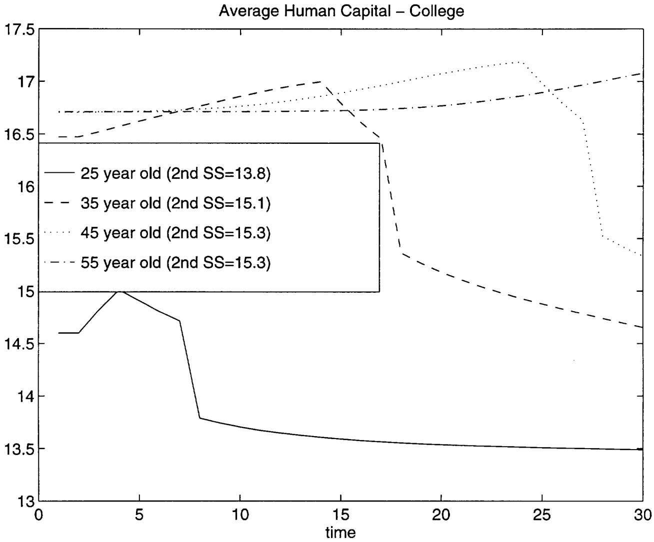


Figure 12B: Estimated trend in α_1 for 30 years.

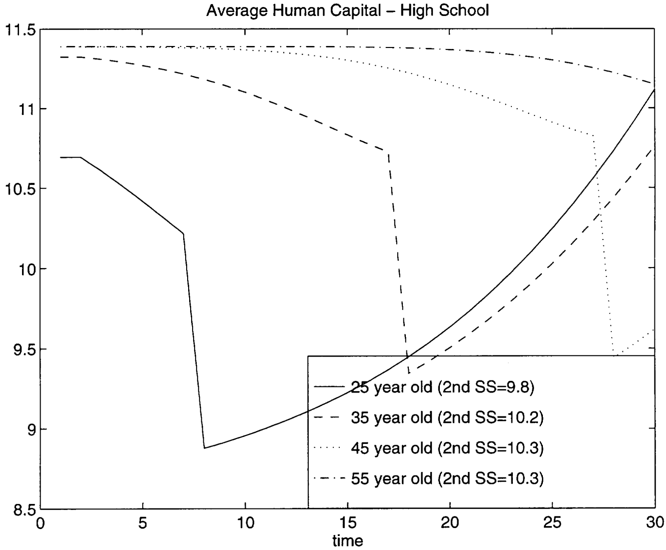


Figure 13: Estimated trend in α_1 for 30 years.

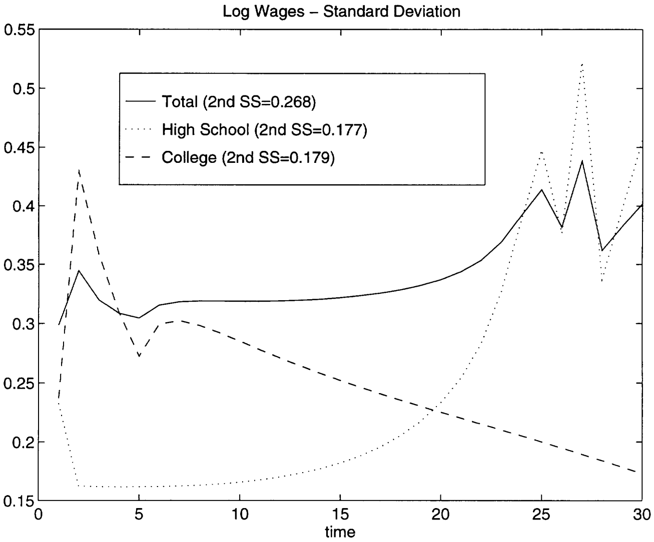


Figure 14: Estimated trend in α_1 for 30 years.

Figure 15A: Estimated trend in α_1 for 30 years.

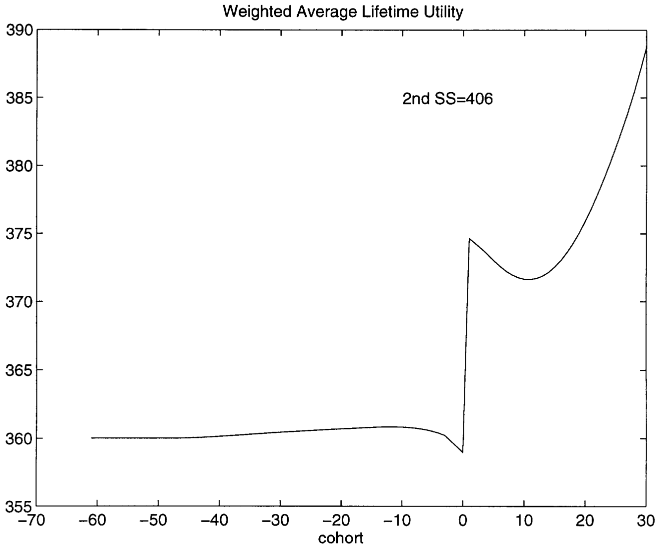


Figure 15B: Estimated trend in α_1 for 30 years.

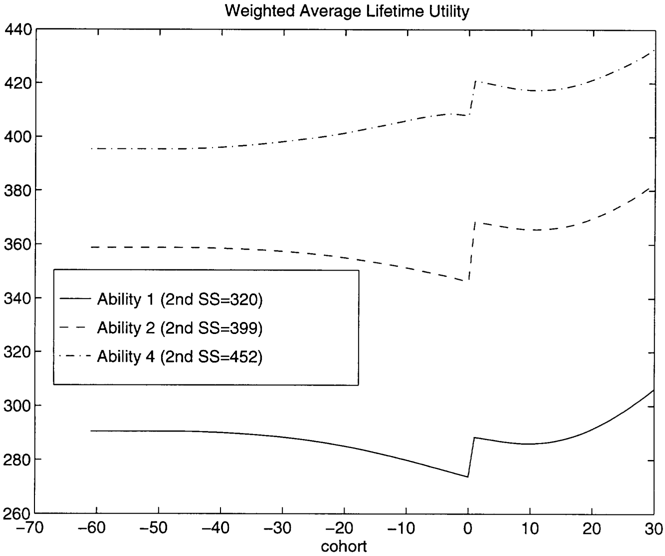


Figure 15C: Estimated trend in α_1 for 30 years.

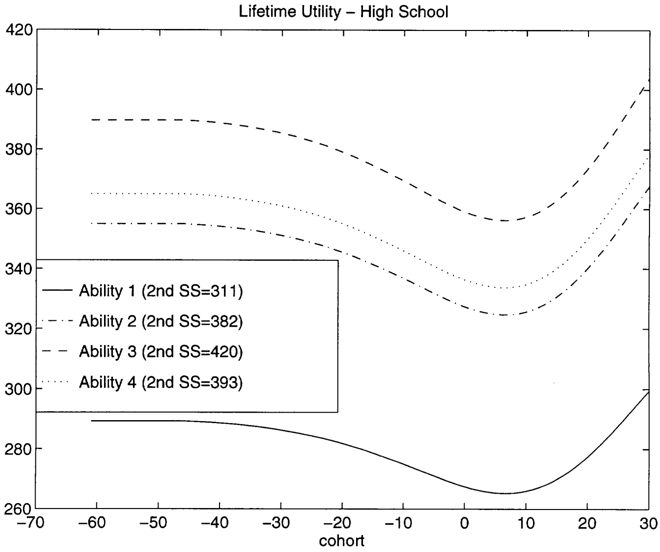


Figure 15D: Estimated trend in α_1 for 30 years.

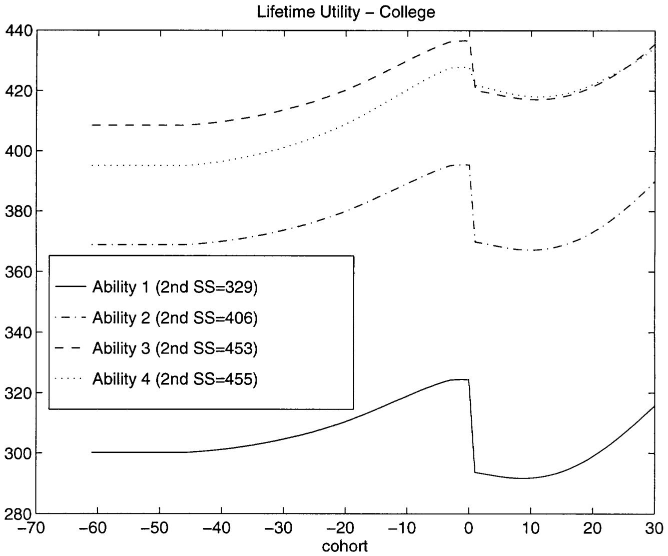


Figure 16: Estimated trend in α_1 for 30 years.

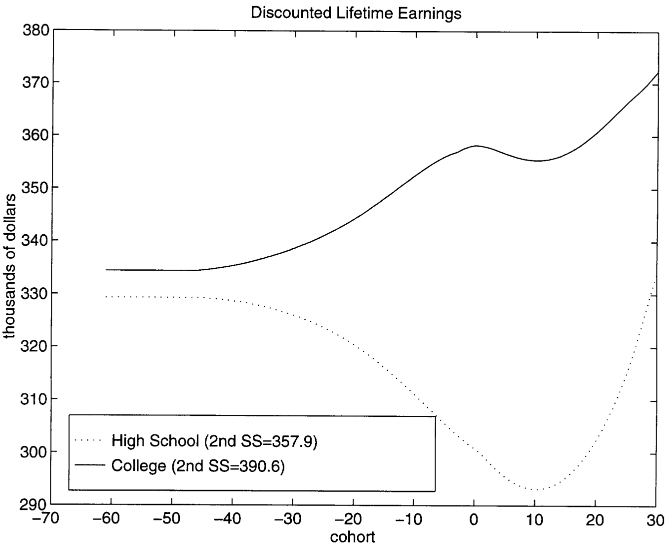


Figure 17: Estimated trend in α_1 for 30 years.

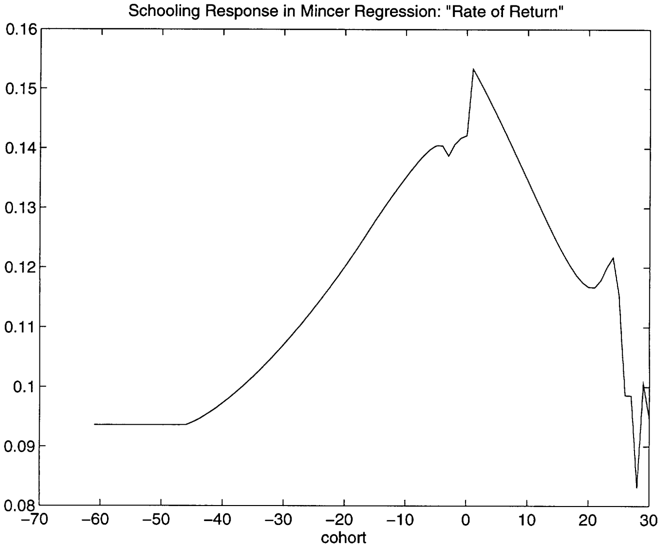


Figure 18: Estimated trend in α_1 for 30 years.

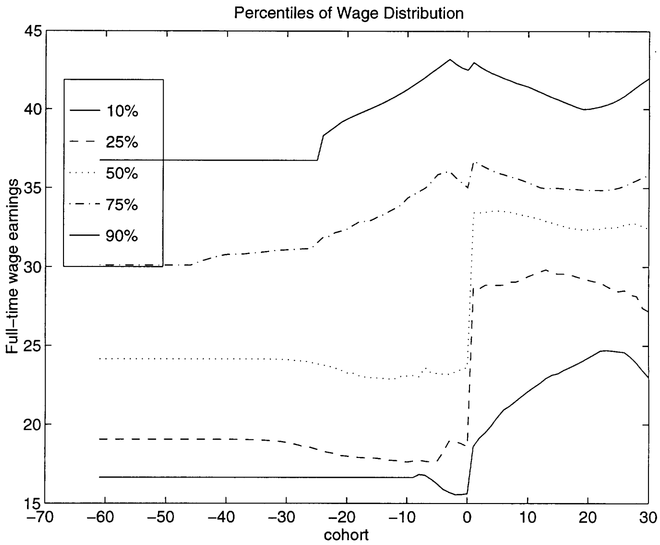


Figure 19: Estimated trend in α_1 for 30 years.
Baby boom (expansion of cohort size by 32%) between years 1965–80

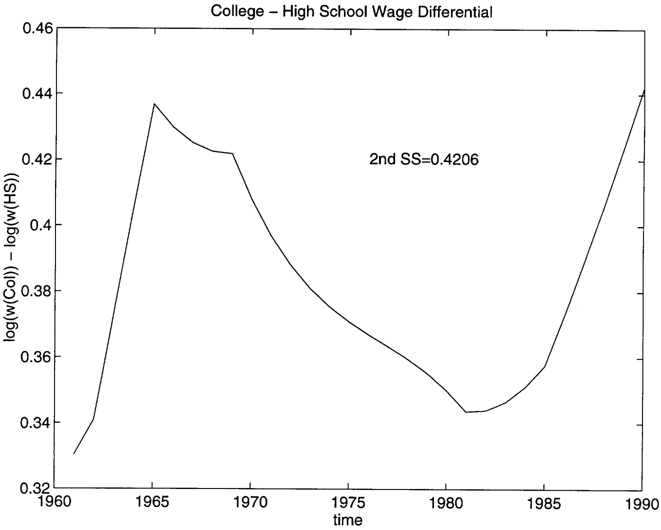


Figure 20: Estimated trend in α_1 for 30 years.
Baby boom (expansion of cohort size by 32%) between years 1965–80