# Wage Equations Part 1: Efficiency Units, Elementary Hedonic Models (Gorman and Lancaster) With and Without Bundling Restrictions

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AEA Continuing Education Program
ASSA Course: Microeconomics of Life Course Inequality
San Francisco, CA, January 5-7, 2016

## Models of wages and the pricing of skills

Standard model of efficiency units

$H =$ human capital measured in efficiency units

$R =$ price per unit efficiency unit

Observed wages are

$W = RH$

(a) Under competition, all workers receive the same price ($R$) per unit human capital

(b) Discrimination, search frictions (including geographical immobility) may create different prices

(c) Workers with different productive characteristics $x$ may have different amounts of human capital
$H = \phi(x)$

(d)
$$\frac{\partial \ln W}{\partial x} = \frac{1}{\phi(x)} \frac{\partial \phi(x)}{\partial x}$$
a purely technological relationship.

(e) Market forces operate only through the intercepts of the log wage equation, not slopes

Widely used in empirical labor economics: Heckman and Sedlacek, Keane and Wolpin, etc.

**Gorman Lancaster Model: Workers have endowments of vectors of traits, each priced like an efficiency unit**

A. Workers have a bundle of traits ($X_i$) for worker $i$.
B. Firms' production functions depend on the aggregate of those traits.

   Let $\hat{X}^j$ be the aggregate of the characteristics of the workforce of firm $j$.
C. $Y^j = f(\hat{X}^j)$

- D Under constant returns to scale, we can represent this as

$$Y^j = N^j f(\bar{X}^j)$$

  where $N^j$ is the number of workers at the firm and $\bar{X}^j$ is the average quality at the firm. We will assume CRS as does the entire literature on the Edgeworth Box (see Mas-Colell, Whinston, and Green, 1995).
- E In the aggregate,

$$Y = G(\hat{X})$$

F. Marginal product of an extra unit of $k$ is

$$\frac{\partial Y}{\partial X_k} = G_k = \pi_k$$

All workers face the same prices;
But now the map between wages and endowments depends on the prices.

G. Labor earnings for worker $i$ are

$$W_i = \sum_{k=1}^{K} \pi_k X_{i,k}$$

$$\ln W_i = \ln\left(\sum_{k=1}^{K} \pi_k X_{i,k}\right)$$

$$\frac{\partial \ln W_i}{\partial X_k} = \frac{\pi_k}{W_i} \qquad\qquad k = 1, \ldots, K$$

Mapping not purely technological;
Suppose that there are two sectors with different skill intensities. (Define skill intensity.) (Same ratios of factors in the two sectors have different productivities.)

The Gorman-Lancaster Model: Two production functions for sectors $A$ and $B$

$$G^A(\hat{X}^A) \qquad\qquad G^B(\hat{X}^B)$$

$$\hat{X}^A + \hat{X}^B = \hat{X}$$

Sectoral productivity of factor $k$ in Sectors $A$ and $B$ are, respectively,

$$\frac{\partial G^A(\hat{X}^A)}{\partial X_k} \qquad\qquad \frac{\partial G^B(\hat{X}^B)}{\partial X_k}$$

As an equilibrium, we know that if workers could unbundle and sell their individual productive characteristics item by item, the law of one price $\Longrightarrow$

$$\frac{\partial G^A(\hat{X}^A)}{\partial X_k} = \frac{\partial G^B(\hat{X}^B)}{\partial X_k}$$

But suppose that skills are bundled?

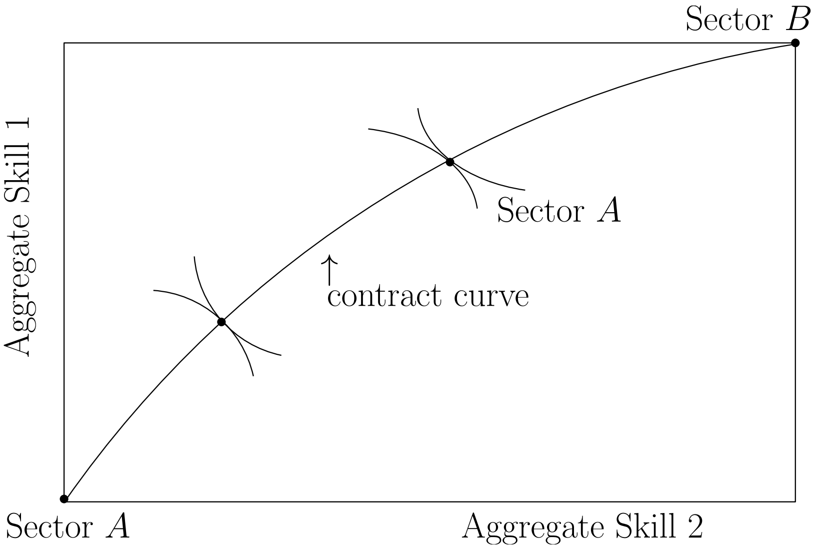
(a) Firm buys a *bundle* of skills

$$X_{i,1}, \ldots, X_{i,k}, \ldots, X_{i,K}$$

when it buys worker $i$.

(b) All skills used in each sector

(c) Consider a case where $K = 2$: Full employment of factors. Draw up an Edgeworth Box: Assume CRS and that workers can unbundle their skills
(Box defines the feasible set)

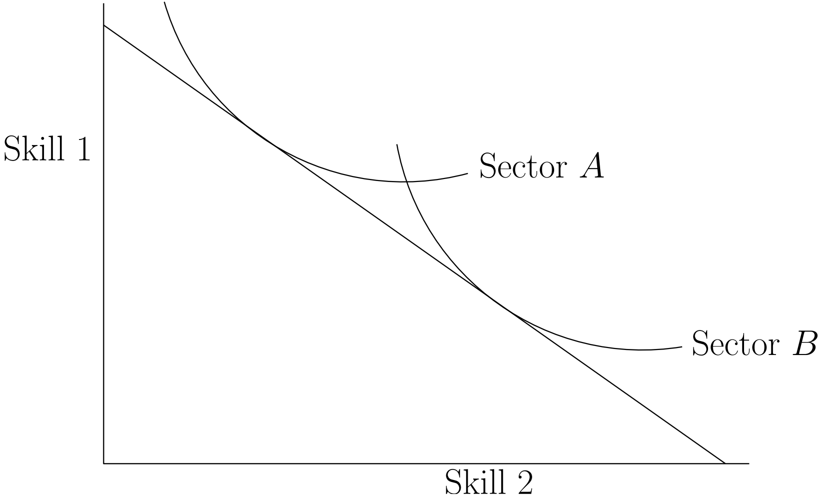

Sector $B$
Aggregate Skill 1
Sector $A$
↑
contract curve
Sector $A$
Aggregate Skill 2

Question: *Why, as you expand Sector A, does the equilibrium price ratio (Skill 1 price to Skill 2 price) flatten (i.e., the price of Skill 1 becomes relatively more expensive)?*
(End of Question.)

Factor Intensities Differ Across Sectors

As drawn, Sector $A$ has greater Skill 1 intensity, i.e. at the same skills price, $\pi = (\pi_1, \pi_2)$, the firm has a bias toward Skill 1.

## An Equilibrium Output: Law of One Price



Notice that as Sector $A$ expands, the only place it can get workers is from Sector $B$.

$\therefore$ it bids up the skill price of 1 in both sectors.

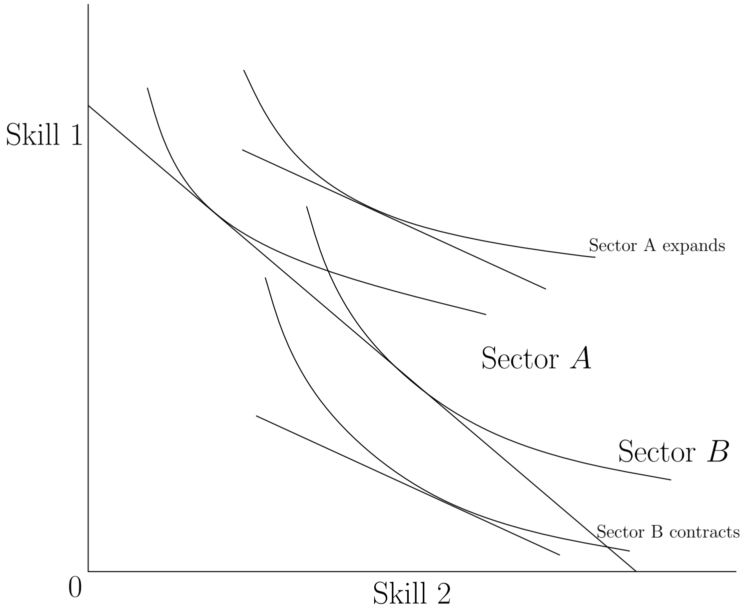
Firms substitute toward Skill 2 (cheaper)

Causes relative price of Skill 1 to expand

Law of one price still applies.

Workers are getting one price.

Workers are indifferent as to which sector they go into.


Skill 1
Sector A expands
Sector $A$
Sector $B$
Sector B contracts
0
Skill 2

*A* is more Skill 1 intensive

Full Employment Assumed:

As the output of Sector *A* expands, Sector *B* contracts.

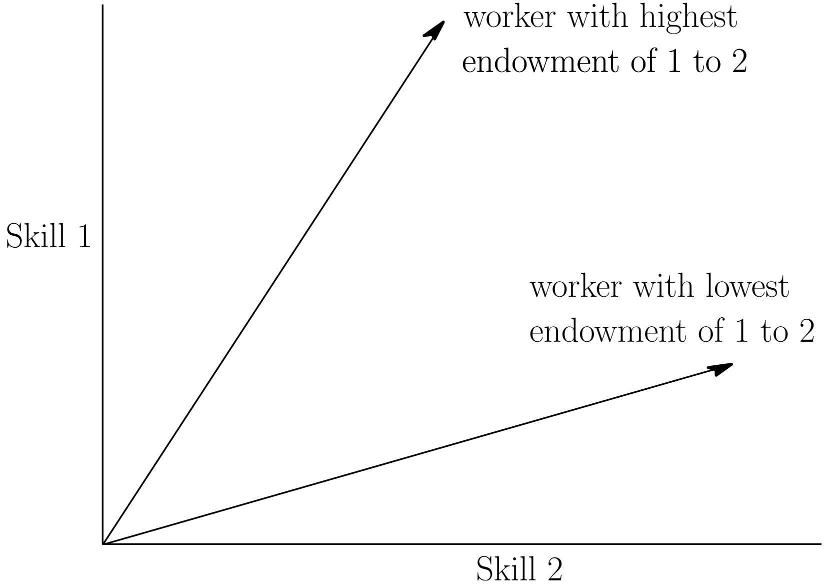
It releases relatively more 2 than 1 because of its skill intensity.

$\therefore$ Skill price of 2 declines relative to 1.

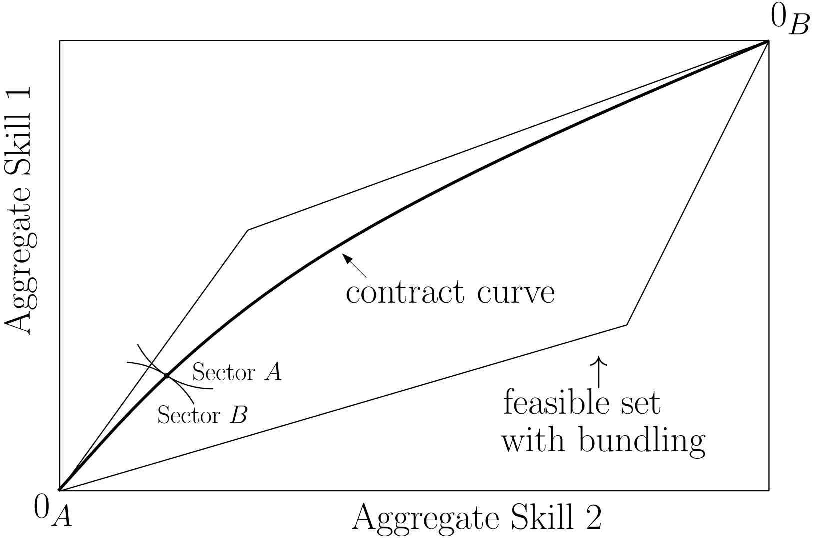
(Remember, we assumed constant returns to scale so we do *not* worry about scale effects which may be important.)

Suppose now, that workers have bundled skill.

Boundaries of Box change: Suppose that range of ratios is as shown

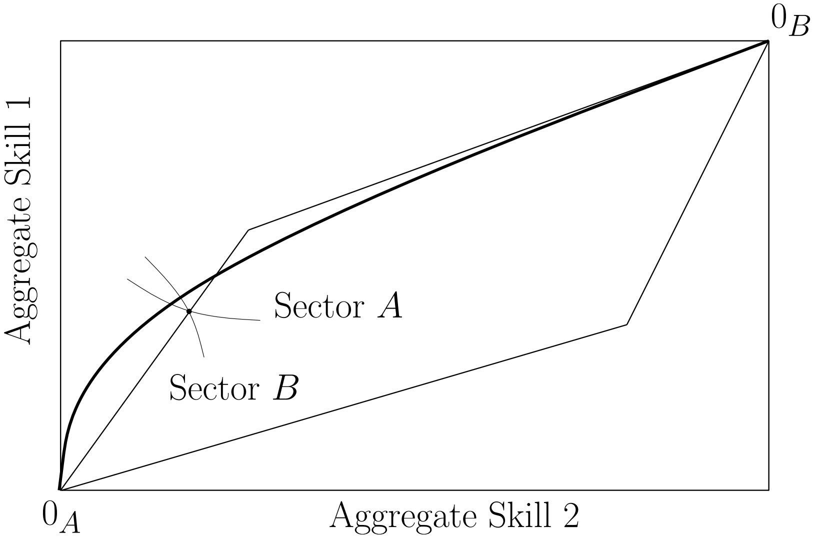

worker with highest
endowment of 1 to 2
Skill 1
worker with lowest
endowment of 1 to 2
Skill 2

This restricts the range of feasible trades



$0_B$
Aggregate Skill 1
contract curve
Sector $A$
Sector $B$
feasible set
with bundling
$0_A$
Aggregate Skill 2

Suppose that the boundaries are binding and Sector $A$ is more skill intensive


$0_B$
Aggregate Skill 1
Sector $A$
Sector $B$
$0_A$
Aggregate Skill 2

If you could unbundle workers (so they could sell their personality or their brawn), contract curve would be dotted line above.

But cannot unbundle.

Relative price of Skill 1 to Skill 2 is higher in Sector $A$.

$\therefore$ unequal prices of skills in the sector

$$\frac{\pi_1^{(A)}}{\pi_2^{(A)}} > \frac{\pi_1^{(B)}}{\pi_2^{(B)}}$$

Now workers care about which sector they go into.

Income maximizing worker $i$ goes into Sector $A$ if

$$\pi^{(A)}X_i > \pi^{(B)}X_i \qquad \text{(Discrete choice model)}$$

Worker at the margin is a person with a bundle $\widetilde{X}$ such that

$$\pi^{(A)}\widetilde{X} = \pi^{(B)}\widetilde{X}$$

$\therefore$ Now sectoral choice and associated price differences are factors that produce income inequality.

(Same factor gets a different price in different sectors.)

Aggregate equilibrium: Workers have

Demand Equal Supply; Workers sort into sectors
(May or may not have equal skill prices)

How to implement this model empirically?

(a) Easy if all components of $X_i$ are observed
(b) Difficult if not
 See Heckman and Scheinkman (1987) on Reading List for empirical work.

- This paves the way to the Roy model of comparative advantage: A basic framework for understanding counterfactuals, wage inequality, and policy variable. Workers have an endowment

$$(X_{iA}, X_{iB})$$

  A worker can use only one skill in any sector. $X_{iA}$ is associated with Sector $A$; $X_{iB}$ is associated with Sector $B$.
- Thus workers have two mutually exclusive endowments.

# The Empirical Importance of Bundling
## A Test of the Hypothesis of Equal Factor Prices Across All Sectors

(From Heckman and Scheinkman, *Review of Economic Studies* 54(2), 1987)

- How to estimate the skill prices across sectors when there are unobserved skill prices?
- How to test equality of skill prices across sectors?
- Unobserved traits may be correlated with observed traits

$$Y_{in} = \underbrace{\underset{\sim}{x}_{io}}_{\text{observed}} \underset{\sim}{w}_{no} + \{ \underbrace{\underset{\sim}{x}_{iu}}_{\text{unobserved}} \underset{\sim}{w}_{nu} + \varepsilon_{in} \}, \qquad (1)$$

$$i = 1, \ldots, I, \quad n = 1, \ldots, N.$$

- Allow for unobserved skills.
- Suppose that persons stay in one sector and we have $T$ time periods of panel data on those persons.
- Stack these into a vector of length $T$.
- Let $\kappa_u$ be the number of unobserved components.
- Let $\kappa_o$ be the number of observed components.

In matrix form we may write these equations for person $i$ as

$$\underset{\sim}{Y}_i = \underset{\sim}{x}_{io} \cdot \underset{\sim}{w}_o + \{\underset{\sim}{x}_{iu} \cdot \underset{\sim}{w}_u + \underset{\sim}{\varepsilon}_i\}, \quad \text{for each sector } n \qquad (2)$$

Following Madansky (1964), Chamberlain (1977) and Pudney (1982), assume $T \geq 2\kappa_u + 1$ and partition (2) into three subsystems:

(i) A basis subsystem of $\kappa_u$ equations from (2)

$$\underset{\sim}{Y}_{(1)} = \underset{\sim}{x}_{io} \cdot \underset{\sim}{w}_{o(1)} + \{\underset{\sim}{x}_{iu} \cdot \underset{\sim}{w}_{u(1)} + \underset{\sim}{\varepsilon}_{(1)}\}, \quad n = 1, \ldots, N \quad (3a)$$

(ii) A second subsystem of equations all of which are distinct from the equations used in (i)

$$\underset{\sim}{Y}_{(2)} = \underset{\sim}{x}_{io} \cdot \underset{\sim}{w}_{o(2)} + \{\underset{\sim}{x}_{iu} \cdot \underset{\sim}{w}_{u(2)} + \underset{\sim}{\varepsilon}_{(2)}\}, \quad n = 1, \ldots, N \quad (3b)$$

(iii) The rest of the equations (at least $\kappa_u$ in number)

$$\underset{\sim}{Y}_{(3)} = \underset{\sim}{x}_{io} \cdot \underset{\sim}{w}_{o(3)} + \{\underset{\sim}{x}_{iu} \cdot \underset{\sim}{w}_{u(3)} + \underset{\sim}{\varepsilon}_{(3)}\}. \quad (3c)$$

Assuming that $\underset{\sim}{w}_{u(1)}$ is of full rank, the first system of equations may be solved for $\underset{\sim}{x}_{iu}$, i.e.,

$$\underset{\sim}{x}_{iu} = [\underset{\sim}{Y}_{(1)} - \underset{\sim}{x}_{io} \cdot \underset{\sim}{w}_{o(1)} - \underset{\sim}{\varepsilon}_{(1)}]\underset{\sim}{w}_{u(1)}^{-1}. \tag{4}$$

Substituting (4) into (3b), we reach

$$\underset{\sim}{Y}_{(2)} = \underset{\sim}{x}_{io}\left[\underset{\sim}{w}_{o(2)} - \underset{\sim}{w}_{o(1)}\underset{\sim}{w}_{u(1)}^{-1}\underset{\sim}{w}_{u(2)}\right] + \underset{\sim}{Y}_{(1)}\underset{\sim}{w}_{u(1)}^{-1}\underset{\sim}{w}_{u(2)} + \underset{\sim}{\varepsilon}_{(2)} - \underset{\sim}{\varepsilon}_{(1)}\underset{\sim}{w}_{u(1)}^{-1}\underset{\sim}{w}_{u(2)}.$$

- Letting $\underset{\sim}{x}_{i0(j)}$ be the observed characteristics for subsystem $j$, we reach our estimating equation

$$\underset{\sim}{Y}_{(2)} = \underset{\sim}{x}_{io(2)} \underset{\sim}{w}_{o(2)} - \underset{\sim}{x}_{io(2)} \underset{\sim}{w}_{o(1)} \underset{\sim}{w}_{u(1)}^{-1} \underset{\sim}{w}_{u(2)} + \underset{\sim}{Y}_{(1)} \underset{\sim}{w}_{u(1)}^{-1} \underset{\sim}{w}_{u(2)} + \underset{\sim}{\varepsilon}_{(2)} - \underset{\sim}{\varepsilon}_{(1)} \underset{\sim}{w}_{u(1)}^{-1} \underset{\sim}{w}_{u(2)}.$$

- Use IV to instrument for $Y_{(1)}$. The natural instruments are $Y_{(3)}$. They are valid as long as $\underset{\sim}{w}_{u(3)}$ are nonzero and the rank condition is satisfied.
- Find a lot of evidence against equality of factor prices across sectors.

## TABLE I

*(Basis described in the appendix)*

| (1) Sector | (2) System MSE | (3) Test | (4) $F$(DFN, DFD) = | (5) Prob > $F$ | (6) Number of observations in each year |
|---|---|---|---|---|---|
| Durable vs. Nondurable | 3·208210 | 1 | (117, 1143) = 1·1448 | 0·1491 | 153 |
| | | 2 | (90, 1143) = 0·9213 | 0·6840 | |
| | | 3 | (27, 1143) = 1·7777 | 0·0087 | |
| Manufacturing vs. Service | 3·447400 | 1 | (117, 3411) = 1·6754 | 0·0001 | 405 |
| | | 2 | (90, 3411) = 0·7336 | 0·9717 | |
| | | 3 | (27, 3411) = 3·0062 | 0·0001 | |
| Blue vs. White Collar | 2·600956 | 1 | (156, 6648) = 2·4197 | 0·0006 | 580 |
| | | 2 | (120, 6648) = 1·2943 | 0·0176 | |
| | | 3 | (36, 6648) = 3·0714 | 0·0001 | |
| North vs. South | 2·299067 | 1 | (156, 7056) = 1·9586 | 0·0001 | 614 |
| | | 2 | (120, 7056) = 1·4981 | 0·0007 | |
| | | 3 | (36, 7056) = 3·0844 | 0·0008 | |
| Manufacturing vs. Non-mfg | 4·746601 | 1 | (117, 5787) = 1·4411 | 0·0015 | 669 |
| | | 2 | (90, 5787) = 1·1062 | 0·2323 | |
| | | 3 | (27, 5787) = 3·0978 | 0·0001 | |

*Notes.*

1. Test 1 tests equality of the coefficients of (12) in both sectors.
   Test 2 tests equality of the coefficients associated with observed characteristics in (12).
   Test 3 tests equality of the coefficients associated with the unobserved characteristics in (12) ($w_{u(1)}^{-1}$, $w_{u(2)}$).

*Notes.*

1. Test 1 tests equality of the coefficients of (12) in both sectors.
   Test 2 tests equality of the coefficients associated with observed characteristics in (12).
   Test 3 tests equality of the coefficients associated with the unobserved characteristics in (12) ($w_{u(1)}^{-1}$, $w_{u(2)}$).
2. Durable: Metal Industries, Machinery including Electrical, Motor Vehicles and other Transportation Equipment, other durables.
   Non Durable: Food, Tobacco, Textile, Paper, Chemical and other Non Durables.
   Manufacturing: All Durable and Non Durable plus "manufacturing unknown".
   Services: Retail Trade, Wholesale Trade, Finance, Insurance, Real Estate, Repair Service, Business Service, Personal Service, Amusement, Recreation and Related Services, Printing, Publishing and Allied Services, Medical and Dental Services, Educational Services, Professional and Related Services.
   North: Conn., Del., Ill., Ind., Maine, Mass., Mich., Minn., N.H., N.J., N.Y., Ohio, Penn., R.I., W. Va., Wis., Vermont.
   South: Alab., Ark., Fla., Geo., Ky., La., Miss., N.C., S.C., Tenn., Tex., Va., Ok.
   White Collar: Professional, Technical and Kindred; Managers, Officials and Proprietors; Self Employed Businessmen; Clerical and Sales Work.
   Blue Collar: Craftsmen, Foremen and Kindred Workers; Operatives and Kindred Workers; Labourers and Service Workers, Farm Labourers.

TABLE II

4 *Factor models*

| (1) Sector | (2) System MSE | (3) Test | (4) $F$(DFN, DFD) = | (5) Prob > $F$ | (6) Number of observations in each year |
|---|---|---|---|---|---|
| Durable vs. Nondurable | 1·480446 | 1 | (144, 1089) = 1·2902 | 0·0166 | 153 |
| | | 2 | (108, 1089) = 1·1722 | 1·1197 | |
| | | 3 | (36, 1089) = 1·3644 | 0·0756 | |
| Manufacturing vs. Service | 1·271277 | 1 | (144, 3357) = 2·6513 | 0·0001 | 405 |
| | | 2 | (108, 3357) = 1·2957 | 0·0231 | |
| | | 3 | (36, 3357) = 6·6334 | 0·0001 | |
| Blue vs. White Collar | 3·830300 | 1 | (192, 6576) = 1·7228 | 0·0001 | 580 |
| | | 2 | (144, 6576) = 1·3400 | 0·0045 | |
| | | 3 | (48, 6576) = 1·8698 | 0·0003 | |
| North vs. South | 2·456318 | 1 | (192, 6984) = 1·9893 | 0·0001 | 614 |
| | | 2 | (144, 6984) = 0·8240 | 0·9381 | |
| | | 3 | (48, 1836) = 2·3018 | 0·0001 | |
| Manufacturing vs. Non-mfg. | 1·617166 | 1 | (180, 1836) = 1·7121 | 0·0001 | 669 |
| | | 2 | (132, 1836) = 1·4107 | 0·0020 | |
| | | 3 | (48, 1836) = 2·0701 | 0·0001 | |

## TABLE III

*5 Factor models*

| (1) Sector | (2) System MSE | (3) Test | (4) $F$(DFN, DFD) = | (5) Prob > $F$ | (6) Number of observations in each year |
|---|---|---|---|---|---|
| Blue vs. White Collar | 1·573852 | 1 | (228, 6912) = 2·0534 | 0·0001 | 580 |
| | | 2 | (168, 6912) = 1·6639 | 0·0001 | |
| | | 3 | (60, 6912) = 3·8733 | 0·0001 | |
| North vs. South | 1·418750 | 1 | (228, 6504) = 3·8840 | 0·0001 | 614 |
| | | 2 | (168, 6504) = 2·2027 | 0·0001 | |
| | | 3 | (60, 6504) = 10·0017 | 0·0001 | |

# APPENDIX

For the 3 factor models we adopt the following basis:

| Years for wages ($\boldsymbol{Y}_{(2)}$) | Basis years |
|---|---|
| 1968, 1969, 1970 | 1971, 1972, 1973 |
| 1971, 1972, 1973 | 1968, 1969, 1970 |
| 1974, 1975, 1976 | 1971, 1972, 1973 |
| 1977, 1978, 1979 | 1974, 1975, 1976 |

For the 4 factor models we adopt the following choice of basis:

| Years for wages ($\boldsymbol{Y}_{(2)}$) | Basis years |
|---|---|
| 1968, 1969, 1970, 1971 | 1972, 1973, 1974, 1975 |
| 1972, 1973, 1974, 1975 | 1968, 1969, 1970, 1971 |
| 1976, 1977, 1978, 1979 | 1972, 1973, 1974, 1975 |

For the 5 factor models we adopt the following choice of basis:

| Years for wages ($\boldsymbol{Y}_{(2)}$) | Basis years |
|---|---|
| 1968, 1969, 1970, 1971, 1972 | 1973, 1974, 1975, 1976, 1977 |
| 1973, 1974, 1975, 1976, 1977 | 1968, 1969, 1970, 1971, 1972 |
| 1978, 1979 | 1968, 1969, 1970, 1971, 1972 |