Some Microfoundations for the Gatsby Curve

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…changes have taken place in ghetto neighborhoods, and the groups that have been left behind are collectively different than those that lived in these neighborhoods in earlier years. It is true that long-term welfare families and street criminals are distinct groups, but they live and interact in the same depressed community and they are part of the population that has, with the exodus of the more stable working- and middle-class segments, become increasingly isolated socially from mainstream patterns and norms of behavior

William Julius Wilson, *The Truly Disadvantaged* (1987)

General Background

Matching, ranging across marriage, neighborhoods, schools, and firms, involves many facets of modern inequality.

Increasing degree of assortative matching has been argued for at least some of these spheres.

Membership theory of inequality can complement human development approach.

Specific Background

1. Work by Miles Corak claimed that for a set of advanced industrialized economies, there is a positive correlation between cross-section inequality and the persistence of status between parents and off-spring. This has been dubbed the Gatsby Curve by Alan Krueger.

2. The Corak claims have been critiqued. Unsurprisingly there are issues associated with data comparability across countries etc.

Our Views

1. We believe that the focus on measurement and cross country comparisons is largely misguided, since there are deep exchangeability questions. Put differently, the bivariate relationship is a type of cross-country growth regression, and as such an element of an empirical methodology that (we believe!) has been shown to deeply flawed.

2. On the other hand, there is a type of Gatsby Curve that we think is more important, namely the relationship between inequality and mobility for a given country. As such, we regard the appropriate focus as intertemporal, i.e. does increased cross-section inequality in one time period affect mobility between that time period and the next?

3. Some empirical evidence exists for this for the US, e.g. Mazumder's work. Chetty et al may be interpreted as challenging this view. We believe there are good reasons to believe that a Gatsby-curve type relationship does exist. We seek to develop a theoretical framework to understand mechanisms underlying the relationship.

Our Approach

We focus on two mechanisms whose interactions produce an intertemporal Gatsby curve.

1. Social influences on individual outcomes.

2. Market frictions.

We construct a social analogue to the Becker-Tomes model, building on Durlauf (1996a,b), Benabou (1993,1996), etc.

In this model, the cross section distribution of income determines the degree of income segregation of families with different incomes across neighborhoods. With "social" determination of human capital formation, this creates mechanism that maps cross-section inequality to intergenerational persistence.

Becker-Tomes type models can produce this relationship via individual-specific heterogeneity in preferences, so that changes in the variance of income affect the distribution of family specific investments. Our approach does not require heterogeneity of preferences.

Model

1. Demography

I dynasties, 2 period overlapping generations model. Agent $i,t+1$ is the member of dynasty i born at time t

Period 1 of life: born, receive human capital

Period 2: become member of neighborhood, produce 1 child, consume

2. Preferences

Utility of i,t is determined in adulthood and depends on consumption $C_{i,t+1}$ and income of the offspring, Y_{it+1}. This is not known at $t+1$, so each agent will maximize expected utility

$$EU_{it} = \pi_1 \log\left(C_{it}\right) + \pi_2 E\left(\log\left(Y_{it+1}\right) \middle| F_t\right) \qquad (1)$$

Cobb-Douglas assumption eliminates heterogeneity in desired fraction of income that is spent on consumption. This renders the political economy of the model trivial. We will explain how to relax.

3. Income and Human Capital

Income in adulthood is determined by human capital received in childhood, H_{nt-1}, and a shock experienced in adulthood ξ_{it}. Human capital is determined at the neighborhood, rather than the individual level.

$$Y_{it} = \phi H_{nt-1} \xi_{it} \qquad (2)$$

The adult shock has both neighborhood and individual components.

$$\xi_{it} = \upsilon_{nt}\gamma_{it} \qquad (3)$$

which allows for social effects outside of human capital. Shocks are assumed to be iid with respect to indices, second moments exist.

4. Decomposition of Income

All educational spending is social, income is split between taxes and consumption.

$$Y_{it} = C_{it} + T_{it} \qquad (4)$$

Taxes are linear in income and neighborhood- and time-specific

$$T_{it} = \tau_{nt} Y_{it} \forall i \in N_{nt} \qquad (5)$$

The total expenditure available for education in neighborhood n at t is

$$TE_{nt} = \sum_{j \in n_t} T_{jt} \tag{6}$$

The implications of these resources will depend on the size of the population of children who will be educated.

5. Educational Expenditure and Educational Investment

Let $s(n_t)$ denotes the population of n_t. The educational input provided by the neighborhood, $ED_{n,t}$ is determined by

$$\frac{TE_{nt}}{f(s(n_t))} = ED_{nt} \qquad (7)$$

Assume $f(s(n_t)) < s(n_t)$ and $f' > 0$.

This means that are returns to scale in education. Captures fixed costs, etc. Not appealing per se. In essence one needs a reason for families to prefer to live together. Could take other routes without any effect on properties of the model.

6. Human Capital

The human capital of a child is determined by a social effect that is a function of average parental education in the neighborhood and the educational input.

$$H_{it} = \Theta\left(Y_{it}, \bar{Y}_{nt}\right) ED_{nt} \qquad (8)$$

$\Theta\left(\bar{Y}_{nt}\right)$ is increasing. Useful to assume that $\Theta\left(\bar{Y}_{nt}\right)$ has an upper bound; simply avoids fissioning of neighborhoods to zero. Could also allow this term to depend negatively on neighborhood size to get the same effect.

Natural to generalize to

$$\Theta\left(Y_{it}, \overline{Y}_{nt}\right)$$

If this function exhibits weak complementarity, then nothing of interest happens. Weak complementarity only provides an additional channel for willingness to pay to be increasing in income.

If the two arguments of the functions are substitutes, then existence of strictly stratified equilibria will depend on whether neighborhoods are supported by core or price differences. More on this below.

7. Political Economy/Market Frictions

1. Neighborhoods are core groupings of families, i.e. all families who want to form a common neighborhood can do so.

The core approach allows us to work without limits on the number of neighborhoods, population requirements for them, etc. Avoid problem of private schools inducing non-single peaked preferences.

The core allocations can be sustained by prices *under our assumptions*. Have not completed proofs on dynamics with prices. We conjecture all theorems hold with prices replacing core rule.

Comment: not clear that core is inferior way to model. May better capture zoning restrictions.

2. Tax rates determined by median voter.

Trivial for Cobb-Douglas preferences; regardless of neighborhood composition or size, the ideal tax rate for each parent is $\tau = \pi_2 / (\pi_1 + \pi_2)$.

3. Neither parents nor communities can borrow. This adds a social analog to the standard borrowing constraint in individual-based models.

Assumptions lead to Simple Formulations of Decisions

Tax preferences defined via

$$\pi_1 \log\left(\left(1-\tau\right)Y_{it}\right) + \pi_2 E\left(\log\left(\phi H_{nt}\left(\tau\right)\xi_{it}\right)\middle| F_t\right) =$$

$$\pi_1 \log\left(\left(1-\tau\right)Y_{it}\right) + \pi_2 \log\left(\tau\phi\Theta\left(\bar{Y}_{nt}\right)\frac{s\left(n_t\right)\bar{Y}_{nt}}{f\left(s\left(n_t\right)\right)}\right)$$

Tax rate defines budget share for neighborhood-specific relative prices for consumption/expected offspring income trade-off.

$$H_{it} = \Theta\left(Y_{it}, \bar{Y}_{nt}\right) ED_{nt}$$

Proposition 1. Effects of Higher Income Neighbors

For a given neighborhood population size $s\left(n_t\right)$,

i. the expected utility of any agent i,t is increasing in monotonic rightward shifts of the empirical income distribution over other families in his neighborhood

ii. the expected income of any agent i,t is increasing in monotonic rightward shifts of the empirical income distribution over other families in his neighborhood.

Key to result: The various assumptions ensure that each i,t adult always prefers his neighbors to have higher incomes than otherwise.

Largely true by assumptions on functions.

The Cobb-Douglas assumption rules out the possibility that differences in preferred tax rates would lead someone to avoid higher income neighbors.

Proposition 2. Existence of Core Allocation of Families

i. At each t for every cross-section income distribution, there exists a core configuration of families across neighborhoods.

ii. At each t, neighborhoods are stratified by income unless all families form a common neighborhood.

Proposition 4. Stochastic Processes for Dynasty-specific Income

Along the equilibrium path for neighborhood compositions,

$$\Pr\left(Y_{it+1} \middle| F_t\right) = \Pr\left(Y_{it+1} \middle| \bar{Y}_{n_t}, s\left(n_t\right)\right)$$
$$\Pr\left(Y_{it+k} \middle| F_t\right) = \Pr\left(Y_{it+k} \middle| F_{Y_t}\right) \text{ if } k > 1$$

Illustrates tricky part in analyzing the long run properties of model, one had to forecast the neighborhood compositions.

Proposition 4. Stratification and Inequality

There exist income levels $\bar{Y}^{high}$ and $\bar{Y}^{low}$ such that families with $Y_{it} > \bar{Y}^{high}$ will not form neighborhoods with families with incomes $\bar{Y}^{low} > Y_{it}$

Proposition 5. Stratification and Effects on Highest and Lowest Income Families

i. Conditional on the income distribution at t, the expected offspring income for the highest family in the population is maximized relative to any other configuration of families across neighborhoods.

ii. Conditional on the income distribution at t, the expected offspring income of the lowest income family in the population is minimized relative to any other configuration of families across neighborhoods that does not reduce the size of that family's neighborhood.

Proposition 6. Inefficiency of Equilibria

Equilibria do not maximize average income over any finite horizon.

Trivial since the model contains spillovers without transfers.

Inefficiency of assortative matching in this context links to related work.

“Dynamic Inefficiency of Assortative Matching” S. Durlauf and A. Seshadri (in progress)

This numerical example illustrates a general idea.

There are 4 agents who are tracked over 3 periods. Each agent is associated with a period-specific characteristic ω_{it}; for concreteness assume that it is educational attainment.

The distribution of period 0 values is 10, 10, 20, 20.

In period 0 and 1, the agents are placed in two person groups, Think of these as classrooms. Agents are placed in pairs $\{i, i'\}$. Pairings can differ between periods 0 and 1.

The value of ω_{it+1} is determined by ω_{it} and $\omega_{i't}$, the value for the agent with whom he is paired, i.e.

$$\omega_{it+1} = \phi\left(\omega_{it}, \omega_{i't}\right)$$

The objective of the policymaker is to maximize $\bar{\omega}_2$. The policy choice is the pair of matching rules for periods 0 and 1.

Suppose that one step ahead transformation function for an agent is the following:

$$\phi\left(\omega_{it+1} \middle| \omega_{it}, \omega_{i't}\right) = f_1\left(\omega_{it}\right) + f_2\left(\omega_{it}, \omega_{i't}\right)$$

such that

$$f_1\left(\omega_{it}\right) =$$

$$0 \text{ if } \omega_{it} \leq 9$$

$$.9\omega_{it} \text{ if } 9 < \omega_{it} \leq 10$$

$$\omega_{it} \text{ if } 10 < \omega$$

$$f_2\left(\omega_{it}, \omega_{i't}\right) =$$

$$\max\left\{\varepsilon\left(\omega_{i't} - 10\right)\omega_{it}, 0\right\} + \eta\omega_{it}\omega_{i't}$$

Result: If η small enough, then exists $\varepsilon > 0$ such that maximization of $\overline{\omega}_2$ leads to reverse assortative matching in period 0 and assortative matching in period 1.

The example has strict increasing differences in the payoff functions. Hence the Becker marriage model result does not hold.

Point: In dynamic models, the mean is not sufficient to characterize effects of matching rule on terminal average outcome.

Proposition 7. Incomes of the Children Higher Income Neighborhoods have Higher Expected Growth Rate than Children in Lower Income Neighborhoods

Let g_{nt+1} denote the average expected income growth between parents and offspring in neighborhood n,t.

For any two neighborhoods n and n' if $\bar{Y}_{nt} < \bar{Y}_{n't}$ $\mu\left(N_{n,t}\right) \geq \mu\left(N_{n',t}\right)$, then $g_{n,t} - g_{n',t} > 0$.

Proposition 8. Possibility of Permanent Income Inequality

For uniformly growing income processes, i.e. income for each family increases in expected value each period, regardless of neighborhood configurations, there exist time t income distributions such that the ratio of the income of the highest family income to the lowest family income never decreases for the descendants of that pair of adults.

$$\Pr\left(\frac{Y_{it+v}^{High}}{Y_{it+v}^{Low}} \geq \frac{Y_{it+v}^{High}}{Y_{it+v}^{Low}} \forall v > 0 \middle| F_{Y_t}\right) > 0;$$

Key: log income differences behave in fashion similar to random walk with drift. Reduction of income ratio is analogous to a random walk with drift hitting an absorbing barrier.

Proposition 8. Gatsby-like Curve

If the variance of income at t, the correlation of parent/offspring income increases.

To be Finished

1. Analysis of dynamics with stratification supported by prices

2. Introduction of richer individual-level heterogeneity. Stratification should be relaxed in presence of heterogeneity in relative weights some parents assign to children.

Further Work

1. Richer family structure is needed for calibration.