Notes on Identification of Genetic Effects

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1. ACE and IGE

An intergenerational form of ACE is

$$l\omega_{it} = aA_{it} + cC_{it} + eE_{it} \tag{1}$$

$$\begin{bmatrix} A_{it} \\ C_{it} \\ E_{it} \end{bmatrix} = \begin{bmatrix} \phi_{AA} & \phi_{AC} & \phi_{AE} \\ \phi_{CA} & \phi_{CC} & \phi_{CE} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{it-1} \\ C_{it-1} \\ E_{it-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{it} \\ \xi_{it} \\ \eta_{it} \end{bmatrix} \tag{2}$$

The restriction in the third row of the AR(1) coefficient matrix in (2) is all 0's means that nonshared environment is assumed to be unpredictable, i.e. the variable corresponds to luck. This can be relaxed.

The MA representation should make it straightforward to calculate covariances for different twin/family type pairs, e.g. monozygotic/separated, etc.

The conventional IGE regression is

$$\omega_{it} = \beta\omega_{it-1} + \psi_{it} \tag{3}$$

The IGE parameter β (identified of course since (3) is a projection) should matter for the study of (1) and (2).

The AR(1) ACE and AR(1) IGE model are not consistent with one another outside of nongeneric cases. (1) and (2) imply

$$\omega_{it} = \begin{bmatrix} a & c & e \end{bmatrix} \begin{bmatrix} A_{it} \\ C_{it} \\ E_{it} \end{bmatrix} =$$

$$\begin{bmatrix} a & c & e \end{bmatrix} \begin{bmatrix} \phi_{AA} & \phi_{AC} & \phi_{AE} \\ \phi_{CA} & \phi_{CC} & \phi_{CE} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{it-1} \\ C_{it-1} \\ E_{it-1} \end{bmatrix} + \begin{bmatrix} a & c & e \end{bmatrix} \begin{bmatrix} \varepsilon_{it} \\ \xi_{it} \\ \eta_{it} \end{bmatrix} = \tag{4}$$

$$\begin{bmatrix} a\phi_{AA} + c\phi_{AC} & a\phi_{CA} + c\phi_{CC} & a\phi_{AE} + c\phi_{CE} \end{bmatrix} \begin{bmatrix} A_{it-1} \\ C_{it-1} \\ E_{it-1} \end{bmatrix} + \begin{bmatrix} a & c & e \end{bmatrix} \begin{bmatrix} \varepsilon_{it} \\ \xi_{it} \\ \eta_{it} \end{bmatrix}$$

Which makes clear that an AR(1) formulation for IGE implies that[1]

[1]We may want to use

$$\begin{bmatrix} a\phi_{AA} + c\phi_{AC} & a\phi_{CA} + c\phi_{CC} & a\phi_{AE} + c\phi_{CE} \end{bmatrix} \begin{bmatrix} A_{it-1} \\ C_{it-1} \\ E_{it-1} \end{bmatrix} = \qquad (5)$$

$$\beta \begin{bmatrix} a & c & e \end{bmatrix} \begin{bmatrix} A_{it-1} \\ C_{it-1} \\ E_{it-1} \end{bmatrix} = \beta \omega_{it-1}$$

This last formulation indicates that equivalence is non-generic. Intuition is simple: aggregation does not preserve AR (1) property of components.

$$\begin{bmatrix} A_{it} \\ C_{it} \end{bmatrix} = \begin{bmatrix} \phi_{AA} & \phi_{AC} \\ \phi_{CA} & \phi_{CC} \end{bmatrix} \begin{bmatrix} A_{it-1} \\ C_{it-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{it} + \phi_{AE}\eta_{it-1} \\ \xi_{it} + \phi_{CE}\eta_{it-1} \end{bmatrix} =$$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \phi_{AA}L & \phi_{AC}L \\ \phi_{CA}L & \phi_{CC}L \end{bmatrix} \right)^{-1} \begin{bmatrix} \varepsilon_{it} + \phi_{AE}\eta_{it-1} \\ \xi_{it} + \phi_{CE}\eta_{it-1} \end{bmatrix}$$

In future calculations.

This is obvious for the special case in which all nondiagonal elements of AR(1) coefficient matrix are zero. For this case, consistency requires

$$\begin{aligned} \phi_{AA} a &= \beta a \\ \phi_{CC} c &= \beta c \end{aligned} \tag{6}$$

which implies that $\phi_{AA} = \phi_{CC}$ if $a, c \neq 0$. This is really just a restatement of the fact that a sum of AR(1) processes is not AR(1). We will want to calculate the value of β as a function of (1) and (2) and use this as an identifying restriction.

2. ACE/IGE and Becker-Tomes

ACE models have been properly criticized because of the assumption that the *A* and *C* components are uncorrelated (Goldberger (1979)). this critique has used to conclude that nature/nurture decompositions are meainingless. (Manski (2011)). It is surely correct that the assumptions in ACE analysis do not lead to interpretable decomposisitions. However, in parallel to the structural VAR literature there is adistinct question concerning whether economic theory can provide credible identifying assumptions.

One way to link economic theory and ACE analysis is via a family investment model which endogenizes shared family environment as done by Becker and Tomes (1979). To link ACE into Becker-Tomes, we will want to start with a variant of the model due to Solon () which uses particular functional forms to produce a the IGE equation as an equilibrium description. Durlauf (1996) does something similar for neighborhoods. Throughout, agent it is born at t. We follow Becker and Tomes by assuming that the outcome of interest is income.

Here is the model:

1. Human capital h_{it} determined by two factors: parental investment I_{it} and a stochastic term ς_{it}. The associated functional form assumption is

$$h_{it} = \theta \log I_{it} + \varsigma_{it} \tag{7}$$

2. Income ω_{it} determined by family investment and a stochastic term ε_{it}; functional form

$$\log \omega_{it} = \mu + w h_{it-1} + \varepsilon_{it} \tag{8}$$

3. Parental investment is determined by maximization of

$$U_{it} = (1-\pi)\log\omega_{it+1} + \pi\log O_{it} \tag{9}$$

where O_{it} is other (I avoid calling it consumption as C is used for shared family environment.) The budget constraint facing the parent is

$$\omega_{it} = I_{it} + O_{it} \tag{10}$$

Assume perfect foresight, i.e. all shocks are known.

What does this model produce? From (7) and (8),

$$\log \omega_{it} = \mu + w\theta \log I_{it} + w\zeta_{it} + \varepsilon_{it} \tag{11}$$

This looks similar to ACE, if one sets

$$\begin{aligned} A_{it} &= w\zeta_{it} \\ C_{it} &= w\theta \log I_{it} \\ E_{it} &= \varepsilon_{it} \end{aligned} \tag{12}$$

We can think of ability as having an educational component $w\theta \log I_{it}$ and a genetic component ζ_{it}. This contrasts with labor market luck ε_{it}. Note that we will want to build in dependence in ε_{it} to capture parent/offspring links and covariance to capture sibling links.

From the Cobb-Douglas implication of constant budget shares, one has the standard IGE model:

$$\log \omega_{it} = \kappa + \pi \log \omega_{it-1} + w\varsigma_{it} + \varepsilon_{it} \tag{13}$$

where $\kappa = \mu + \gamma \log\left(\frac{\pi\theta w}{1-\pi\left(1-\theta w\right)}\right)$

Variance decompositions can be based on this framework.