1. Role of Income
The Role of Income in Human Development

• Strong correlation between child outcomes and family income
• Less clear evidence of a causal role of income
  ○ Pure income effects vs effects due to family background factors
  ○ Correlated family factors (parental ability, education...) influence both permanent income and child development
• Need to distinguish effects of income by source and between pure income and price (substitution) effects (tuition a price effect)
Figure 1: Children of NLSY
Average percentile rank on anti-social behavior score, by income quartile

Source: Cunha et al. (2006)
Figure 2: Children of NLSY
Adjusted average anti-social behavior score percentile by income quartile*

* Adjusted by maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age

Source: Cunha et al. (2006)
Each score standardized within observed sample. Using all observations and assuming data missing at random. Source: Brooks-Gunn et al. (2006).
2. Technology of Skill Formation
\[ \theta_{t+1} = f(t)(\theta_t, I_t, \theta_{P,t}). \]
• $f(t)$ twice continuously differentiable, increasing in all arguments and concave in $I_t$.

• Dimension of $\theta_t$ and $f(t)$ likely increases with the stage of the life cycle $t$, as does the dimension of $I_t$.

• New skills emerge along with new investment strategies.

• The technology is stage-specific, allowing for critical and sensitive periods in the formation of capabilities and the effectiveness of investment.

• This technology accommodates the family formation of child preferences, as in Becker and Mulligan (1997), Becker et al. (2012), Bisin and Verdier (2001), and Doepke and Zilibotti (2012).
• Crucial concept: complementarity between skills and investments at later stages \((t > t^*)\) of childhood:

\[
\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} > 0, \quad t > t^*.
\]

• Empirical literature consistent with notion that:

\[
\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} \leq 0, \quad t < t^*, \quad \left(\text{or } \epsilon > \frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} > 0, \text{ for "small" } \epsilon\right)
\]

• Complementarity increases with age:

\[
\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t'} \uparrow \quad t \uparrow.
\]
• Dynamic complementarity (Cunha and Heckman, 2007, 2009)
• $l_t \uparrow \Rightarrow \theta_{t+1} \uparrow$
• Because of self-productivity, $\theta_{t+1} \uparrow \Rightarrow \theta_{t+s} \uparrow, s \geq 1$
• Thus:

$$\frac{\partial^2 \theta_{t+s+1}}{\partial l_t \partial l'_{t+s}} > 0, \quad s \geq 1.$$ 

• Investments in period $t+s$ and investments in any previous period $t$ are always complements as long as $\theta_{t+s}$ and $l_{t+s}$ are complements, irrespective of whether $l_t$ and $\theta_t$ are complements or substitutes in some earlier period $t$. 
3. Determinants of Investments
Determinants of Investments

- Analysis of causal role of income related with study of borrowing constraints
- \( V(t, \theta_t^P, \theta_t, s_t, \epsilon_t) \): Parental value function

\[
\mathbb{E}_{I_t} \left[ \frac{\partial f(\theta_t, I_t, \theta_t^P)}{\partial I_t} \right] V_{\theta_t+1} = (1 + r) \ p \ \mathbb{E}_{I_t} \left[ V_{s_t+1} \right] + \mu_t
\]  

- Optimal \( I_t^* \) which solves the equation
  - Tighter borrowing constraints induce a wedge in optimal condition (\( \mu_t \))
  - Heterogeneous preferences (\( V \)), technologies (\( f \) via \( \theta_t^P \) and \( \theta_0 \)) and information sets (\( I_t \)) imply differences in investment independent from borrowing constraints
  - Borrowing constraints are not necessarily associated with or the cause of low level of investments
  - All can be targeted by policies: compare magnitude of changes in \( I_t \)
Timing of Investments

- Not only contemporaneous credit constraints matter
  - Expectation of future constraints reduces current investments
- Dynamic complementarity interacts with credit constraints
  - Constraints in the early years are especially harmful
4. Late Constraints
Evidence on Credit Constraints: College

- Little evidence of an important role of credit constraints in college enrollment
  - (Keane and Wolpin 2001, Carneiro and Heckman 2002, Cameron and Taber 2004)
    - Effects of income disappear once ability is controlled for
    - (Carneiro and Heckman 2002)
- Evidence that the role of income has become stronger over time
  - (Belley and Lochner 2007, Bailey and Dynarski 2011, Lochner and Monge-Naranjo 2012)
Changes Over Time in College Attendance

Source: Belley and Lochner (2007). College attendance by AFQT and family income quartiles. Data from NLSY79 and NLSY97 placed on one graph.
Increased Role of Income Over Time (?)

- General increase in college attendance
- Increase is highest for less able children with richer parents
- Is this evidence of greater role for income than in the past?

1. Role of changes in income distribution
   - If education is an income elastic merit good and
   - changes in the income distribution concentrated in right tail
   - $\Rightarrow$ Expect children of richer families to enroll more

2. Role of parental transfers
   - Tied parental transfers explain much of the effects of parental income
   - Returns to college are negative for low ability students
   - $\Rightarrow$ No efficiency reasons to increase college attendance of the less able
5. Early Constraints
Credit Constraints in Sensitive Periods

- Ability is the major determinant of college attendance
- But ability is shaped early in life through a process characterized by dynamic complementarity
- Therefore it can be affected by constraints experienced in earlier periods
- Studies have addressed the issue by:
  1. Try to isolate the causal effect of income in the early years
  2. Analyze the interaction of technology and constraints through structural models
Evidence from Income Transfer Programs

• Policy variation in EITC (Dahl and Lochner 2012):
  ○ $1000 increase in transfer produces a 6% of a std increase in test scores
  ○ Cost is large: $1000 for each year the average family expects to be in the program
  ○ Effect of EITC on labor supply not directly modeled, but evidence that parental time with children matters

• Randomized welfare-to-work interventions (Duncan et al 2011):
  ○ $1000 increase in income produces a 6% of a std increase in test scores
  ○ No distinction between alternative sources of income
  ○ No control for endogeneity of receipt of welfare and labor supply
  ○ Out of 16 programs studied only 2 show significant effects
Evidence from Income Transfer Programs

• Casino revenue (Akee et al. 2010):
  ○ $4000 increase in family income: +15% hs graduation, -22% p of being arrested at 16 and 17
  ○ Direct incentive for high school graduation: children became eligible for bonus themselves at 18 only if had graduated hs
  ○ No effect on crime reduction once older: incapacitation effect of schooling attendance

• Negative income tax experiments (various studies):
  ○ Income and substitutions effect are conflated
  ○ Effects are mixed varying by age and outcome studied

• Child benefit programs in Canada (Milligan and Stabile 2011):
  ○ Effects driven by Quebec where income transfers are complemented by other programs (such as child care subsidies)
Other Interventions Target Parenting

- Conditional cash transfers
- Supplementing information and knowledge
- Provide investments directly through interventions

⇒ Effects seem strong on multiple outcomes and robust to proper statistical testing
Structural Estimates of Parental Credit Constraints

• Caucutt and Lochner (2011) study the interaction of financial constraints, timing of receipt of income, precautionary saving behavior, and child investments
• Constraints stronger for parents in the early years of child’s life
• Constraints are pervasive (more than 50% of the population)
• Families affected are often high skill (68% constrained among college graduates)
• Benefits from relaxation of constraints are concentrated in the short run and among college-graduate parents
• No equalization in investments across families from relaxing constraints
• **Endogenous fertility is ignored and can bias the results**
6. Conclusion of Part I
Lessons from the Literature

- Higher levels of parental resources promote child outcomes
- In the current literature, not clear separation between income flows, parental environment and investments
- First order importance of ability for college going
- Income gradient in college attendance is stronger at low ability levels, but can depend on parental preferences
- Promising results analyzing the interaction of dynamic complementarity, the receipt of income and credit constraints
- Further research should isolate income effects from substitution effects affecting multiple margin of choice such as labor supply and fertility
“The Role of Income and Credit Constraints in Human Development” Part II

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Rasmus Landersø
Aarhus University

HCEO Social Mobility Conference
The University of Chicago
November 5th, 2014
1. Introduction
• This paper analyzes the mechanisms that drive differences and similarities in social mobility between the U.S. and Denmark.
• First, the paper rests on and contribute to the literature on human development through childhood and early adolescence.
• Second, we add to the literature on cross-country comparisons of income and social mobility.
• Third, we add to the literature on credit constraints and educational attainment.
Changes Over Time in College Attendance

Source: Belley and Lochner (2007). College attendance by AFQT and family income quartiles. Data from NLSY79 and NLSY97 placed on one graph.
Figure 1: High school completion in the U.S. and Denmark

(a) High school completion, U.S.

(b) High school completion, Denmark

Note: Figures from Denmark shows high school completion rates by grades from the written math exam.
Figure 1: College Attendance in the U.S. and Denmark

(c) College attendance, U.S.  
(d) College attendance, Denmark

Note: Figures from Denmark shows high school rates by grades from the written math exam.
2. Framework
• We follow a simplified version of the technology of skill formation, where childhood only lasts for one period.
• Children are born with birth endowments $\theta_0$.
• Parents are altruistic and invest in their children. Parental investments take two dimensions: $I_1^N$ is investments as nurture and time spent with the child and $I_1^S$ is investment in schooling.
• After observing $\theta_0$, parents choose levels of investments in their child at age 10-13, based on endowments and their own income $Y_1^P$ and wealth $W_1^P$:

$$I_1^k = f(\theta_0, Y_1^P, W_1^P), \ k = \{N, S\}$$
• Skills (age 15-16) evolve as a function of birth endowments and parental investments.

\[ \theta_1 = g(\theta_0, I_1^N, I_1^S) \]

• We consider two skill dimensions; cognitive and non-cognitive skills: \( \theta_1 = (\theta_1^C, \theta_1^{NC}) \).

• Schooling outcomes \( S^C \) are a function of child’s own skills \( \theta_1 \), parental income and wealth \( Y_1^P, W_1^P \) and demographic characteristics \( X_1 \):

\[ S^C = h(\theta_1, Y_1^P, W_1^P, X_1) \]
• A linearized version of the technology:
• Effects of birth endowments and parental income and wealth on nurture and schooling investments at age 10-13:

\[ I_1^N = \gamma_1^N + \gamma_1^N \theta_0 + \gamma_2^N Y^P + \gamma_3^N W^P \]
\[ I_1^S = \gamma_0^S + \gamma_1^S \theta_0 + \gamma_2^S Y^P + \gamma_3^S W^P \]
• Formation of skills at age 15-16:

\[ \theta_1^k = \gamma_0^k + \gamma_1^k \theta_0 + \gamma_2^k I_1^N + \gamma_3^k I_1^S, \quad k = \{C, NC\} \]

• The effect of skills and parental resources on the child’s schooling outcome \( S^C \):

\[ S^C = \gamma_0^Y + \gamma_1^Y \theta_1^C + \gamma_2^Y \theta_1^{NC} + \gamma_3^Y Y^P + \gamma_3^Y W^P + X' \gamma_4^Y \]
• $M_{ij}^{BE}, M_{ij}^{I}, M_{ij}^{C}, M_{ij}^{NC}$: measures of $\theta_0, I^N, \theta_1^C, \theta_1^{NC}$

\[
M_{j}^{BE} = \eta_{0j}^{BE} + \eta_{1j}^{BE} \theta_0 + Z' \mu_{2j}^{BE} + \epsilon_{j}^{BE}
\]
\[
M_{j}^{I} = \eta_{0j}^{I} + \eta_{1j}^{I} I_1^N + Z' \mu_{2j}^{I} + \epsilon_{j}^{I},
\]
\[
M_{j}^{k} = \eta_{0j}^{k} + \eta_{1j}^{k} \theta_1^k + Z' \mu_{2j}^{k} + \epsilon_{j}^{k}, \ k = \{C, NC\}
\]
Anchor the model to high school completion for both countries in order to meaningfully compare the estimation (Cawley et al., 1998, Cunha and Heckman, 2008 and Cunha et al., 2010):

$$HS_2 = \delta_0 + \delta_1 \theta_1^C + \xi$$

(2)

The anchoring uniquely determines an interpretable scale used for cross-country comparisons to the extent that high school completion is comparable between the U.S. and Denmark.
3. Data and Summary Statistics
U.S. Data
• CLSNY

Danish Data
• Cohort of 1987; register/administrative data
• DALSC (Danish Longitudinal Study of Children) linked to register data
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>U.S., CNLSY</th>
<th>Denmark, 1987</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gender (boy=1)</strong></td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.50)</td>
</tr>
<tr>
<td><strong>Minority/immigrant</strong></td>
<td>0.16</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(0.16)</td>
</tr>
<tr>
<td><strong>Siblings</strong></td>
<td>1.53</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>(1.17)</td>
<td>(0.86)</td>
</tr>
<tr>
<td><strong>Mother’s age at birth</strong></td>
<td>23.58</td>
<td>28.05</td>
</tr>
<tr>
<td></td>
<td>(4.21)</td>
<td>(4.60)</td>
</tr>
<tr>
<td><strong>Mother high school</strong></td>
<td>0.92</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.48)</td>
</tr>
<tr>
<td><strong>Mother college</strong></td>
<td>0.32</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(0.49)</td>
</tr>
<tr>
<td><strong>Parental income (1,000 2010USD)</strong></td>
<td>124.69</td>
<td>105.357</td>
</tr>
<tr>
<td></td>
<td>(180.51)</td>
<td>(53.56)</td>
</tr>
<tr>
<td><strong>Parental wealth (1,000 2010USD)</strong></td>
<td>329.76</td>
<td>289.05</td>
</tr>
<tr>
<td></td>
<td>(553.15)</td>
<td>(44.89)</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>3,268</td>
<td>33,956</td>
</tr>
</tbody>
</table>
• We consider high school completion and college attendance

• U.S.:
  ○ We define high school completion as having a high school diploma/GED.
  ○ College attendance is defined by report of either full or part time enrollment in college until age 21.

• Denmark:
  ○ High school completion is defined as 12 years of completed schooling.
  ○ College attendance is defined as enrollment in medium or long tertiary educations at community colleges, colleges, and universities
Table 2: Summary of Schooling Variables

<table>
<thead>
<tr>
<th></th>
<th>U.S., CNLSY</th>
<th>Denmark, 1987</th>
</tr>
</thead>
<tbody>
<tr>
<td>High school completion</td>
<td>0.75</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td>(0.49)</td>
</tr>
<tr>
<td>College attendance</td>
<td>0.43</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.49)</td>
</tr>
<tr>
<td>Observations</td>
<td>3,268</td>
<td>33,956</td>
</tr>
</tbody>
</table>
• Birth endowments: Birth weight, gestation length, length of child.

• Cognitive measures:
  ◦ U.S.: PIAT scores; math, reading recognition, reading comprehension.
  ◦ Denmark: Exam grades on Math and Science.

• Non-cognitive measures
  ◦ U.S.: BPI scores; anti-social, head strong, and hyperactivity.
  ◦ Denmark: Grades on organization/neatness.

• Nurture investments ages 10-13: Questions on time spent helping with homework, time spent reading for/with the child, time spent playing with the child

• Investments via private schooling ages 10-13: Measured as a dummy for whether the child attends private schooling or not.
Distributions of measures at birth, U.S. 

Distributions of measures at birth, Denmark
Distributions of cognitive skill measures, U.S.

Distributions of cognitive skill measures, Denmark
Histogram of non-cognitive skill measures (BPI scores), U.S.

Histogram of non-cognitive skill measures (grades), Denmark
Histogram of investment measures, U.S.

Histogram of investment measures, Denmark

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Figure 2: Schooling and parental income

(a) High school completion, U.S.

(b) High school completion, Denmark
Figure 2: Schooling and parental income

(c) College attendance, U.S.

(d) College attendance, Denmark
Figure 3: High school completion by parental income and wealth

(a) U.S.  
(b) Denmark
Figure 4: College attendance by parental income and wealth

(a) U.S.  
(b) Denmark
Figure 5: Schooling by mother’s education

(a) High school completion

Note: US is CNLSY sample. Sample size 3,268. DK is Danish register data. Sample size is 33,956. P(US=DK) for mom no high school: 0.862. P(US=DK) for mom high school: 0.000. P(US=DK) for mom college: 0.348.
Figure 5: Schooling by mother’s education

(b) College attendance

Note: US is CNLSY sample. Sample size 3,268. DK is Danish register data. Sample size is 33,956. P(US=DK) for mom no high school: 0.000. P(US=DK) for mom high school: 0.072. P(US=DK) for mom college: 0.001.
4. Empirical Estimates
Educational Attainment
Figure 6: High school completion by parental income and wealth skills $\perp \theta^C, \theta^{NC}$ and $X$

(a) U.S.  
(b) Denmark
Figure 7: High school completion by cognitive and non-cognitive skills, age 15-16

Parental income, wealth, and $X$

(a) U.S.  (b) Denmark
Figure 8: College attendance by parental income and wealth skills $\perp \perp \theta^C, \theta^{NC}$ and $X$

(a) U.S.  
(b) Denmark
Figure 9: College attendance by cognitive and non-cognitive skills, age 15-16 ⊥⊥ Parental income, wealth, and X

(a) U.S.  

(b) Denmark
Birth Endowments, Investments, and Skill Formation
**Figure 10:** Birth endowments $\theta_0$ by mother’s education and parental income

(c) By parental income, levels, U.S.  
(d) By parental income, levels, Denmark

Note: Dashed lines on parental income figure indicate 95% confidence intervals. Figure shows predicted birth endowments $\hat{\theta}_0$, from the estimated model. Birth endowments are anchored to $P($high school completion$)$ and the y-axis may be interpreted as such.
Table 3: Effect of $\theta_0$ and parental income/wealth on parental investments

<table>
<thead>
<tr>
<th></th>
<th>Schooling investments $I^S_1$</th>
<th>Nurture investments $I^N_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S.</td>
<td>Denmark</td>
</tr>
<tr>
<td>Birth endowments</td>
<td>0.0009 (0.0014)</td>
<td>-0.0008 (0.0008)</td>
</tr>
<tr>
<td></td>
<td>0.0110** (0.0040)</td>
<td>0.0389 (0.0280)</td>
</tr>
<tr>
<td>Parental income</td>
<td>0.0154*** (0.0032)</td>
<td>0.0036* (0.0017)</td>
</tr>
<tr>
<td></td>
<td>0.0029*** (0.0004)</td>
<td>0.0019 (0.0016)</td>
</tr>
<tr>
<td>Parental wealth</td>
<td>0.0111*** (0.0031)</td>
<td>0.0052*** (0.0017)</td>
</tr>
<tr>
<td></td>
<td>0.0046*** (0.0004)</td>
<td>0.0023 (0.0016)</td>
</tr>
</tbody>
</table>

Note: All estimates are anchored to $p$(high school completion). Income and wealth is measured as percentiles (0-1).
Figure 11: Cognitive skills at age 15-16 $\theta_1^C$ by Birth endowments $\theta_0$ and parental nurture investments $I_1^N \perp \perp$ schooling investments $I_1^S$

(a) U.S.  

(b) Denmark

Cognitive skills are anchored to $P$(high school completion) and the y-axis may be interpreted as such.
Figure 12: Non-cognitive skills at age 15-16 $\theta_{1}^{NC}$ by Birth endowments $\theta_0$ and parental nurture investments $I_1^N$ $\perp\perp$ schooling investments $I_1^S$

(a) U.S.  
(b) Denmark

Non-cognitive skills are anchored to $P$(high school completion) and the y-axis may be interpreted as such.
Table 4: Effect of parental schooling investments at age 10 - 13 on cognitive and non-cognitive skills at age 25

<table>
<thead>
<tr>
<th>Schooling investments (0/1)</th>
<th>Cognitive skills $\theta^C_1$</th>
<th>Non-cognitive skills $\theta^{NC}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. D</td>
<td>Denmark (U.S. D)</td>
</tr>
<tr>
<td>0.2995*** (0.0487)</td>
<td>0.0058 (0.0038)</td>
<td>0.2175*** (0.0377) (0.0017)</td>
</tr>
<tr>
<td>0.0021</td>
<td>0.0377 (0.0017)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Cognitive and non-cognitive skills are anchored to p(high school completion). Schooling is measured as a dummy 0/1.
Figure 13: Cognitive skills at age 15-16 $\theta_1^C$ by parental income and wealth percentiles

(a) U.S.  
(b) Denmark

Cognitive skills are anchored to P(high school completion) and the y-axis may be interpreted as such.
Other Sources of Schooling Investments
Figure 14: Mean level of birth endowments $\theta_0$ for peers in pre-school, by parental income levels, Denmark

Figure shows school 'leave-one-out' means of predicted birth endowments $\hat{\theta}_0$ from the estimated model by pre-school level. The endowments are anchored to $P(\text{high school completion})$ and the y-axis may be interpreted as such.
Figure 15: Mean skill level of peers in school, by parental income levels, Denmark

(a) Cognitive skills, age 15-16 $\theta_1^C$

Figure shows school 'leave-one-out' school means of predicted cognitive skills $\hat{\theta}_1^C$ from the estimated model. The skills are anchored to $\text{P(high school completion)}$ and the $y$-axis may be interpreted as such.
Figure 15: Mean skill level of peers in school, by parental income levels, Denmark

(b) Non-cognitive skills, age 15-16 $\theta_{1}^{NC}$

Figure shows school 'leave-one-out' school means of predicted non-cognitive skills $\hat{\theta}^{NC}$, from the estimated model. The skills are anchored to $P(\text{high school completion})$ and the y-axis may be interpreted as such.
Figure 16: Mean birth endowments of children, by property value in school catchment-area, Denmark

Figure shows school means of predicted birth endowments \( \theta_0 \) from the estimated model. The skills are anchored to \( P(\text{high school completion}) \) and the y-axis may be interpreted as such.
Figure 17: Mean cognitive skills of children age 15 - 16, by property value in school catchment-area, Denmark

Figure shows school means of predicted cognitive skills $\hat{\theta}^C$ from the estimated model. The skills are anchored to $P$(high school completion) and the y-axis may be interpreted as such.
Figure 18: Mean level of non-cognitive skills of children age 15 - 16, by property value in school catchment-area, Denmark

Figure shows school means of predicted non-cognitive skills $\hat{\theta}^{NC}$ from the estimated model. The skills are anchored to $P(\text{high school completion})$ and the y-axis may be interpreted as such.
5. Summary & Conclusions
Figure 19: Skills at age 15-16 by mother’s schooling

(a) Cognitive skills $\theta^C_1$, U.S.

(b) Cognitive skills $\theta^C_1$, Denmark

Note: Figure shows predicted cognitive skills $\hat{\theta}^C$ and non-cognitive skills $\hat{\theta}^C$, from the estimated model. The skills are anchored to $P($high school completion$)$ and the x-axis may be interpreted accordingly.
Figure 19: Skills at age 15-16 by mother’s schooling

(c) Non-cognitive skills $\theta_1^{NC}$, U.S.  
(d) Non-cognitive skills $\theta_1^{NC}$, Denmark

Note: Figure shows predicted cognitive skills $\hat{\theta}^C$ and non-cognitive skills $\hat{\theta}^{NC}$, from the estimated model. The skills are anchored to $P$(high school completion) and the $x$-axis may be interpreted accordingly.
Figure 20: Appendix to Figure 3
Figure 21: Appendix to Figure 4
Figure 22: Appendix to Figure 6
Figure 23: Appendix to Figure 8