Models of Directed Search - Labor Market Dynamics, Optimal UI, and Student Credit

Florian Hoffmann, UBC

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Why Equilibrium Search Theory of Labor Market?

- Theory of
  - Unemployment
  - worker mobility, wage dynamics and residual inequality
- Econometric framework to
  - quantify search frictions
  - quantify importance of skill vs. human capital vs. luck
- Framework to study optimal UI and taxation

Why Directed Search?

- Random search well known
- Endogenizes search frictions (to some extent ...)
- Overcomes some inefficiencies in random search economies
- Some empirical evidence that workers "direct" their search
- Random search models cannot generate much residual inequality $\longrightarrow$ can directed search models?
- Computational tractability
  - block recursivity
  - non-stationarity
  - worker and firm heterogeneity.

Outline of Talk

1. Game-theoretic foundations of directed search (very brief)
2. Axiomatic approach to directed search ("competitive search")
3. Worker Heterogeneity
4. Optimal UI
5. On-the-job-search, dynamics and human capital
6. Incorporating education and student credit: A proposal

Directed Search: Basic Framework

- N workers and M firms, both risk-neutral&homog., with $N, M < \infty$, try to match to produce output y
- Each firm m can employ one worker
  - more than 1 worker may contact firm $m \Rightarrow$ job is allocated randomly
- $2-$stage game:
  - 1^{st} stage: each seller m posts wage w_m
  - 2^{nd} stage: each buyer n chooses probabilities ρ_{nm} of visiting each seller m
- Utility of buyer: w_m if matched with seller m and 0 otherwise
- Profit of seller: $y - w_m$ if matched and 0 otherwise

Symmetric mixed-strategy equilibria

- Pure strategy equilibria require lots of coordination
  - $\Rightarrow$ focus on symmetric mixed-strategy equilibria
- **Def:** A symmetric mixed-strategy equilibrium is a wage-fct. $w_m(M, N)$ and an application strategy $\rho_{nm}(M, N)$ s.t., given strategies of all other players:
  - $\rho_{nm}(M, N)$ max. expected utility of each n
  - $w_m(M, N)$ max. expected profits of each m
  - $\rho_{nm}(M, N) = \rho_m(M, N)$ for all n
  - $w_m(M, N) = w(M, N)$ for all m.

Properties of Equilibria

- **Prop**: (Burdett, Shi, Wright, 2001): There is a unique symmetric mixed-strategy eqm. with the following properties:
  1. $\rho_m(M, N) = \frac{1}{M}$
  2. $w(M, N)$ has $w_1(M, N) > 0$ and $w_2(M, N) < 0$
  3. The endog. matching fct. $\theta(M, N)$ has DRS.
  4. Fix $\gamma = \frac{N}{M}$ and let $M, N \to \infty$. Then $\theta(M, N) \to \theta^*(M, N)$ and $\theta^*(.)$ is CRS.
- Endogenous ("search") frictions due to lack of coordination
  - some vacancies left unfilled, some workers left unemployed.

Directed Search: Axiomatic Approach

- Property 4 motivates axiomatic approach
  - e.g. Montgomery (1991), Moen (1997), Acemoglu&Shimer (1999), Shi (2009), Menzio&Shi (2011a,b)
- Specifies a CRS matching technology $\theta^*(.)$ as model primitive.
- Economy partitioned into submarkets indexed by w, associated with queue length $q(w)$
- Some advantages of axiomatic approach:
  - generally more tractable
  - dynamics
  - on-the-job-search.

Axiomatic Approach: Model Description

- Submarkets indexed by w :
  - $q(w) = \frac{n(w)}{m(w)}$, where $n(w)$: # of workers; $m(w)$: # of vacancies
  - workers applying to w match with prob.

$$\frac{\theta^*(m(w), n(w))}{n(w)} = \theta^* \left(\frac{1}{q(w)} \right) \equiv \mu \left(q(w) \right)$$

  - firms posting w match with prob. $\eta \left(q(w) \right) = q(w) * \mu \left(q(w) \right)$
  - $\mu' < 0$ and $\eta' > 0$.

Axiomatic Approach: Decision Problems

1. Worker's optimal application:

$$U^* = \max_w \mu\left(q(w)\right) * w + \left[1 - \mu\left(q(w)\right)\right] * 0$$

2. Firm's profit maximization and free entry:

$$\eta\left(q(w)\right) * (y - w) \leq \kappa \text{ and } q(w) \geq 0.$$

with complementary slackness.

Axiomatic Approach: Equilibrium

- **Def:** An eqm. is a set of wages W and a queue length function $q^*(w)$ s.t. (1) and (2) are jointly satisfied.
- Note: $q^*(w)$ is defined on $\mathbb{R}_+$, not W.
- **Prop** (Moen; Acemoglu&Shimer): Any eqm. allocation solves

$$\max_{w,q} \mu(q) * w + [1 - \mu(q)] * 0$$
$$s.t.\ \eta(q) * (y - w) = \kappa$$

- **Proposition** (Moen): The eqm. allocation attains the first-best allocation in the sense that it maximizes aggregate production net of vacancy creation costs. If there is more than one eqm., they are all equivalent in terms of welfare.

Random Search vs. Directed Search

- The following holds in the model above (but not in random search models):
  - search frictions are endogenous
  - wage posting can implement the first best
  - even though firms post wages, they are generally above the monopsony wage
- However:
  - efficiency depends on risk neutrality of workers
  - in finite economies ($M, N < \infty$) the equilibria derived using the game-theoretic and axiomatic approach may not coincide.

Introducing Worker Heterogeneity

- Adding worker het. in terms of ability surprisingly straightforward $\longrightarrow$ can be used to introduce (stochastic) skill accumulation (Hoffmann&Shi, 2012)
- Let there be L worker types, where type l produces y_l when matched
- Result: A submkt $(w, q(w))$ cannot be visited by different types of workers:

$$\eta(q) * (y_l - w) > \eta(q) * (y_{l-1} - w) = \kappa$$

  violating free entry.
- Instead, can construct an eqm. with L types as a collection of L autarkic equilibria $\{W_l; q_l^*(w)\}_{l=1}^{L}$.

Optimal UI: Acemoglu&Shimer

- Modify the model above:
  - homog., risk averse workers with utility $u(c)$ and initial assets A
  - match produces $f(k)$, k: capital with price $R = 1$
  - firm becomes active after making ex-ante investments $k > 0$
  - unempl. benefits b financed by lump-sum tax τ
- Definition of eqm. needs to be modified:
  - set of investment levels K
  - wage correspondences $W(k)$
  - budget balance: $b = (1 - \mu(q^*(w))) * \tau$.

Optimal UI: Acemoglu&Shimer

- **Prop**: Any eqm. allocation solves

$$\max_{w,q,k} \mu(q) * u(A - \tau + w) + [1 - \mu(q)] * u(A - \tau + b)$$
$$s.t.\ \eta(q) * (f(k) - w) - k = \kappa$$

With CARA-preferences, eqm. is unaffected by A.

- Set $\kappa = 0$. We have:

$$\begin{aligned} \eta(q^*) * f'(k^*) &= 1 \\ &\Rightarrow w^* = f(k^*) - k^* f'(k^*) \end{aligned}$$

$\Rightarrow$ free entry condition generates upward sloping relationship in $\{q, w\}$ −space.

Optimal UI: Acemoglu&Shimer

- **Prop**: If agents are risk-averse, eqm with $b = 0$ is not output maximizing. However, there is a moderate $b > 0$ that can implement the output maximizing allocation.
- Without sufficient insurance, workers apply to low-q submarkets $\Rightarrow$ prob. that a vacancy gets filled is low $\Rightarrow$ firms are not willing to make large ex-ante investments.
- Interpretations:
    - type of moral hazard
    - redistribution between successful and unsuccessful searchers important.

Comparison with Random Search

- Focus on different types of incentives: Search effort vs. "job quality"
- In random search, wage offers often taken as exogenous
  - Hopenhayn&Nicolini (1997); Shimer&Werning (2008)
- Relationship between ex-ante capital investments and labor mkt. tightness in directed search.

Dynamics, On the Job Search and Human Capital Accumulation

- Job-to-job transitions frequent in data
- Source of wage dynamics and residual inequality
- Introduces scope for wage taxation
- Need dynamics
- How to incorporate human capital accumulation?

Model Modifications

- infinite horizon, discrete time
- risk neutral workers, discount factor β
- exogenous job breakups at rate δ
- employed can send applications with prob. λ_e in each period
- unemployed can send applications with prob. $\lambda_u = 1$
- match produces output y
- Timing: production $\longrightarrow$ separations $\longrightarrow$ search

Value Posting and contracting

- Submkts characterized by promised expected life-cycle earnings x and tightness $q(x)$
- Firm can deliver this value in a lot of different ways
- Follow Hoffmann&Shi (2012) and assume firms pay in terms of output shares ω.

Search Problem

- Search problem of worker with status-quo value V:

$$R(V) = \max_{x} \; \mu\left(q(x)\right) * (x - V)$$

  with policy fct. $s(V)$.

- $s'(V) > 0 \Rightarrow$endogenous worker separations ("wage ladder").

- This is what separates directed from random search.

Value Functions of Workers

- Unemployed:

$$V^u = \beta * [b + V^u + R(V^u)]$$

- Employed:

$$V(\omega) = \beta * [\omega * y + \delta * V^u + (1 - \delta)(V(\omega) + \lambda^e * R(V(\omega)))]$$

- This generates a relationship $\omega(V)$.

Value Functions of Firms

- Filled vacancy in submarket x:

$$\begin{aligned}\frac{J(x)}{\beta} &= (1-\omega(x)) * y \\ &\quad + (1-\delta) * (1-\lambda^e * \mu(q(s(x))) * J(x))\end{aligned}$$

- Note: Promise keeping constraint of firm embedded in $\omega(x)$
- Free Entry:

$$\begin{aligned}\eta(q(x)) * J(x) &\leq \kappa; \\ q(x) &\geq 0 \quad w.c.s.\end{aligned}$$

Equilibrium

- Defn of eqm. analogues to Moen (1997) and Acemoglu&Shimer (1999)
- wage contracts inefficient!
- Endogenous wage ladder and worker separation: given V and $\omega(V)$, workers apply to unique submkt.
- Different firms play different strategies:
  - $\Rightarrow$ frictional (residual) wage inequality.

Block Recursivity

- In contrast to e.g. Moen (1997), eqm. characterization using a dual problem not possible
- However: There exists a block recursive eqm.
- This is already embedded in the value fcts above: do not depend on endog. value distribution $G(x)$
- $G(x)$ can be simulated using policy functions and some initialization.

Block Recursivity and Empirical Directed Search

- Block recursivity has computational advantages.
- It is fairly straightforward to introduce:
  - worker heterogeneity
  - match heterogeneity
  - human capital accumulation
  - non-stationary productivity process
  - multiple sectors (Hoffmann&Shi, 2012)'
- Model can be solved along transition paths.

Stochastic Human Capital Accumulation (Hoffmann&Shi)

- Worker het. $\alpha \in \{\alpha_1, ..., \alpha_L\}$
- Output $y_l = \alpha_l * y$.
- Learning by doing:
  - while empl., $\alpha' \sim \Gamma(\alpha', \alpha)$
  - for simplicity, assume $\alpha = \alpha_1$ if unempl.
- Timing: update α after separation, before search.

Stochastic Human Capital Accumulation (Hoffmann&Shi)

- Value Functions of Workers:

$$
\begin{aligned}
R(V,\alpha) &= \max_x \; \mu\left(q(x,\alpha)\right) * (x - V) \\
V^u &= \beta * \left[b + V^u + \textstyle\sum_{\alpha'} \Gamma\left(\alpha',\alpha\right) * R(V^u,\alpha') \right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{V(\omega,\alpha)}{\beta} &= \omega * \alpha * y + \delta * V^u + (1-\delta) * \\
& \quad \textstyle\sum_{\alpha'} \Gamma\left(\alpha',\alpha\right) * \left(V(\omega,\alpha') + \lambda^e * R(V(\omega,\alpha'))\right) \\
\Rightarrow & \quad w(V,\alpha)
\end{aligned}
$$

Stochastic Human Capital Accumulation (Hoffmann&Shi)

- Value Functions of Firms:

$$\frac{J(x,\alpha)}{\beta} = (1-\omega(x,\alpha))*\alpha*y+(1-\delta)* \sum_{\alpha'}\Gamma\left(\alpha',\alpha\right)*\left(1-\lambda^{e}*\mu\left(q(s(x,\alpha'))\right)*J(x,\alpha')\right)$$

$$V(\omega(x,\alpha),\alpha)=x$$

$$\begin{aligned}\eta\left(q(x,\alpha)\right)*J(x,\alpha) &\leq \kappa; \\ q(x,\alpha) &\geq 0 \quad w.c.s.\end{aligned}$$

Comparison with Random Search Models w. Wage Posting

- Random search models with on-the-job-search not block-recursive.
- Workers draw randomly from $G(x) \rightarrow$ even conditional on contacting a vacancy, match forms with prob. $G(V) \leq 1 \rightarrow$ $G(x)$ enters all value fct.s
- Imposes restrictions on econometric modeling of eqm. search that do not exist in directed search.
- What about empirical properties?
  - to be explored.

Optimal UI and private student credit

- How do UI benefits affect the market for private student credits?
- Go back to quasi-static Acemoglu&Shimer model.
- UI affects workers' search for risky jobs:
  - may $\uparrow$ output and hence may $\uparrow$ student credit in eqm.
  - particularly plausible if effect of UI particularly strong for high types
- Eqm interactions between student credit and UI may be complicated: Creditors care about $\mu * w$
  - however, $\uparrow$ in UI leads to $\mu \downarrow$ and $w \uparrow \Rightarrow$ optimal (w, μ)?
- What if unemployment benefits cannot be used to repay debt (e.g. food stamps)?

Adding Student Credit to Acemoglu&Shimer

- Adopt limited commitment and imcomplete mkt assumption as in Lochner&Monge-Naranjo
- Some challenges
  - discrete types, but preferably continuous education variable
  - discrete wage distribution in eqm
  - decisions depend on asset holdings and its distribution
  - how to model education stage?
- Acemoglu&Shimer: with CARA-utility, decisions do not depend on assets, even in dynamic economy
  - $\rightarrow$ With limited commitment, this is not true anymore.

Proposed Model: Technology and Types

- L types of workers, producing $y_l = \alpha_l * f(k)$ when matched
- Assume α_l are grid points on a uniform grid $[\alpha_1, \alpha_L]$ with $\Delta = \alpha_l - \alpha_{l-1}$
- Can invest into education $e \in \mathbb{R}_+$ at some cost
  - student credit $Q(D, \alpha, e)$, D : amount to be repaid
  - default cost $\phi * (income)$
- Returns to education: with prob. $h(e)$, ind. has prod.gain of $\Delta = \alpha_l - \alpha_{l-1}$
  - $h(e) \in [0, 1]$ with $h'(e) \geq 0$
  - initial discrete distribution over types $p_0(\alpha)$ with $p_0(\alpha_L) = 0$

Decision Problem of Worker: Search Stage

- Divide labor mkt. into L directed search economies
- Assume firms post output shares
- Search Problem:

$$V(\alpha, D) = \max_{w} \left\{ \begin{array}{c} \mu\left(q(w,\alpha)\right) * V^{e}(w,\alpha,D) \\ +\left(1-\mu\left(q(\omega,\alpha)\right)\right) * V^{u}(D) \end{array} \right\}$$

$$V^{e}(w,\alpha,D) = \max \left\{ \begin{array}{c} \left(u(\omega * \alpha * f(k) - D - \tau)\right); \\ u\left(\phi * \omega * \alpha * f(k)\right) \end{array} \right\}$$

$$V^{u}(D) = \max\left\{\left(u(b-D-\tau)\right); u\left(\phi * b\right)\right\}$$

Decision Problem of Firm

- Free entry and profit maximization:

$$\begin{aligned} \eta\left(q(w,\alpha)\right) * (1-w) * \alpha * f(k) &\leq k \\ q(w,\alpha) &\geq w.c.s. \end{aligned}$$

Education Stage

- On the search stage, worker provides one unit of labor or is unemployed
- Furthermore, labor market has frictions
- What about education stage?
- Continuation value of type$-l$ worker with (e, D) is

$$\begin{aligned} V_{+1}(e, \alpha_l, D) &= h(e) * V(\alpha_{l+1}, D) \\ &\quad + (1 - h(e)) * V(\alpha_l, D) \end{aligned}$$

- One option (in the spirit of Lagos-Wright): Assume labor mkt on education stage is frictionless.

A Frictionless Education Stage

- Follow Lochner&Monge-Naranjo:

$$\begin{aligned} & \max_{e,Q} u(\alpha_I * w_0 * (1-e) + Q) + V_{+1}(e, \alpha_I, D) \\ s.t.Q &= D - E\left[loss\right]. \end{aligned}$$

- Here, $E\left[loss\right]$ is determined by default decision on the matching stage.