Notes on Dynamic Optimal Taxation with Endogenous Human Capital Formation

Marek Kapička, UC Santa Barbara

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In this note I review some results in a multiperiod Mirrleesean economy where agents have private information about their ability and human capital is endogenous. I focus on dynamic aspects of the optimal tax reform: would the sequence of marginal taxes that individuals face be increasing/decreasing, and what are the forces that determine their dynamics. I start from the simplest deterministic model with observable human capital and then keep adding various frictions to see how the answers change.

I focus on three economies: a deterministic economy with observable human capital and schooling, a deterministic economy with unobservable human capital and schooling, and an economy with stochastic rates of return on human capital. Human capital in the last economy is observable but both the rates of return on human capital and schooling effort are unobservable. This introduces a moral hazard problem, on top of the private information problem. In some ways, the last economy is a hybrid of the first two: incentives to accumulate human capital must be provided indirectly (like in the deterministic economy with unobservable human capital) but taxes can depend on human capital (like in the deterministic economy with observable human capital).

There are several factors that determine the intertemporal profile of optimal taxes. First, as is well known from a static optimal taxation literature, the elasticity of labor supply is one of the key determinants. The main modification here is that the elasticity of labor supply

1 A Deterministic Economy

Consider the following economy. Agents live for $J > 1$ periods where J can be infinite. They like to consume, dislike working and schooling, and have preferences given by

$$\sum_{j=1}^{J} \beta^{j-1} \left[U(c_j) - V(l_j, s_j) \right], \quad 0 < \beta < 1, \tag{1}$$

where j is age, c_j is consumption, l_j is labor, and s_j is schooling. The function U is strictly increasing, strictly concave, and differentiable. The function V is strictly increasing, strictly convex, and differentiable in both arguments.

The agent's earnings are determined by agent's ability θ, current human capital h_j, current labor l_j and the rental rate of human capital w:

$$y_j = w\theta h_j l_j \tag{2}$$

Human capital next period h_{j+1} depends on current human capital h_j, and on current schooling s_j:

$$h_{j+1} = F(h_j, s_j) \tag{3}$$

where the function F is strictly increasing, strictly concave, and differentiable in both arguments. I invert it to express schooling as a function of current and future human capital, $s_j = S(h_j, h_{j+1})$.

Each individual is associated with an ability level $\theta \in [\underline{\theta}, \bar{\theta}] = \Theta$, which is constant over time. Agent's ability and human capital determine her skills θh_t and, together with labor supply, her output $y_t = \theta h_t l_t$. The ability is drawn from a distribution function Q which is differentiable and has density $q(\theta)$.

1.1 Informational Structure

Agent's abilities, as well as her labor supply, are private information of the agent, and are not observed by the social planner. Consumption and output are, on the other hand, both publicly observable. I will cover cases with both observable and unobservable human capital and highlight the differences.

1.2 Observable Human Capital

I start with the simplest possible case where human capital is observable. Since the agent's ability level is private information, the social planner needs to elicit the agent's type from her. At the beginning of period 0 the agent reports her type to the social planner. The utility of a θ - type agent who reports $\hat{\theta}$ is given by

$$W_{y,c,h}(\hat{\theta}|\theta, h_1) = \sum_{j=1}^{J} \beta^{j-1} \left[U(c_j(\hat{\theta})) - V\left(\frac{y_j(\hat{\theta})}{\theta h_j(\hat{\theta})}, S(h_j(\hat{\theta}), h_{j+1}(\hat{\theta})) \right) \right],$$

where $(c, y, h) = \{c_j, y_j, h_{j+1}\}_{j=1}^{J}$ is an *allocation*, chosen by the social planner. The observability of human capital is reflected by the fact that an agent who reports $\hat{\theta}$ must choose a human capital sequence of a $\hat{\theta}$ agent as well.

The incentive compatibility constraint requires the allocation to be such that an agent with ability θ prefers to report her own type to any other report: for all $\theta \in \Theta$,

$$W_{y,c,h}(\theta|\theta, h_1) \geq W_{y,c,h}(\hat{\theta}|\theta, h_1) \quad \forall \hat{\theta} \in \Theta. \tag{4}$$

A necessary condition for an allocation to be incentive compatible is given by the envelope condition:

$$W_{y,c,h}(\theta|\theta, h_1) = \int_{\underline{\theta}}^{\theta} \sum_{j=0}^{J} \beta^{t-1} V_{l,j}(\varepsilon) l_j(\varepsilon) \frac{d\varepsilon}{\varepsilon} + W_{y,c,h}(\underline{\theta}|\underline{\theta}, h_1), \tag{5}$$

where $V_{l,j} = V_l(l_j, s_j)$. Equation (5) shows how the agent's period utility varies with her type. The variation in period utility is proportional to the informational rent the agent obtains from having a given ability level.

It is shown in **?** that the envelope condition (5) is also sufficient, as long as both schooling $s_t(\theta)$ and the income to human capital ratio $\frac{y_t(\theta)}{h_t(\theta)}$ are increasing in θ for all periods. A similar condition in the static optimal taxation literature requires that income must be increasing in abilities. The analogue is partial because then envelope condition and the monotonicity condition in the static model is both necessary and sufficient for incentive compatibility while in the dynamic model it is only sufficient. The fact that the monotonicity conditions are not necessary for an allocation to be incentive compatible has one important implication in the subsequent analysis: unlike in static models, the monotonicity conditions cannot be imposed as a constraint on the social planner's problem. Therefore, we solve a *relaxed social planner's problem* where the incentive compatibility constraint (4) is replaced only by the envelope condition (5) and check whether the resulting allocation satisfies the monotonicity conditions.

Suppose that $U(c) = c$. The optimality condition for the planning problem, which I do not derive here[1] expresses the *intratemporal wedge*

$$\tau_j \equiv 1 - \frac{V_l(l_j, s_j)}{\theta h_j U_c(c_j)}$$

as a function of the Frisch elasticity of labor supply $\nu_j(\theta)$, and a *cumulative distortion* $X(\theta) \geq 0$:

$$\frac{\tau_j(\theta)}{1-\tau_j(\theta)} = \left[1 + \nu_j(\theta)^{-1}\right] X(\theta) \tag{6}$$

This condition is similar to the one in static models (see **?**). The cumulative distortion $X(\theta)$ represents the planner's desire to redistribute resources, and itself depends on the distribution of skills, as well as on the social welfare function. An example, in case of a social planner with Rawlsian preferences, would be $X(\theta) = \frac{1-Q(\theta)}{\theta q(\theta)}$. Also, if the distribution of skills is bounded then one has $X(\overline{\theta}) = 0$ which leads to $\tau(\overline{\theta}) = 0$ (no distortion at the

[1] For that, see **?**.

top). From the perspective of dynamic optimal taxation the important property is that X is constant over time. This follows from the assumption that θ is constant over time. The dynamics of the intratemporal wedge is thus driven mainly by the Frisch elasticity of labor supply $\nu_j(\theta)$, which is in general endogenous, and will be discussed later..

In addition, the efficient allocation features an *intertemporal human capital wedge* which is the wedge in the Euler equation for investment in human capital:

$$\Delta_j \equiv \frac{V_{s,j}}{F_{s,j}} - \beta\left(V_{l,j+1}\frac{l_{j+1}}{h_{j+1}} + V_{s,j+1}\frac{F_{h,j+1}}{F_{s,j+1}}\right) \tag{7}$$

The first term on the right-hand side denotes the private marginal costs of investing in human capital, expressed in units of output. The second term are the private marginal benefits of investing in human capital. They consist of higher productivity tomorrow, and of highrr human capital tomorrow. In general, the social planner will want to put a wedge between the private marginal costs and private marginal benefits.

1.2.1 Example I

Assume that the disutility of labor and schooling is additively separable, utility is linear in consumption, and human capital fully depreciates each period:

Assumption 1 $V(l,s) = \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}} + \frac{s^{1+\epsilon^{-1}}}{1+\epsilon^{-1}}$, $U(c) = c$ *and* $F(h,s) = s$.

Here ν is the Frisch elasticity of labor supply, a constant, and ϵ determines how responsive schooling is to changes in taxes. It follows from (6) that the planner will set

$$\frac{\tau_j(\theta)}{1-\tau_j(\theta)} = (1+\nu^{-1})X(\theta) \tag{8}$$

in all periods. That is, the intratemporal wedge is constant over time, and is equal to the intratemporal wedge in a static model. At the same time, the social planner equalizes marginal cost of schooling to social marginal benefits. It is easy to see that the social planner sets

$$\Delta_j(\theta) = \beta(1+\nu^{-1})X(\theta)\frac{l_{j+1}(\theta)^{1+\nu^{-1}}}{s_j(\theta)} \geq 0.$$

Schooling subsidies are therefore needed to increase incentives to invest in human capital. In the absence of schooling subsidies the agent does not invest efficiently in human capital because with distorting taxes private returns are smaller than social returns. The schooling subsidy corrects for that. Note also that schooling subsidies are zero whenever the cumulative distortion is zero.

1.2.2 Example II

More realistically, the utility is not additively separable in labor supply and schooling. We assume the following:

Assumption 2 *$V(l,s) = \frac{(l+s)^{1+\nu^{-1}}}{1+\nu^{-1}}$, $U(c) = c$ and $F(h,s) = s$.*

The elasticity of labor supply is

$$\nu_j(\theta) = \nu\left(1 + \frac{s_j(\theta)}{l_j(\theta)}\right).$$

The key difference from the previous example is that the elasticity of labor supply in (6) is now endogenous. Higher schooling relative to labor supply implies higher elasticity and lower intratemporal wedge. In **?** we solve for an infinite horizon economy and find the changes in elasticity to be relatively small. However, this is likely to be overturned in a life-cycle economy where $s_J = 0$ and the elasticity of labor supply necessarily decreases at least at the end of the life-cycle. This leads to the following conjecture:

Conjecture 3 *In a life-cycle economy with J finite the intratemporal wedge is increasing in age.*

The reason why schooling subsidies are strictly positive is simply that it simply corrects for the fact that, due to the positive marginal income tax rate, private benefits from investment in human capital are smaller than social benefits from investment in human capital.

The same argument is used in ?. In ? we show that in an infinite horizon economy the human capital wedge is positive whenever schooling is increasing over time, and in steady state.

A generalization of this result is provided by ?. They show that the "purely corrective" role of educational subsidies is modified when human capital investments are useful in separating people of different skills. For instance, if human capital investments benefit more those who pretend to have low productivity, rather than those who truly have low productivity, human capital investments can be discouraged.

1.3 Unobservable Human Capital

The social planner now not only needs to motivate the agent to provide a truthful information not only about her type, but also needs to ensure that the agent follows her recommendations regarding the human capital accumulation. The incentive compatibility constraint now requires

$$h = \arg\max_{\hat{h}} W_{c,y,\hat{h}}(\theta|\theta, h_1) \tag{9}$$

$$W_{c,y,h}(\theta|\theta, h_1) \geq \max_{\hat{h}} W_{c,y,\hat{h}}(\hat{\theta}|\theta, h_1) \qquad \forall \hat{\theta} \in \Theta. \tag{10}$$

Necessary conditions for incentive compatibility include the envelope condition (5) and an Euler equation for the investment in human capital:

$$\frac{V_{s,j}}{F_{s,j}} = \beta \left(V_{l,j+1} \frac{l_{j+1}}{h_{j+1}} + V_{s,j+1} \frac{F_{h,j+1}}{F_{s,j+1}} \right) \tag{11}$$

The Euler equation for investment in human capital equates marginal costs of investing in human capital with private marginal benefits. Essentially, unobservability of human capital investments forces the intertemporal human capital wedge (7) to be zero. The sufficiency of those conditions (10) and (11) must in general be checked numerically.

Since the human capital wedge must be zero, the social planner must provide incentives to accumulate human capital in different ways. The optimality condition (6) no longer applies.[2] As we shall see, one way to do it will be to decrease marginal income taxes over time.

1.3.1 Example I

Assume again that assumption (1) holds and so the disutility of labor and schooling is additively separable, utility is linear in consumption, and human capital fully depreciates each period.

I show in **?** that if Assumption (1) holds then the optimal marginal income tax rates are given by

$$\frac{\tau_1(\theta)}{1-\tau_1(\theta)} = (1+\nu^{-1})X_\mu(\theta)$$
$$\frac{\tau_j(\theta)}{1-\tau_j(\theta)} = (1+\hat{\nu}^{-1})X_\mu(\theta), \quad j = 2\ldots J,$$

where $\hat{\nu} = \frac{2+\nu^{-1}+\epsilon^{-1}}{\nu^{-1}\epsilon^{-1}-1}$.

One way of proving this result is to show that an economy with endogenous human capital formation is isomorphic to an economy with no human capital and a higher elasticity of labor supply $\hat{\nu}$ from the second period onwards. Rearranging the terms, one can also express a direct relationship between the tax rate in period zero and all the future periods as

$$\frac{\tau_j(\theta)}{1-\tau_j(\theta)} = \frac{1+\epsilon^{-1}}{2+\nu^{-1}+\epsilon^{-1}}\frac{\tau_1(\theta)}{1-\tau_1(\theta)}, \quad j = 2\ldots J.$$

The size of the decrease in the marginal tax rates is independent of the cumulative distortion, and is therefore the same for all agents. The optimal marginal tax rate schedule thus simply shifts down from period 1 on. If the elasticity of schooling is larger or if the elasticity of labor supply is smaller, then the drop in the marginal tax rates is larger. Note also that the marginal income tax rate in period zero is, conditional on the cumulative distortion, identical to the marginal income tax rate one would obtain in a static economy with no human capital, and in a dynamic economy with observable human capital.

[2] Although the no distortion at the top result still holds.

Additional insights into the social planner's problem can be obtained by analyzing the social planner's trade-offs directly. In the absence of any connection between human capital and labor supply the social planner would set the marginal tax rates according to (8) in all periods, not just in the initial period, and provide incentives to accumulate human capital separately. That is not possible anymore. However, in periods $t \geq 2$ there is a complementarity between labor supply and schooling. It is optimal for the social planner to increase labor supply above what he would have chosen in the absence of human capital considerations. The reason is that individuals only take into account private benefits from schooling, and private benefits are smaller than social benefits because of a positive marginal income tax.[3] The discussion in the previous paragraph indicates that unobservability of human capital is essential in obtaining the decreasing time profile of the marginal income tax rates.

Finally, note that full commitment to the optimal tax schedule is essential in generating the results. If the social planner lacks commitment, he will, at the beginning of each period, reoptimize, and choose the marginal income tax rate according to the first period formula $\theta h_t l_t^{\nu} - 1 = (1 + \nu^{-1})X$. Thus, the marginal income tax rates will be constant over time. They will, however, be accompanied by a lower investment in human capital, relative to the case of observable human capital.

1.3.2 Example II

If Assumption (2) holds and the utility is not additively separable in labor and schooling, I show in **?** for the case of $J = 3$ that

$$\tau_1(\theta) > \tau_2(\theta) > \tau_3(\theta) > 0.$$

[3]One can show that the marginal social benefit from additional one percent of schooling is $\theta h_t l_t^{-\nu^{-1}} - 1 > 0$. It follows from the agent's Euler equation that a one percent increase in labor supply in period $j \geq 2$ increases schooling by $\frac{1+\nu^{-1}}{1+\epsilon^{-1}}$ percent in period $j - 1$. Taking this into account, the social planner will optimally set $(1 + \frac{1+\nu^{-1}}{1+\epsilon^{-1}})(\theta h_t l_t^{-\nu^{-1}} - 1) = (1 + \nu^{-1})X$. Rearranging, one gets the optimality condition.

The strictly decreasing pattern of the marginal income taxes is a result of two factors, changes in the substitutability and complementarity patterns between labor supply and schooling over time, and changes in the labor supply elasticity over time. In example I only the second effect was present. The first effect works in the following way: initial period labor supply is a substitute for initial period schooling, last period labor supply is a complement to schooling in the intermediate period, and labor supply in the intermediate period is a substitute to schooling in the intermediate period, and a complement to schooling in the initial period. Relative to what the social planner would choose in the absence of human capital considerations (conditional on the labor supply elasticity), taxes in the last period are therefore lower (because of the complementarity of the last period labor supply with intermediate period schooling). Similarly, taxes in the initial period are higher (because of substitutability between initial period labor supply and initial period schooling). Both substitutability and complementarity effects the marginal income taxes in the middle period. The changes in labor supply elasticity work in the opposite direction, since labor supply elasticity decreases over time. The proof shows that this effect is not strong enough to revert the decreasing pattern of taxes over time. Numerical simulations show that the same conclusion holds for an infinite horizon economy with Ben-Porath human capital production function, see **?**. It is, however, possible, that this could be reverted in a finite horizon with more general human capital production function. Since the importance of human capital accumulation diminishes at the end of the life-cycle, it is possible that the elasticity effect could then dominate and one would have:

Conjecture 4 *In a more general life-cycle economy with J finite the intratemporal wedge could be U-shaped in age.*

2 A Stochastic Economy

Consider now an economy that is identical to the economy in the previous section, with one exception. Human capital next period h_{j+1} depends, in addition to current human capital

h_j, and on current schooling s_j, on idiosyncratic human capital depreciation shock z_j:

$$h_{j+1} = e^{z_{j+1}} F(h_j, s_j). \tag{12}$$

The idiosyncratic human capital shock is serially uncorrelated, but its density can depend on age j. As is standard in the moral hazard literature, it is useful to transform the state-space representation of the problem to work directly with the distribution induced over h_j. To that end, we construct a probability density function of human capital in period $j + 1$ conditional on $f_j = F(h_j, s_j)$, and denote it by $p_{j+1}(h_{j+1}|f_j)$. We also construct a probability density function of a sequence of human capital shocks $h^j = (h_1, \ldots, h_j)$ for a given history of schooling choices s^{j-1} and initial human capital h_1. It is given by

$$P^j(h^j|h_1, s^{j-1}) = p_2(h_2|F(h_1, s_1)) \ldots p_j(h_j|F(h_{j-1}, s_{j-1})) \quad j = 1, \ldots, J.$$

This economy is identical to **?**, with two exceptions. First, this model includes leisure. That is essential for thinking about optimal taxation. Second, the ability θ affects earnings directly, rather than indirectly through the human capital production function. That is irrelevant in an incomplete markets economy studied by **?** if the human capital production function takes the Ben-Porath form:

$$F(h, s) = h + (hs)^\alpha. \tag{13}$$

2.1 Informational Structure

Agent's abilities, as well as her labor supply, are private information of the agent, and are not observed by the social planner. Consumption and output are, on the other hand, both publicly observable. In the next section, human capital is assumed to be observable, but both schooling effort and rates of return on human capital are unobservable. I then provide some remarks regarding the case when human capital is unobservable as well.

2.2 Observable Human Capital, Unobservable Schooling Effort

The optimal tax problem now combines a standard Mirrleesean private information friction arising from unobservability of individual abilities with a moral hazard friction arising from unobservability of schooling effort.

Let $s_j(h^j)$ be schooling in period j after a history of human capital realizations h^j and let $s = \{s_j(h^j)\}_{j=1}^J$ be an arbitrary state contingent schooling plan. Define the utility of type θ agent who reports $\hat{\theta}$ and chooses schooling plan s by

$$W_{c,y,s}(\hat{\theta}|\theta, h_1) = \sum_{j=1}^{J} \beta^{j-1} \int_{H^{j-1}} \left[U(c_j(\hat{\theta}, h^j)) - V\left(\frac{y_j(\hat{\theta}, h^j)}{ah_j}, s_j(h^j) \right) \right] P^j \left(h^j | h_1, s^{j-1}(h^{j-1}) \right) \, dh^j,$$

where the allocation is $(c, y, s) = c\{c_j(\theta, h^j), y_j(\theta, h^j), s_j(\theta, h^j)\}_{j=1}^J$ is chosen by the social planner. Incentive compatibility requires that the agent prefers to tell the truth about her ability and that the schooling choice maximizes his utility:

$$s = \arg\max_{\hat{s}} W_{c,y,\hat{s}}(\theta|\theta, h_1) \tag{14}$$

$$W_{c,y,s}(\theta|\theta, h_1) \geq \max_{\hat{s}} W_{c,y,\hat{s}}(\hat{\theta}|\theta, h_1) \quad \forall \theta, \hat{\theta} \in \Theta \forall h_1 \in H \tag{15}$$

Necessary conditions for incentive compatibility are, again, of two types. First, an envelope condition, saying how the lifetime utility needs to vary with ability in order to deter the agent from misreporting his type:

$$W_{c,y,s}(\theta|\theta, h_1) = W_{c,y,s}(\underline{\theta}|\underline{\theta}, h_1) + \int_{\underline{\theta}}^{\theta} \sum_{j=1}^{J} \beta^{j-1} \int_{H^{j-1}} \left[V_l \left(\frac{y_j(\varepsilon, h^j)}{\varepsilon h_j}, s_j(\varepsilon, h^j) \right) \frac{y_j(\varepsilon, h^j)}{\varepsilon h_j} P^j(h^j | h_1, s^{j-1}(\varepsilon, h^{j-1})) \, dh^j \right] \frac{d\varepsilon}{\varepsilon}. \tag{16}$$

The second one is the first-order condition in schooling and says that, at the optimum, the marginal costs of schooling (given by the disutility from spending an additional unit of time by schooling) must be equal to the expected marginal benefit of schooling (given by the additional utility arising from the fact that the distribution of future human capital shocks is now more favorable):

$$V_s\left(\frac{y_j(\theta,h^j)}{\theta h_j}, s_j(\theta,h^j)\right) = \sum_{i=1}^{J-j}\beta^i \int_{H^i}\left[U(c_{j+i}(\theta,h^j,\zeta^i)) - V\left(\frac{y_{j+i}(\theta,h^j,\zeta^i)}{\theta\zeta_i}, s_{j+i}(\theta,h^j,\zeta^i)\right)\right] \times \frac{P_{s_j}{}^{j+i}\left(h^j,\zeta^i|h_1, s^{j+i-1}(\theta,h^j,\zeta^{i-1})\right)}{P^j\left(h^j|h_1, s^{j-1}(\theta,h^{j-1})\right)} d\zeta^i \tag{17}$$

for all histories $h^j \in H^j$. This constraint is a generalization of condition (11).

2.3 Example I

I now return to Example 1 with one exception: I assume that $U(c)$ is strictly concave. It is easy to see that, due to the moral hazard friction, the Inverse Euler Equation holds. In addition, we have the following sharp characterization of the intratemporal wedges, which is proven in **?**:

Proposition 5 *Suppose that* $V(l,s) = \frac{l^{1+\gamma}}{1+\gamma} + g(s)$. *Then*

$$\frac{1}{\tau_j(\theta,h^j)} = \int_H \frac{1}{\tau_{j+1}(\theta,h^{j+1})} p_{j+1}\left(h_{j+1}|F(h_j, s_j(\theta,h^j))\right) dh_{j+1}.$$

The result is due to several facts. First, the tax revenue of an $\theta-$type agent is proportional to $\frac{\tau_j(\theta,h^j)}{1-\tau_j(\theta,h^j)}$ (**?**). Second, if the assumptions of Proposition 2 hold then (since the ability shock is permanent) the social planner wants to keep the tax revenue valued at the utility cost $\frac{1}{U'(c_j(\theta,h^j))}$ constant over time and state. Hence the expression $\frac{1}{U'(c_j(\theta,h^j))}\frac{\tau_j(\theta,h^j)}{1-\tau_j(\theta,h^j)}$ is constant over time and state. Since $\frac{1}{U'(c_j(\theta,h^j))}$ follows a random walk, the result follows. Jensen's inequality then implies that the average intratemporal wedge is increasing over time,

$$\tau_j(\theta,h^j) < \int_H \tau_{j+1}(\theta,h^{j+1}) p_{j+1}\left(h_{j+1}|F(h_j, s_j(\theta,h^j))\right) dh_{j+1}.$$

and so the intratemporal wedge is on average increasing over time. Second , I conjecture that since the intratemporal wedge is between zero and one, in an infinite horizon economy the intratemporal wedge will converge to one.

Finally, the no distortion at the top result holds for this economy as well.

2.4 Unobservable Human Capital, Unobservable Schooling Effort

This is, the hardest problem to be solved, as human capital is now a persistent hidden variable with a continuous support. The results exist only for very special cases. In particular, **?** study a case when investment in human capital is made only in the initial period, and is subject to stochastic depreciation shocks later on. In addition, leisure is fixed in the initial period, the depreciation shocks can take only two values, and the low value is absorbing.[4] Relative to the case with observable human capital, they show the following results.

First, unlike in all the previous results, the no distortion at the top does not hold. People with the highest skill level face a *negative* marginal income tax rate. The intuition follows from the nature of incentive constraints in their model: the binding incentive constraints are those that prevent, in any period, the high human capital agents from pretending to have low human capital. Since the low human capital level is absorbing, this cane be done only onece over one's lifetime. If an agent plans to deviate in period j (i.e. incorrectly report low human capital), he will invest less in human capital initially at time zero. But lower human capital also means that the deviating agent will have to work more than the truthtelling high human capital agent in all the periods before $1, \ldots, j-1$. Since the disutility from working is increasing, encouraging high human capital agents to work more is especially harmful to the deviating agents. It therefore helps to relax the incentive constraints. Encouraging the truthtelling high human capital agent is done through negative intratemporal wedge.

Second, consumption becomes more front-loaded than in the case of observable human capital. This is again designed to relax the incentive constrains. If a deviating agent in-

[4]More specifically, the initial investment in human capital s_0 yields either human capital $h_1 = 0$ or $h_1 = 1$. In all subsequent periods, the human capital is either $h_{j+1} = h_j$ or $h_{j+1} = 0$. The second case is absorbing.

vests less in human capital, he will enjoy more consumption in the initial period than the truthtelling agent. Since the marginal utility of consumption is decreasing, the benefit will be lower if the initial consumption is higher to start with.

It is not obvious to me if those results will extend to more general environments. Especially the first result seems to rely heavily on the assumption that there are only two human capital levels and the low state is absorbing. If there were more human capital levels, the deviating agent who has invested less in human capital initially would have more options to misreport his type, and would not necessarily need to replicate the thruthtelling high human capital agent's behavior. As a result, the allocations of the thruthtelling high human capital agent would continue being undistorted.

3 Conclusions

More work needed...

References