Optimal Taxation in a Life-Cycle Economy with Endogenous Human Capital Formation: A Review

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Markets Group, Chicago
June 5, 2012
Introduction

This presentation: Overview of selected results on dynamic optimal taxation in an environment where
- Human capital is endogenous
- Individual’s abilities are unobservable and permanent

I will cover several cases:
- Human capital is observable and deterministic
- Human capital is unobservable and deterministic
- Human capital is observable and has unobservable stochastic returns
- Human capital is unobservable and has unobservable stochastic returns

I will focus on the case when the cost of accumulating human capital is time, not physical resources

This presentation will focus on qualitative results. However, one goal of this research is to allow for quantitative characterization of the optimal tax policies
0. Static Optimal Taxation

- A standard static optimal taxation model (Mirrlees 1971):
  - Question: How should a government design a tax system that maximizes some utilitarian social welfare function?
  - Individuals have abilities $\theta \in \Theta$
  - Abilities are private information: The government only knows their distribution $Q(\theta)$
  - People choose labor supply $l$ and consumption $c$. Income is $y = \theta l$.
  - $c$ and $y$ is observable, $l$ and $\theta$ is not.
  - Taxation principle: Anything that can be achieved by an income tax function can be achieved in a direct revelation mechanism with incentive constraints
0. Static Optimal Taxation

- The planner’s problem:
  - Maximize some social welfare function by choosing \{c(θ), y(θ)\}
  - subject to an incentive constraint

\[
U(c(θ)) - V\left(\frac{y(θ)}{θ}\right) \geq U(c(\hat{θ})) - V\left(\frac{y(\hat{θ})}{\hat{θ}}\right),
\]

- and a resource constraint

\[
\int_Θ [c(θ) - θl(θ)]dQ(θ) \leq 0.
\]

- Replace the incentive constraint by an envelope condition:

\[
U(c(θ)) - V(l(θ)) = w_0 + \int_θ^Θ V(l(ε))l(ε)\frac{dε}{ε},
\]
0. Static Optimal Taxation

Diamond (1998) shows that with $U(c) = c$ the optimal intratemporal wedge $\tau(\theta) = 1 - \frac{V'(l)}{\theta}$ satisfies

$$\frac{\tau(\theta)}{1 - \tau(\theta)} = (1 + \nu^{-1})X(\theta)$$

where $\nu$ is the elasticity of labor supply, $X(\theta)$ is given by

$$X(\theta) = \frac{1 - Q(\theta)}{\theta q(\theta)} C(\theta)$$

and $C(\theta)$ depends on the social welfare function.

For instance, if the planner is Rawlsian then $C(\theta) = 1$.

If the shock support is finite, $X(\overline{\theta}) = 0$ (no distortion at the top)
1. Deterministic Human Capital

- Agents live for \( J > 1 \) periods. They consume, work, and learn.
- Preferences:

\[
\sum_{j=1}^{J} \beta^{j-1} [U(c_j) - V(l_j, s_j)], \quad 0 < \beta < 1,
\]  

(1)

- Labor Earnings:

\[
y_j = \Theta h_j l_j
\]  

(2)

- Human capital formation:

\[
h_{j+1} = F(h_j, s_j)
\]  

(3)

- \( c \) is consumption, \( l \) is labor, \( s \) is schooling effort, \( h \) is beginning of period human capital.
- \( c \) and \( y \) are always observable. \( \Theta \) is always unobservable.
1.1. Deterministic and Observable Human Capital

- Allocation: \((c, y, h) = \{c_j(\theta), y_j(\theta), h_{j+1}(\theta)\}_{j=1}^J\)

- Lifetime utility:
  \[
  W_{y,c,h}(\hat{\theta}|\theta, h_1) = \sum_{j=1}^J \beta^{j-1} \left[ U(c_j(\hat{\theta})) - V \left( \frac{y_j(\hat{\theta})}{\theta h_j(\hat{\theta})}, S(h_j(\hat{\theta}), h_{j+1}(\hat{\theta})) \right) \right]
  \]

- Incentive compatibility:
  \[
  W_{y,c,h}(\theta|\theta, h_1) \geq W_{y,c,h}(\hat{\theta}|\theta, h_1) \quad \forall \hat{\theta} \in \Theta. \tag{4}
  \]

- Resource constraint
1.1. Deterministic and Observable Human Capital

- Replace the incentive constraint by an envelope condition:

\[ W_{y,c,h}(\theta|\theta, h_1) = \int_0^\theta \sum_{j=0}^J \beta^{t-1} V_{l,j}(\varepsilon) l_j(\varepsilon) \frac{d\varepsilon}{\varepsilon} + W_{y,c,h}(\theta|\theta, h_1), \tag{5} \]

where \( V_{l,j}(\theta) = V_l(l_j(\theta), s_j(\theta)) \)

- Characterize the optimum by the following:
  - The *intratemporal wedge*

\[ \tau_j \equiv 1 - \frac{V_l(l_j, s_j)}{\theta h_j U_c(c_j)} \]

  - The *human capital wedge*

\[ \Delta_j \equiv \frac{V_{s,j}}{F_{s,j}} - \beta \left( V_{l,j+1} \frac{l_{j+1}}{h_{j+1}} + V_{s,j+1} \frac{F_{h,j+1}}{F_{s,j+1}} \right) \tag{6} \]
1.1. Deterministic and Observable Human Capital

**Example 1**

**Assumption (Example 1)**

\[ V(l, s) = \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}} + \frac{s^{1+\epsilon^{-1}}}{1+\epsilon^{-1}}, \quad U(c) = c \quad and \quad F(h, s) = s. \]

- The intratemporal wedge

\[ \tau_j(\theta) \left(1 - \tau_j(\theta) \right) = \left(1 + \nu^{-1}\right)X(\theta) \quad (7) \]

- The intertemporal wedge

\[ \Delta_j(\theta) = \beta(1 + \nu^{-1})X(\theta)\frac{l^{j+1}(\theta)^{1+\nu^{-1}}}{s_j(\theta)} \geq 0. \]

- Main implications:
  - Intratemporal wedge is the same as in the static model
  - Schooling subsidies provided to encourage investment in human capital
1.1. Deterministic and Observable Human Capital

Example 1

- Separable utility drives the constant intratemporal wedge

- Human capital wedge corrects for the fact that private benefits from investment in human capital are smaller than social benefits from investment in human capital

- DaCosta and Maestri (2007) show that the result is modified if human capital can help to separate people of different skills
1.1. Deterministic and Observable Human Capital

Assumption (Example 2)

\[ V(l, s) = \frac{(l + s)^{1 + \nu} - 1}{1 + \nu - 1}, \quad U(c) = c \text{ and } F(h, s) = s. \]

- The intratemporal wedge

\[
\frac{\tau_j(\theta)}{1 - \tau_j(\theta)} = (1 + \left[\nu(1 + \frac{s_j(\theta)}{l_j(\theta)})^{-1}\right]X(\theta) \tag{8}
\]

- The elasticity term is now endogenous

- Higher schooling to labor ratio decreases the intratemporal wedge
  - Makes the tax system initially more regressive relative to a static economy
  - Intratemporal wedge should be increasing over the life-cycle
1.2. Deterministic and Unobservable Human Capital

The incentive compatibility constraint:

\[ h = \arg \max_{\hat{h}} W_{c,y,\hat{h}}(\theta|\theta, h_1) \]  \hspace{1cm} (9)

\[ W_{c,y,h}(\theta|\theta, h_1) \geq \max_{\hat{h}} W_{c,y,\hat{h}}(\theta|\theta, h_1) \quad \forall \hat{\theta} \in \Theta. \]  \hspace{1cm} (10)

Necessary conditions for incentive compatibility:

- Envelope condition (the same as before)
- Euler equation in HC investment:

\[ \frac{V_{s,j}}{F_{s,j}} = \beta \left( V_{l,j+1} \frac{l_{j+1}}{h_{j+1}} + V_{s,j+1} \frac{F_{h,j+1}}{F_{s,j+1}} \right) \]  \hspace{1cm} (11)

Incentives to accumulate human capital must now be provided differently.
1.2. Deterministic and Unobservable Human Capital

Example 1 cont’d

Assumption (Example 1)

\[ V(l, s) = \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}} + \frac{s^{1+\epsilon^{-1}}}{1+\epsilon^{-1}},\ U(c) = c \text{ and } F(h, s) = s. \]

- The intratemporal wedge is given by
  \[ \frac{\tau_1(\theta)}{1 - \tau_1(\theta)} = (1 + \nu^{-1})X(\theta) \]
  \[ \frac{\tau_j(\theta)}{1 - \tau_j(\theta)} = (1 + \Psi^{-1})X(\theta), \quad j = 2 \ldots J, \]

  where \( \Psi = \frac{2+\nu^{-1}+\epsilon^{-1}}{\nu^{-1}\epsilon^{-1}-1-1} > \nu. \)

- Lower wedge for \( j \geq 2 \): labor supply is complementary with previous human capital investment
1.2. Deterministic and Unobservable Human Capital

Example 2 cont’d

Assumption (Example 2)

\[ V(l, s) = \frac{(l+s)^{1+\nu} - 1}{1+\nu} \]

\[ U(c) = c \] and \[ F(h, s) = s. \]

- For \( J = 3 \) one can show that the intratemporal wedge satisfies

\[ \tau_1(\theta) > \tau_2(\theta) > \tau_3(\theta) > 0. \]

- Complementarity of current schooling with future labor and substitutability of current schooling with current labor imply decreasing wedge

- Changes in labor elasticity imply increasing wedge

- The first effect dominates
Bohacek/Kapicka (2007): A calibrated infinite horizon economy

\[ h' = (1 - \delta)h + (hs)^\alpha \]
2. Stochastic Human Capital

- Human capital accumulation

\[ h_{j+1} = e^{z_{j+1}}(h_j + (h_j s_j)^\alpha), \quad z_j \sim i.i.d. \]

- \( z \) and \( s \) are both private information

- Private information from unobservability of individual abilities and labor

- Moral hazard problem from unobservable schooling effort and rates of return

- Can be mapped into Huggett, Ventura, Yaron (AER, 2011)
2.1. Stochastic and Observable Human Capital

- Social planner chooses \( (c, y, s) = \{c_j(\theta, h^j), y_j(\theta, h^j), s_j(\theta, h^j)\} \)
- Lifetime utility

\[
W_{c,y,s}(\hat{\theta}|\theta, h_1) = \sum_{j=1}^{J} \beta^{j-1} \int_{H_{j-1}} \left[ U(c_j(\hat{\theta}, h^j)) - V \left( \frac{y_j(\hat{\theta}, h^j)}{ah^j}, s_j(h^j) \right) \right] P^j(h^j|h_1, s^{j-1}(h^{j-1})) dh^j
\]

- Incentive compatibility

\[
s = \arg \max_{\hat{s}} W_{c,y,\hat{s}}(\theta|\theta, h_1) \quad \text{(12)}
\]

\[
W_{c,y,s}(\theta|\theta, h_1) \geq \max_{\hat{s}} W_{c,y,\hat{s}}(\hat{\theta}|\theta, h_1) \quad \forall \theta, \hat{\theta} \in \Theta \forall h_1 \in H \quad \text{(13)}
\]
2.1. Stochastic and Observable Human Capital

Relaxed Planning Problem in a 2 period version

Assume 2 periods, expected utility maximization, and let $W(\theta) = W_{c,y,s}(\theta|\theta, h_1)$

$$\max \int_\Theta W(\theta) q(\theta) d\theta$$

subject to

$$\int_\Theta \left[ c_1(\theta) - y_1(\theta) + R^{-1} \int_H [c_2(\theta, h_2) - y_2(\theta, h_2)] P(h_2|s_1) dh_2 \right] q(\theta) d\theta \leq 0.$$

$$V_s \left( \frac{y_1(\theta)}{h_1 \theta}, s_1 \right) = \beta \int_h V_{\ell} \left( \frac{y_2(\theta, h_2)}{\theta h_2}, 0 \right) P_s(h_2|s_1) dh_2$$

$$W(\theta) = W(\theta) + \int_\Theta \left[ V_{\ell} \left( \frac{y_1(\varepsilon)}{\varepsilon h_1}, s_1(\varepsilon) \right) \frac{y_1(\varepsilon)}{\varepsilon h_1} + \beta \int_h V_{\ell} \left( \frac{y_2(\varepsilon, h_2)}{\varepsilon h_2}, 0 \right) \frac{y_2(\varepsilon, h_2)}{\varepsilon h_2} P(h_2|s_1(\varepsilon)) dh_2 \right] \frac{d\varepsilon}{\varepsilon}$$
2.1. Stochastic and Observable Human Capital

Inverse Euler Equation

**Proposition**

\[
\frac{1}{U'(c_1(\theta))} = \int_H \frac{1}{U'(c_2(\theta, h_2))} P(h_2|s_1(\theta)) \, dh_2
\]

*by Jensen’s Inequality,*

\[
U'(c_1(\theta)) < \int_H U'(c_2(\theta, h_2)) P(h_2|s_1(\theta)) \, dh_2
\]
2.1. Stochastic Observable Human Capital

Example 1 cont’d

**Proposition**

Suppose that \( V(l, s) = \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}} + \frac{s^{1+\epsilon^{-1}}}{1+\epsilon^{-1}}. \) Then

\[
\frac{1}{\tau_1(\theta)} = \int_H \frac{1}{\tau_2(\theta, h_2)} P(h_2|s_1(\theta)) \, dh_2.
\]

by Jensen’s Inequality,

\[
\tau_1(\theta) < \int_H \tau_2(\theta, h_2) P(h_2|s_1(\theta)) \, dh_2
\]

Intratemporal wedge increases over time!
Intratemporal Wedges

**Figure:** Intratemporal Wedges
Figure: Intratemporal Wedge in the Second Period
2.2. Stochastic Unobservable Human Capital

- The most complicated: large number of deviations
- Grochulski and Piskorski (2007): solution for a special case:
  - investment in human capital only in the initial period
  - no labor supply in the initial period
  - depreciation shocks take only two values $e^z \in \{0, 1\}$
  - the low shock is absorbing
2.2. Stochastic Unobservable Human Capital

- Two novel results:
  - No distortion at the top does not apply: negative intratemporal wedge at the top
  - Stronger front-loading of consumption

- Both features relax the incentive constraints

- Not obvious if the first result survives in an economy with e.g. continuum of shocks

- The second result is similar to the higher tax in the initial period
3. Conclusions

- Main results:
- Corrective schooling subsidies whenever schooling effort is observable
- If not possible, use intertemporal variations in taxes to provide incentives to accumulate human capital
- The exact pattern depends on
  - substitutability/complementarity of human capital and schooling
  - changes in labor supply elasticity
  - riskiness of human capital investments
- More generally, one of the important questions is if human capital investments are useful in separating people of different skills