These lectures are designed to provide an overview of the study of intergenerational mobility.

I will focus on three topics:

1. Measurement
2. Theory
3. Normative implications
Within economics, the empirical analysis of intergenerational mobility has given particular prominence to the intergenerational elasticity (IGE) which relates the logarithm of parent’s income (or wages or any other socioeconomic variable of interest) to the corresponding value for a child.
The IGE is calculated using pairs \((Y_{i,p}, Y_{i,o})\) in which, for family \(i\). Let \(X_i\) denote any control variables the analyst wishes to include such as the ages of the parent and offspring. The IGE corresponds to the coefficient \(\rho\) in the regression

\[
\log Y_{i,o} = \alpha + \rho \log Y_{i,p} + \beta X_i + \varepsilon_{i,o}
\]

Note that \(\varepsilon_{i,o}\), the regression residual, captures unobserved heterogeneity that is uncorrelated with either the parent’s income or the control variables.
For clarity, I will set $X_i = 0$ in the subsequent discussion. The reason for introducing it was to signal that one needs to think about the location of parents and offspring in their respective life cycles, but the vector is not germane to the subsequent discussion and so its omission simply makes the algebra simpler.

Estimates for the US and China have been increasing over time. For the US, the interpretation of increasing estimates is prosaic and concerns what is meant by the log income of the parent and the offspring. US measures are now regarded as approximately .6 (Mazumder (2005)) to .44 (Fan, Yi, and Zhang (2013)).
Specifically, the US literature has been concerned with intergenerational relations between persistent components of income. To formalize this intuition, economists distinguish between permanent and transitory income.
The simplest way to understand this concept is to assume that for parent $i$, income at time $t$ divides into

$$\log Y_{i,p,t} = \log Y^p_{i,p} + Y^T_{i,p,t}$$

and that for an offspring, the same division applies:

$$\log Y_{i,o,t} = \log Y^p_{i,o} + \log Y^T_{i,o,t}$$

I will assume that $\log Y^T_{i,p,t}$ and $\log Y^T_{i,o,t}$ are uncorrelated across time and across individuals, have expected value 0, and possess constant variances.
• Why is the permanent and transitory income distinct so important?

• The basic idea is that transitory income reflects luck and is not something affected by parents; similarly a parent’s luck is regarded as having second order importance in terms of the effect of the luck on children.
Formally, suppose that the intergenerational transmission mechanism involves permanent income. Notice I have shifted from a statistical to a behavioral model.

$$\log Y_{i,o}^P = \alpha + \rho \log Y_{i,p}^P + \varepsilon_{i,o}$$

Without loss of generality, assume that $\log Y_{i,p}^P$ and $\varepsilon_{i,o}$ are independent. This might be justified if permanent income is determined by education and a parent’s permanent income determines an offspring’s education. But this is a story, not a formal model of how parents affect their children, which is in fact one of the most important gaps in mobility research.
The difficulty with a permanent income model is obvious: it is not observable. As a result, one is forced to construct proxies for permanent income. One way to do this is to simply employ the realized income of parents and offspring at times $t$ and $t+K$. Hence one runs the cross-section (i.e. across $i$) regression

$$\log Y_{i,o,t+k} = \alpha + \beta \log Y_{i,p,t} + \eta_{i,o,t+k}$$
How does this regression relate to the “true” permanent income model?

The permanent income model can be rewritten

$$\log Y_{i,o,t+k} - \log Y_{i,o,t+k}^T = \alpha + \rho \left( \log Y_{i,p,t} - \log Y_{i,p,t+k}^T \right) + \varepsilon_{i,o} \Rightarrow$$

$$\log Y_{i,o,t+k} = \alpha + \rho \log Y_{i,p,t} + \varepsilon_{i,o} + \log Y_{i,o,t+k}^T - \rho \log Y_{i,p,t+k}^T$$

The regression error

$$\eta_{i,o,t} = \varepsilon_{i,o} + \log Y_{i,o,t+k}^T - \log Y_{i,p,t+k}^T$$

is correlated with the regressor in the standard IGE regression.
The projection parameter $\beta$ is related to the behavioral parameter $\rho$ by

$$
\beta = \rho \left( \frac{\text{var}(Y_P^p)}{\text{var}(Y_P^p) + \text{var}(Y_T^p)} \right) < \rho.
$$

which reveals the standard result on the downward bias associated with measurement error.
• The recognition that transitory income can create inconsistent estimates of the IGE that generated the subsequent literature that attempted to address this problem. This has led researchers to conclude that the IGE is substantially higher than previously believed, .2 versus .6.

• How is the presence of transitory income addressed? The trick that has been employed by a number of authors is based on the idea of replacing one year’s income with a multi-year average.
In other words, the empirical analysis is conducted with averaged incomes for parents and offspring,

\[
\log \bar{Y}_{i,p,t} = \frac{1}{L} \sum_{l=0}^{L-1} \log Y_{i,p,t-l}
\]

and

\[
\log \bar{Y}_{i,o,t} = \frac{1}{L} \sum_{l=0}^{L-1} Y_{i,o,t-l}
\]

respectively.
How does this help? Note that, under the various assumptions we have made

\[ \bar{Y}_{i,p,t} = Y_{i,o}^P + \frac{1}{L} \sum_{l=0}^{L-1} Y_{i,o,t-l} \]

Hence the measurement error for permanent income \( \frac{1}{L} \sum_{l=0}^{L-1} Y_{i,o,t-l} \) has variance \( \frac{1}{L^2} \sigma_{Y_p}^2 \). Notice that the fast diminution of the measurement error would be attenuated if the transitory income terms were correlated.
An Alternative Perspective on the IGE

There is another way of thinking about the IGE. Suppose one rewrote the behavioral model of the evolution of permanent income as

\[
\log Y_{i,o}^P - \log y_{i,p}^P = \alpha + (\rho - 1)\log Y_{i,p}^P + \varepsilon_{i,o}
\]

If \( \beta < 1 \), then \( \rho < 0 \). In words, the gap between offspring and parent log permanent income is negatively related to the parent’s income. This means that, in expectation, higher income families will experience decreases in income between generations when compared to low income ones.
Why is this important?

Suppose we consider two separate families, $i$ and $j$. Suppose for a pair of contemporary parents, $\log Y_{i,p} > \log Y_{j,p}$.

If $\rho < 0$, then in expectation, $\log Y_{i,o}^P - \log Y_{j,o}^P$ will be smaller than $\log Y_{i,p}^P - \log Y_{j,p}^P$. This means that contemporary inequality between families will, in expected value, diminish across time. One can show, for this linear structure, that eventually any contemporary differences will disappear, so long as $\alpha$ is common across families.

For this reason, $\rho$ is known as the *convergence* coefficient.
Focus on $\rho$ rather than $\alpha$ is odd. The latter involves permanent inequality.

Roberts (2013) show how ignoring regional heterogeneity can lead to spuriously high estimates of the IGE for the United States.

Bernard and Durlauf (1996) show that linear assumption can mask poverty traps.
Suppose that the true behavioral model is

$$\log Y_{i,o}^P = \bar{\alpha} + \alpha_i + \rho \log Y_{i,p}^P + \varepsilon_{i,o}$$

Interpret $\alpha_i$ as indexing steady states. May represent poverty and affluence traps, i.e. indexed by some income, ethnic, etc. measure.
The statistical IGE will equal

$$\beta = \rho + \frac{\text{cov}(\alpha_i, \log Y_{i,p})}{\text{var}(\log Y_{i,p})}$$

Sign can still be positive.

In fact, $\beta > \rho$ is possible.
Work by Chetty uses ranks to measure mobility. Idea is that one regresses percentile of child against parent.

\[ R_{i,o} = \alpha + \beta \log R_{i,p} + \varepsilon_{i,o} \]

Argued to better address issue of zeroes in income.

Measures relative mobility. Different from other concepts.
Theory
• Becker and Tomes (1993) and Loury (1989) are the sources of what may be regarded as the classical intergenerational mobility models.

• These models focus on income as the outcomes whose intergenerational persistence is of interest.

• Substantively, the model treats the transmission process as intrafamily. Parents invest in the human capital of their offspring.
I distinct family dynasties, denoted by $i$.

Individuals are assumed to live 2 periods. The first period is childhood, denoted as $c$ and the second period is adulthood, denoted as $a$. $t$ denotes time of birth.

Individuals reproduce asexually; this allows us to ignore issues of intermarriage across dynasties and is inessential for the purposes of these notes.
In childhood, an individual in dynasty $i$ born at time $t$ makes no choices but receives a human capital investment $l_{i,c,t}$ from her parent.

This investment produces a level of education attainment for the child $E_{i,c,t}$. 
Realized education is determined by the process

\[ E_{i,c,t} = e(I_{i,c,t}, \zeta_{i,c,t}) \]

\( \zeta_{i,c,t} \) denotes unobserved heterogeneity across children; can think of as latent ability. Treat as observable to parent. It is natural to allow it to be correlated within families for both genetic and environmental reasons.

Assume that \( \zeta_{i,c,t} \) identically distributed across all agents with probability measure \( \mu_\zeta(\cdot) \). Assume that parents observe \( \zeta_{i,c,t} \) when they make their investment decisions; this assumption is nontrivial.
As an adult, this same individual $i$ born at $t$ works and receives income $Y_{i,a,t+1}$; note that the time index has changed as we have moved from childhood to adulthood. Income is determined by the process

$$\log Y_{i,a,t+1} = f\left(E_{i,c,t}, \varepsilon_{i,a,t+1}\right)$$

In this equation, $\varepsilon_{i,a,t+1}$ represents unobserved heterogeneity associated with adults; it is assumed to be identically distributed across agents with probability measure $\mu_{\varepsilon}(\cdot)$. Given unobserved childhood heterogeneity, perhaps $\varepsilon_{i,a,t+1}$ is most naturally interpreted as labor market luck. I impose additive separability of the observable and unobservable heterogeneity. This is done to mimic the way that the IGE regressions treat unobserved heterogeneity.
I place the following three assumptions on $\varepsilon_{i,a,t+1}$

\[
E(\varepsilon_{i,a,t+1}) = 0
\]

\[
\text{cov}(\varepsilon_{i,a,t} \varepsilon_{i,a,t'}) = 0 \text{ if } t \neq t'
\]

\[
\text{cov}(\varepsilon_{i,a,t} \varepsilon_{i',a,t'}) = 0 \text{ if } i \neq i'
\]

\[
\text{cov}(\varepsilon_{i,a,t} \zeta_{i',c,t'}) = 0 \quad \forall i, i', t, t'
\]

Substantive content is that parent has no information on $\varepsilon_{i,a,t+1}$. 
Where does choice appear in this model? Choice appears via parental investment decisions. Each adult splits her income between the consumption, $C_{i,a,t+1}$ and the human capital investment in the child $I_{i,c,t+1}$, i.e.

$$Y_{i,a,t+1} = C_{i,a,t+1} + I_{i,c,t+1}$$

This is more than an identity as it means that a parent cannot borrow to raise the human capital investment of her child. To do so would require that a parent can create a legal obligation for a child to pay her debts.
(Obviously, the parent will not be around to repay any loan!) Hence all investment in a child must come from the parent’s income.

This structure is sufficient to provide a description of intergenerational income transmission. Since each adult agent is solving an identical decision problem, if a solution exists for the optimal human capital investment for each value of $Y_{i,a,t+1}$ and $\zeta_{i,y,t+1}$ it must be the case that it can be expressed as

$$l_{i,c,t+1} = g\left(\log Y_{i,a,t+1}, \zeta_{i,c,t+1}\right)$$
One more substitution yields the law of motion for family income:

$$\log Y_{i,a,t+1} = f\left( e\left( g\left( \log Y_{i,a,t}, \zeta_{i,c,t} \right), \zeta_{i,c,t} \right), \varepsilon_{i,a,t+1} \right)$$

This equation provides a description of the evolution of income across generations.
The nature of the evolution will depend on the three functions $e(\cdot, \cdot)$, $f(\cdot, \cdot)$ and $g(\cdot, \cdot)$.

The first function $e(\cdot, \cdot)$ characterizes the relationship between human capital investment and education.

The second function, $f(\cdot, \cdot)$ characterizes how education is converted into income.

These two functions represent the technology of the economy.
The third function, \( g(\cdot, \cdot) \), will depend on the technology of the economy as well as the preferences of adults.

Notice that this general structure does not produce the regression that is the basis of the IGE literature.

To do so, one needs additional assumptions. I now provide functional form assumptions on primitives that generate the standard IGE equation.
First, assume that human capital is additive in investment and ability.

\[ E_{i,c,t} = \theta \log l_{i,c,t} + \zeta_{i,c,t} \]

As before, \( \zeta_{i,c,t} \) denotes unobserved ability heterogeneity. Recall that parents are assumed to know the value of \( \zeta_{i,c,t} \) when human capital investment decisions are made.
Second, income is determined by

\[ Y_{i,a,t+1} = \mu + w \log E_{i,c,t} + \epsilon_{i,a,t+1} \]

so that

\[ Y_{i,a,t+1} = \mu + w\theta \log E_{i,c,t} + w\zeta_{i,c,t} + \epsilon_{i,a,t+1} \]
Third, the utility of the adult $i$ born at time $t-1$ is

$$U_{i,a,t} = (1 - \pi) \log C_{i,a,t} + \pi \log Y_{i,a,t+1}$$
Equilibrium Law of Motion

\[ \log Y_{i,a,t+1} = \mu + \gamma \log \left( \frac{\pi \theta w}{1 - \pi (1 - \theta w)} \right) + \pi \log Y_{i,a,t-1} + w\zeta_{i,c,t+1} + \varepsilon_{i,a,t+1} \]
Note that it is not natural to assume that $\zeta_{i,c,t+1}$ is independent across time if there is a genetic component to ability, for example. Suppose that

$$
\zeta_{i,c,t+1} = \delta + \lambda \zeta_{i,c,t} + \nu_{i,a,t+1}
$$

where $\nu_{i,a,t+1}$ is uncorrelated, then for the regression

$$
\log Y_{i,a,t+1} = \alpha + \beta \log Y_{i,a,t-1} + \zeta_{i,a,t+1}
$$

$$
\beta = \frac{w\theta + \lambda}{1 + \lambda w\theta}
$$
• The frontier in understanding family and other influences on individuals moves away from the focus on income towards cognitive skills and personality traits which together represent a broad conception of skills.

• This work has been pioneered by James Heckman; the surveys Borghans, Duckworth, Heckman, and B. ter Weel (2008) and Almlund, Duckworth, Heckman, and Kautz (2011) are comprehensive surveys.
• Focus is to a large extent on family, but other factors are naturally included.

• Early childhood investment is one obvious example.

• Note that the objects of interest in intergenerational mobility are now changed.
• The new economics of skills has two critical features.

• First, it employs a broad definition of skills. In particular, it differentiates between cognitive and noncognitive skills. In this respect, the economics of skills has followed the psychology literature, in which intelligence and personality studied as distinct aspects of the mind.

• Many psychologists dislike the term “noncognitive” skills since these skills are also part of the mind, and so in their view are cognitive. Nevertheless, I will follow the language of economists.
• Second, the literature focuses on development across the childhood and adolescence.

• It therefore directly challenges the 2-period overlapping generations paradigm.
• Cognitive skills are those that one associates with intelligence and human capital. Both of these components, in turn, may be understood as possessing subcomponents.

• *Crystal* intelligence refers to the ability of an individual to draw on his knowledge and experience to solve recurring problems.

• *Fluid* intelligence refers to the ability to draw on knowledge and experience to solve novel problems.
Noncognitive skills may be thought of as personality traits. In the psychology literature, a standard way of conceptualizing personality is the so-called 5-factor model:

- Openness (which captures curiosity and receptivity to new situations), Conscientiousness (which includes whether one is well organized and/or efficient), Extraversion (which includes friendliness and whether one is high energy), Agreeableness which includes friendliness and compassion), and Neuroticism (which includes self-confidence and sensitivity to stress). The acronym for these is OCEAN.
From the perspective of understanding intergenerational mobility, the importance of this broad conception of skills is that an individual is associated with a vector of cognitive skills, $\theta^C_i$ and a vector of noncognitive skills $\theta^{NC}_i$ which evolve over childhood and adolescence.

While they do not stop at age 18, the family’s role can be understood as influencing the evolution during this time.
James Heckman has developed formal models of skill evolution in a large number of papers; in terms of notation I essentially follow Cunha and Heckman (2007) which is an especially accessible treatment; the main difference is that I index investments so that an investment at $t$ affects skills at $t$. 
The formal model of skill evolution, in abstract terms, is simply a difference equation that describes the values of $\theta_{i,t}^C$ and a vector of noncognitive skills $\theta_{i,t}^{NC}$ as the outcomes of dynamic processes; for $t > 0$

\[
\theta_{i,t}^C = f_t^C \left( \theta_{i,t-1}^C, \theta_{i,t-1}^{NC}, l_{i,t}, h_i \right)
\]

and

\[
\theta_{i,t}^{NC} = f_t^{NC} \left( \theta_{i,t-1}^C, \theta_{i,t-1}^{NC}, l_{i,t}, h_i \right)
\]
In these equations, $I_{i,t}$ denotes a vector that captures everything that affects individual $i$ at time $t$. Cunha and Heckman refer to these effects as investments, which corresponds to the idea that $I_{i,t}$ creates a change in the stock of skills.

Under this very broad definition, $I_{i,t}$ would therefore include everything from parental inputs to schooling to luck (e.g. whether one has been healthy during the time period). The term $h_i$ denotes a vector of initial conditions for $i$. Cunha and Heckman mention parental characteristics such as IQ and education as components of $h_i$. 
There are two things to observe about the dynamic processes.

First, notice that $\theta_{i,t}^C$ is a function of $\theta_{i,t-1}^{NC}$ and $\theta_{i,t}^{NC}$ is a function of $\theta_{i,t-1}^C$. This captures the idea that one type of skills facilitates the acquisition of the other type. It is easy to think of intuitive examples. A smart child who is not conscientious may not acquire much knowledge.

Second, the functions $f_t^C$ and $f_t^{NC}$ have time indices. This captures the idea that mapping from the arguments of the functions to skill levels can depend on age. This is one way, at least in terms of notation, to allow for the possibility that the plasticity of skills changes over childhood and adolescence. We have talked about the standard example of differential plasticity: language acquisition ability.
Cunha and Heckman show that if one engages in recursive substitution, to eliminate the dependence of $\theta_{i,t}^C$ and $\theta_{i,t}^{NC}$ on $\theta_{i,t-1}^C$ and $\theta_{i,t-1}^{NC}$, one can reformulate the dynamic skills models as

$$\theta_{i,t}^C = m_t^C (I_{i,t}, \ldots, I_{i,1}, h_i)$$

and

$$\theta_{i,t}^{NC} = m_t^{NC} (I_{i,t}, \ldots, I_{i,1}, h_i)$$

This formulation emphasizes the idea that the stocks of skills at $t$ are determined by initial conditions and the series of investments up to that time.
The properties of the investment are important in thinking about policy.

One such property is dynamic complementarity. Suppose that $I_{i,j,t}$ is an investment that the government can augment. If

$$\frac{\partial^2 f^{C}_t}{\partial \theta^C_{i,t-1} \partial I_{i,j,t}} > 0$$

$$\frac{\partial^2 f^{C}_t}{\partial \theta^{NC}_{i,t-1} \partial I_{i,j,t}} > 0$$

then skills exhibit dynamic complementarities in the sense that the marginal product of increasing $I_{i,j,t}$ is increasing in the stocks of skills.
It may be that the efficacy of later investments, e.g. high school, depends on skills acquired earlier in life. A related notion of dynamic complementarity is

\[
\frac{\partial^2 m^c_t (l_{i,t}, \ldots, l_{i,1}, h_i)}{\partial l_{i,j,t} \partial l_{i,t-k}} > 0, \ 0 < k \leq t - 1
\]

which involves complementarities across investments. This captures the idea that schooling investments at different ages are interrelated.
Cunha and Heckman also define the notion of critical periods of development.

Let $I_{i,-j,t}$ denote the vector of investments in $i$ at $t$ other than investment type $j$. 
A critical development period with respect to investment type $j$ is a time $T$ such that

$$\frac{\partial m_t^c(l_{i,j,t}, \overline{l}_{i,-j,t}, \overline{l}_{i,t-1}, \ldots, \overline{l}_{i,1}, \overline{h}_i)}{\partial l_{i,j,t}} \approx 0, \text{ if } t \neq T$$

and

$$\frac{\partial m_t^c(l_{i,j,t}, \overline{l}_{i,-j,t}, \overline{l}_{i,t-1}, \ldots, \overline{l}_{i,1}, \overline{h}_i)}{\partial l_{i,j,t}} = 0, \text{ if } t = T$$
• Notice that this is a distinct idea from that of dynamic complementarity.

• The critical period idea says that there are particular times when investments are efficacious.

• Complementarity suggests that investments are interconnected.
Skills and Intergenerational Mobility

The skills approach is qualitatively different from the family income investment approach.

1. Different objects of interest and richer set of mechanisms.
2. Life cycle is explicit.
3. Parent and offspring skill lifecycles interact in ways not available to 2 generation OG. Example (due to Heckman Pres. Lecture). Credit constraints of parents in early childhood are key.
• Will discuss in second lecture
One possible source of intergenerational persistence of socioeconomic status is via genes. There is, unsurprisingly, little question that an individual’s cognitive skills play an important role in determining socioeconomic outcomes.

That said, there is great controversy (and in my judgment no clear evidence) on the role of genes in determining cognitive and noncognitive skills.
• This is not to say that there is no role for genes in transmitting socioeconomic status from parents to children, but rather that empirical social science, despite many strong claims, has failed to establish the empirical salience of genes in explaining intergenerational mobility and cross-sectional inequality.

• Further, it is important to keep in mind that even if genes play a first order role, this has no bearing on whether government policies can affect inequality. To use a famous example due to Arthur Goldberger (1979), eyesight may be purely determined by genes, but this does not affect the efficacy of wearing glasses.

• Finally, cannot link to group inequality!!
The classical model used to measure the respective roles of nature, nurture, and luck can be described as follows. Note that the calculations are expressed in terms of variances instead of covariances; the latter is more standard in the literature, but there is no substantive difference between the approaches. The use of variances makes explicit how contrasts between different types of individuals are the basis of measuring how genes matter.
Twins Studies

The canonical mode of for nature nurture decompositions is

\[ \omega_{it} = aA_{it} + cC_{it} + eE_{it} \]  \hspace{1cm} (1)

\( \omega_{it} \) = outcome of offspring i in generation t;

\( A_{it} \) = genetic component

\( C_{it} \) = shared environment

\( E_{it} \) = nonshared environment (luck)
The objective of twins studies is to identify the contributions of the three factors to overall variance, in particular the genetic coefficient, $a$, since this coefficient is the basis for measuring the roles of nature versus nurture. To do this, it is first assumed that the different determinants of $\omega_i$ are uncorrelated with one another, i.e.

$$\text{cov} (A_i,C_i) = \text{cov} (A_i,E_i) = \text{cov} (C_i,E_i) = 0.$$  \hspace{1cm} (2)

This renders a variance decomposition meaningful.
Second, covariances of components of outcomes between twins are determined by combination of assumptions and laws of genetics.

By assumption, parallel to (2)

\[
\text{cov} (A_i, C_{i'}) = \text{cov} (A_i, E_{i'}) = \text{cov} (C_i, E_{i'}) = 0 \text{ if } i \neq i' \quad (3)
\]

By assumption children in same house experience same shared environment.
Genetic covariation is determined by twin type.

\[ \text{cov}(C_i, C_{i'}) = 1 \text{ if } i \neq i' \] \hspace{1cm} (4)

\[ \text{cov}(A_i, A_{i'}) = 1 \text{ if } i \neq i', \ (i,i') \text{ monozygotic} \]
\[ \text{cov}(A_i, A_{i'}) = .5 \text{ if } i \neq i', \ (i,i') \text{ dizygotic} \] \hspace{1cm} (5)
Three moments are observable: variances of outcomes for individuals and covariances by twin type.

\[
\text{var} \left( \omega_i | m \right) = \text{var} \left( \omega_i | d \right) = a^2 + c^2 + e^2 \tag{6}
\]

\[
\text{cov} \left( \omega_i, \omega_i | m \right) = a^2 + c^2 \tag{7}
\]

\[
\text{cov} \left( \omega_i, \omega_i | d \right) = .5a^2 + c^2 \tag{8}
\]

These identify the variance components. The genetic component can be written as

\[
a^2 = \text{var} \left( \omega_i - \omega_i | d \right) - \text{var} \left( \omega_i - \omega_i | m \right) \tag{9}
\]
In a long series of papers, Arthur Goldberger systematically critiqued twins studies. His work, in addition to careful deconstruction of specific studies, made a general methodological criticism in that he challenged two uncorrelatedness of genes and environment

\[
\text{cov}(A_i, C_i) \neq 0; \quad \text{cov}(A_i, C_i | m) \neq \text{cov}(A_i, C_i | d)
\]

and the equal environment assumption for monozygotic twins

\[
\text{cov}(C_i, C_i | m) \neq \text{cov}(C_i, C_i | d)
\]
This leads to a system of 4 equations

\[
\text{var}(\omega_i | m) = a^2 + c^2 + e^2 + 2ac \text{cov}(A_i, C_i | m) \tag{10}
\]

\[
\text{var}(\omega_i | d) = a^2 + c^2 + e^2 + 2ac \text{cov}(A_i, C_i | d) \tag{11}
\]

\[
\text{cov}(\omega_i, \omega_{i'} | m) = a^2 + c^2 \text{cov}(C_i, C_{i'} | m) + 2ac \text{cov}(A_i, C_{i'} | m) \tag{12}
\]

\[
\text{cov}(\omega_i, \omega_{i'} | d) = .5a^2 + c^2 \text{cov}(C_i, C_{i'} | d) + 2ac \text{cov}(A_i, C_{i'} | d) \tag{13}
\]

and 8 unknowns, so identification fails.
Note that, in isolation, gene environment interaction leads to partial identification. Suppose that one allows for gene environment correlation

\[
\text{cov} (A_i, C_i | m) = \text{cov} (A_i, C_i | d) = r
\]

preserves the identical environment for twins assumption, i.e.

\[
\text{cov} (C_i, C_i | m) = \text{cov} (C_i, C_i | d) = 1
\]

In this case, the system of second moments is
\[
\text{var}(\omega_i|m) = a^2 + c^2 + e^2 + 2acr 
\]

(14)

\[
\text{cov}(\omega_i,\omega_r|m) = a^2 + c^2 + 2acr 
\]

(15)

\[
\text{cov}(\omega_i,\omega_r|d) = .5a^2 + c^2 + 2acr 
\]

(16)

While there are 3 equations in 4 unknowns \((a^2, c^2, e^2, r)\). The nonlinearity in the system means that \(a = 0\) is testable, since if it holds,

\[
\text{cov}(\omega_i,\omega_r|m) = \text{cov}(\omega_i,\omega_r|d) 
\]
Generalized Kinship Models

- Within behavioral genetics, twins models have been extended. One popular variant is children of twins models.

- There is an impossibility theorem for identification across kinship groups that can be based on Goldberger’s approach to twins.
Suppose there are $K$ distinct pairs. In the case of twins, there are two: monozygotic and dizygotic. In the case of children of twins, there are two types of cousins, children of monozygotic and children of dizygotic.

Let

$$\Omega_1 = E\left( (A_i, C_i, E_i)(A_i, C_i, E_i)' | k \right)$$  \hspace{1cm} (17)$$

$$\Omega_2 = E\left( (A_i, C_i, E_i)(A_{i''}, C_{i''}, E_{i''})' | k \right)$$  \hspace{1cm} (18)$$

Then
This means that every kinship pair type that is added to a set of initial kinship variances and covariances introduces 12 unknown elements of \( \Omega_{k1} \) and \( \Omega_{k2} \).
Coefficients \((a, c, e)\) are not identified for any collection of kinship types, without restrictions on \(\Omega_{k1}\) and \(\Omega_{k2}\).
(Relatively) Credible Restrictions

Assumption 1: Restriction on nonshared environment. Many studies treat nonshared environment as luck. This means

\[ \text{cov}(A_i, E_i | k) = \text{cov}(C_i, E_i | k) = \text{cov}(A_i, E_i | k) = \text{cov}(C_i, E_i | k) = 0 \]

and

\[ \text{var}(E_i | k) \text{ independent of } k \]
In words, nonshared environment is uncorrelated with other factors and it variance is the same for all population types.

Assumption 2: Restriction on distribution of genes across kinship types

\[ \text{var}(A_i | k) \] constant across \( k \).

This rules out differences in mating patterns.
Assumption 3: Restriction on distribution of environment across kinship types

$$\text{var}(C_i|k) \text{ constant across } k.$$ 

It is not clear what this assumption means if shared environment endogenous.
Even with these assumptions, $\text{cov}(A_iC_i|k)$ and $\text{cov}(A_jC_j|k)$ appear in each variance covariance relationship for kinship types. As a result, increases in observable second moments by addition of new types of kinship types, cannot difference in number of unknown parameters and moments.

**Impossibility Theorem 2:**

Conditional on Assumptions 1,2, 3, no set of kinship relationships can identify $(a,c,e)$

This is the generalized Goldberger critique.
Epigenetics

• One of the most exciting developments in genetic research involves what is called epigenetics.

• Epigenetics refers to the way that the environment influences the expression of genes.

• Particular attention has focused on how the environment experienced by a mother affects gene expression during fetal development.
• A famous example of this concerns the effects of a famine in the Netherlands during 1945 on fetal development. The important finding is that as adults, the offspring of mothers whose pregnancies overlapped with the famine exhibit greater obesity than others.

• The explanation of this finding is that the expression of genes in the developing fetus was influenced by the fact that the mother was experiencing a calorie-deprived environment. This experience caused genes to be expressed that led offspring to crave calories. The triggering mechanism can be explained by evolutionary arguments if our distant ancestors evolved in an environment in which famines, for example, were multigenerational, which seems plausible.
• Remarkably, there is even evidence that epigenetic effects can be transgenerational in some species, although nothing is known at this point with respect to humans; see Youngson and Whitelaw (2008) for a survey. But the evidence suggests the possibility that environmental effects on a mother can have persistent consequences across generations.
My discussion so far has treated $A$ as unobservable. With the emergence of genomic data, social scientists are beginning to examine whether such data are predictive of socioeconomic outcomes. The use of genetic data in this way is called a genome-wide association study (GWAS).
Unfortunately, the GWAS methodology cannot identify gene complexes, which one would expect are the source of emergent properties such as intelligence. The problem is that the DNA sequence is so complicated, that the identification of gene-gene interactions represents the frontier of the literature.

This barrier exists in understanding the genetics of complex diseases such as tuberculosis. In fact, for a number of diseases, geneticists have referred to the lack of GWAS evidence as the missing heritability problem.

A second problem is that attributes such as intelligence are, in my view, likely to be emergent properties from overlapping gene complexes, so studies relating a single gene to politics, etc. are not interpretable.
• There are good reasons to believe that genes matter, yet social scientists have yet to develop persuasive ways to measure the influence.

• The development of more credible ways to conceptualize and measure the role of genes is, in my view, very important!
• How does the study of intergenerational mobility interact with ethical considerations?
• By this, my question is how positive research on intergenerational mobility matters normatively.
One obvious ethical consideration that is closely linked to intergenerational mobility is equality of opportunity.

How should one operationalize equality of opportunity?

Following ideas due to John Roemer and others, one objective of public policy is to reduce the dependence of individual outcomes on factors for which an individual is not responsible. This type of equality of opportunity respects the agency of individuals.
How might one formalize Roemer’s idea? Consider a socioeconomic outcome of interest, denote this as $\omega_i$. Suppose this outcome is determined by two vectors of observable characteristics $X_i$ and $Z_i$, and a scalar unobservable characteristic $\varepsilon_i$.

$$\omega_i = \phi(X_i, Z_i, \varepsilon_i)$$
Suppose that we believe that an individual is not responsible for $Z_i$ but is responsible for $X_i$ and $\varepsilon_i$.

An empirical analyst could construct the conditional probability of the outcome $\omega_i$ given the observable characteristics.

One could then say that perfect equality of opportunity with respect to $\omega$ exists, if the following conditional probabilities hold:

$$\forall i, j \quad \mu(\omega_i | X_i, Z_i) = \mu(\omega_j | X_j, Z_j) \text{ if } X_i = X_j$$
• In words, equality of opportunity means that so long as two individuals have the same values for the variable for which they are responsible, the probabilities of their outcomes are not affected by the variables for which they are not responsible.

• This formulation first appears in Durlauf (1996) although it seems an obvious idea given Roemer (1993).

• This naturally links to mechanisms I have described for intergenerational transmission of socioeconomic status. Children are not responsible for their parents, their social environment, or their genes.
• However, there are several limitations of the definition of equality of opportunity I have presented.

• Here I note two.
First, the analysis assumed that the unobservable source of heterogeneity $\varepsilon_i$ is something for which individuals should be held responsible.

If this assumption is wrong, then a policy implementation of my equality of opportunity definition could exacerbate the failure of equality of opportunity.
Second, the dichotomy between variables for which one is or is not responsible is problematic. (Roemer recognizes this).

If my family and background has led me to adopt values that are not conducive to economic success, e.g. I do not work hard in school, is my effort something for which I am or I am not responsible?

Desert and responsibility are distinct notions. Compare differences in wages due to discrimination versus genetic ability.

Deep issue: how to respect human beings as agents?
A second reason for concerns about the mechanisms that generate intergenerational persistence is that they are linked to the ability of an individual to lead a flourishing life.

Amartya Sen has developed a view of distributive justice that is premised on the idea that society should maximize capabilities of its members. The concept is, by his admission, undertheorized, but involves the capacity to make reflective judgments, etc.

These are closely related to Heckman’s skills approach. Heckman/Sen respects agency of persons.