Credit and Insurance for Investments in Human Capital

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June, 2012
1 Disclaimer

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2 Introduction

Extensive literature on credit constraints and human capital:

Micro:

structural and IV estimates.

Macro:

social mobility, inequality;

cross-country differences.

Most papers use simple, ad-hoc models of credit constraints.
This paper: Simplest Human Capital Investment Model

*Credit limitations and lack of insurance*

distorted investments and consumption; lower welfare.

*Incentive problems*

*limited commitment* (complete and incomplete markets)

*moral hazard* (in school, in labor markets)

*costly state verification*
Main message:

Useful to move beyond standard notions of credit constraints and look at endogenous constraints on financing.

cross section variation; response to economy-wide changes.

default highlights the importance of insurance and incentives.

Ongoing/Future work:

Integrate endogenous labor market risk (e.g. unemployment, disability) with optimal investment in human capital.
3 Government Student Loans and Limited Commitment

GSL programs

lending is directly tied to investment.

upper loan limits

extended enforcement vis-a-vis private loans.

\[ d_g \leq \min \{ \tau h, \bar{d} \}. \] (1)
Private Lending

punishment for default (credit bureaus, costly avoidance actions)

foreseen by rational lenders

A fraction \( 0 < \tilde{\kappa} < 1 \)

\[
d_p \leq \tilde{\kappa} R^{-1} a_f(h). \tag{2}
\]
Overall credit:

Simple two-period model:

\[ d = d_g + d_p \leq \min \{ h, \bar{d} \} + \tilde{\kappa} R^{-1} a f (h). \quad (3) \]

Life-cycle model

\[ d_p \leq \kappa_1 \Phi a h^\alpha + \kappa_2 d_g, \quad 0 \leq \kappa_1 \leq 1 \text{ & } \kappa_2 > -1. \quad (4) \]
Empirical Implications.

(1) Schooling is strongly positively correlated with ability over time.

(2) The correlation between schooling and family income (conditional on ability and family background) has grown since the early 1980s.

(3) There has been a sharp increase in the fraction of undergraduates borrowing the maximum amount from GSL programs since the 1990s.

(4) There has been a dramatic rise in student borrowing from private lenders since the mid-1990s.
A Rise in the Costs of and Returns to Schooling

Aim: compare the U.S. economy in 1980s vs. 2000s.

- increase the cost of investment and the returns to human capital
- keep constant GSL borrowing limits
- keep constant private enforcement
Comparison with Exogenous Constraints Model (EX.C)

calibration as in the 1980s

comparable credit limits as with our model

Results:

EX.C model compatible with higher impact of family wealth in the 2000s, but...

it also implies a negative ability-investment relationship
4 Uncertainty, Default and other Incentives

A Simple Environment

stochastic second period price of human capital

\[ i = 1, \ldots, N, \text{ realizations, } p_i = \text{prob. } w_{1,i}; w_{1,1} < w_{1,2} < \ldots < w_{1,N}. \]

preferences

\[ U = u(c_0) + \beta \sum_{i=1}^{N} p_i u(c_{1,i}), \]

where \( c_{1,i} \) is the second period consumption in realization \( i \).
$D_i$ (possibly negative) debt in second period

$$c_0 = W - h + \sum_{i=1}^{N} q_i D_i,$$

$$c_{1,i} = a f(h) w_{1,i} - D_i, \quad i = 1, \ldots, N.$$ 

Arrow prices: $q_i = \beta p_i$. 
Unrestricted optima

Let \( \bar{w}_1 \equiv \sum_{i=1}^{N} p_i w_{1,i} \) be the expected price of skill at \( t = 1 \)

**Optimal investment:**

\[
\bar{w}_1 a f' \left[ h^U (a) \right] = \beta^{-1},
\]

neither preferences nor initial wealth \( W \) are factors.

**Optimal consumption:**

\[
u' (c_0) = u' \left( c_{1,i} \right), \text{ for all } i = 1, \ldots, N.
\]
Limited Commitment with Complete Markets

Participation constraints: \( u \left[ w_{1,i}af(h) - D_i \right] \geq V^D (w_{1,i}, a, h) \). This limits the set of assets/debts individuals can hold as well as their ability to insure against some future states.

Letting \( \lambda_i \geq 0 \) denote the LM. (discounted) multipliers

\[
u'(c_0) = (1 + \lambda_i) \, u'(c_{1,i}).\]

Assume: \( V^D (w_{1,i}, a, h) = u [(1 - \kappa) w_{1,i}af(h)] \).

Then: solvency constraints \( D_i \leq \kappa w_{1,i}af(h) \) for all \( i = 1, ..., N \).
Optimal human capital investment $h^{LC}(a, W)$ satisfies

$$w_1 a f'[h^{LC}(a, W)] \left[ \frac{\sum_{i=1}^{N} p_i w_{1,i} \left( \frac{1+\lambda_i \tilde{\kappa}}{1+\lambda_i} \right)}{\bar{w}_1} \right] = \beta^{-1}.$$ 

...underinvestment.

........but not really a model for default.
Limited commitment with incomplete markets

Same temptation for default.

Non-contingent debt.

Threshold: \( \tilde{w}_1(D, a, h) \equiv \frac{D}{\tilde{\kappa} a f(h)} \).

Probability of default, \( \Pr \left[ w_{1,i} < \tilde{w}_1(D, a, h) \right] \).
Credit:

\[ Q(D, a, h) = \beta \left\{ D - \sum_{w_1, i < \tilde{w}_1} p_i [D - \tilde{\kappa} w_{1, i} a f(h)] \right\}. \]

A 'hard' borrowing constraint is given by \( \sup_D \{ Q(D, a, h) \} < \infty \).
**Consumption**: first order condition for $D$:

$$u'(c_0) = E[u'(c_{1,i}) | w_{1,i} \geq \tilde{w}_1].$$

**Human capital**: optimal $h$ is

$$\tilde{w}_1 a f'(h) \left[ \sum_{i=1}^{N} p_i u'(c_{1,i}) w_{1,i} - \tilde{\kappa} \sum_{w_{1,i} < \tilde{w}_1} p_i u'(c_{1,i}) w_{1,i} \right] \frac{\tilde{w}_1 u'(c_0) (1 - Q_h)}{\tilde{w}_1 u'(c_0)(1 - Q_h)} = \beta^{-1},$$

where $0 < Q_h < 1$ is the partial derivative (subgradient) of $Q$ wrt $h$. 
Investment: (relative to full insurance)

investment expand credit, which encourages investment.

some benefits of investment lost when default, discourages investment.

a precautionary motive, which may or may not encourage investment.
Default:

can occur in equilibrium.

if it happens, it is for low realizations of $w_{1,i}$

serves an insurance.

may enhance investments.

**Interest rates:** $R(D, a, h) \equiv D / Q(D, a, h)$.

premium for default.

reduced by ability and investment, increased by debt. ($Q_a > 0$, $Q_h > 0$, $Q_{ah} > 0$; $Q_D < 1$).
Pros:

model of default and interest rates

provides a more interesting policy trade-off

Cons:

Conceptual: Exogenous incompleteness.

Applicability: Abstracts from potential interesting incentive problems.

Question:

Quantitative: Can the model explain observed data?
Moral Hazard (while investing)

Assume $z$ is a continuous rv, support $Z = [0, \infty)$.

Earnings $w_1(z) = w_1z$

Effort (while in school) is:

- costly: if $e_0 < e_1$, then disutility $v(e_1) > v(e_0)$.

- productive: $e_1 > e_0$, then $\Phi(z|e_1) \leq \Phi(z|e_0)$ (first order dominance).

For now: $e \in \{e_0, e_1\}$. 
**Optimal Contract**

\[
\max_{h,e,d,\{R(z)\}} u[W - h + d] - v(e) + \beta \int_Z u[w(z) af(h) - R(z)] \phi(z|e) dz
\]

BEC of the lender (with the LM \( \lambda \)):

\[
[\lambda] : -d + \beta \int_Z [R(z) \phi(z|e_H)] dz \geq 0,
\]

ICC (assuming \( e_H \) is optimal; LM \( \mu \geq 0 \))

\[
[\mu] : -v(e_H) + \beta \int_Z [w(z) af(h) - R(z)] \phi(z|e_H) dz \\
\geq -v(e_L) + \beta \int_Z [w(z) af(h) - R(z)] \phi(z|e_L) dz.
\]
FOCs:

WRT $R(z)$:

$$u'[c_0] = \left[1 + \mu \left(1 - \frac{\phi(z|e_L)}{\phi(z|e_H)}\right)\right] u'[c_1(z)],$$

WRT $h$:

$$u'[c_0] = \beta a f'(h) w_1 \int_0^\infty z u'[c_1(z)] \phi(z|e_H) dz$$

$$+ \mu \left[\int_0^\infty z u'[c_1(z)] \left[\phi(z|e_H) - \phi(z|e_L)\right] dz\right]$$
Key implication: As long as $e = e_H$ optimal investment is first best:

$$\beta^{-1} = af'(h)w_1 \left[ \int_{0}^{\infty} z\phi(z|e_H)dz \right].$$

i.e. as long as the first best effort is implemented, then the first best level of investment is also implemented.

But, consumption must be ‘distorted’.
Impact of $W$ : income effects vs. ICC.

Implementing $e = e_H$ may not be feasible for too low or too high $W$ levels

If $e = e_L$ and $\mu = 0$ and

full insurance: $u'[c_0] = u'[c_1(z)]$,
underinvestment: $\beta^{-1} = af'(h) w_1 \left\{ \int_0^\infty z\phi(z|e_L)dz \right\}$,

i.e. consumption is not distorted but investment is.
This model of moral hazard while investing:

Either investment or consumption is distorted. But not both.

interesting interactions of wealth, ability and investment.
Costly State Verification

Assume no incentive problems in inducing $e_H$.

However, cost $\vartheta$ to verify a borrower's outcome.

Verification: threshold $\bar{z}$

Verification $(z < \bar{z})$: Full insurance $c_1(z) = c_0$

$$R(z) = zw_1af(h) - [W + d - h].$$

No verification $(z > \bar{z})$: borrower repays $\bar{R}$. 
Maximum repayment,

\[ \tilde{R} = \tilde{z} \text{w}_{1a}(h) - [W + d - h] \quad \text{or} \quad \tilde{z} = \frac{\bar{R} + W + d - h}{\text{w}_{1a}(h)}. \]

The BEC for the lender:

\[ \beta \left\{ w_{1a}(h) \int_{0}^{\tilde{z}} z \phi(z|e_{H}) dz - (W + d + \vartheta - h) \Phi(\tilde{z}|e_{H}) \right. \\
\left. \quad + \bar{R} \left[ 1 - \Phi(\tilde{z}|e_{H}) \right] \right\} \geq d. \]
The Lagrangean boils down to

\[
L = \max_{\{h,d,\bar{z}\}} \min_{\lambda} \left\{ \begin{array}{c}
\lambda \left[ u [W - h + d] \right. \\
+ \beta \left[ u [W - h + d] \Phi(\bar{z} | e_H) \\
+ \int_{\bar{z}}^\infty u[(z - \bar{z}) w_1 a f(h) + W + d - h] \phi(z | e_H) dz \right] \\
+ \lambda \left[ w_1 a f(h) \left[ \int_0^{\bar{z}} z \phi(z | e_H) dz + \bar{z} [1 - \Phi(\bar{z} | e_H)] \right] - \psi \Phi(\bar{z} | e_H) - W + h \right] \\
- d (1 + \beta) \end{array} \right. \}
\]
Consumption:

with verification:

\[ c_1(z) = c_0. \]

no verification:

\[ c_1(z) = w_1 z a f(h) - \bar{R} > c_0. \]
Underinvestment in Human Capital:

Can show from FOCs:

\[ \beta^{-1} \leq w_1 a f'(h) \{ \Gamma E(z|e_H) + (1 - \Gamma) E[\min \{z, \bar{z}\}|e_H] \}, \]

where \( \Gamma \equiv \frac{(1+\beta)E_{z>\bar{z}, e_H}[u'[c_1(z)]]}{u'(c_0)+\beta\{\Phi(\bar{z}|e_H)u'[c_0]+[1-\Phi(\bar{z}|e_H)]E_{z>\bar{z}, e_H}[u'[c_1(z)]]\}} \)

We have \( 0 < \Gamma < 1 \) because \( c_1(z) > c_0 \) for \( z > \bar{z} \).

*Underinvestment unless \( \vartheta \to 0 \implies \bar{z} = \infty \) i.e. full insurance.*
Pros:

endogenous incompleteness.

model of default and interest rates

provides interesting framework for policies on $\mathcal{O}$

Cons:

full insurance in case of default!
Moral Hazard with Costly State Verification

Optimal Contract

\[
\max_{\bar{z}, h, e, d, \{\bar{R}, R(z)\}} \quad u[W - h + d] - v(e) + \beta \int_{\mathbb{Z}} u[w(z) a f(h) - R(z)] \phi(z | e) dz
\]

BEC of the lender (LM \(\lambda\)): \(-d + \beta \int_{\mathbb{Z}} [R(z) - \varphi \chi(z)] \phi(z | e_H) dz \geq 0\),

the optimal ‘auditing’ or verification incentive compatibility:

\( R(z) \leq \bar{R} \) for \( z < \bar{z} \)
ICC for $e = e_H$ (LM $\mu \geq 0$)

$$-v(e_H) + \beta \int_Z u[w(z) af(h) - R(z)] \phi(z|e_H) dz$$

$$\geq -v(e_L) + \beta \int_Z u[w(z) af(h) - R(z)] \phi(z|e_L) dz.$$
Consumption:

with verification:

\[ u'[c_0] = \left[ 1 + \mu \left( 1 - \frac{\phi(z|e_L)}{\phi(z|e_H)} \right) \right] u'[c_1(z)] \]

no verification:

\[ c_1(z) = w_1 z a f(h) - \bar{R} \]
Pros:

endogenous incompleteness.

model of default and interest rates

provides interesting framework for policies on $\vartheta$

no full insurance in case of default!

Cons:

abstracts from other incentive problems

Question:

Quantitative implications.
Moral Hazard: effort in labor markets

second period utility: \( E[u(c_1)] - v(s) \).

earnings distribution \( \phi[y|a, h, s] \)

equilibrium exertion of effort: given \( D(y, a, h) \)

\[
s^* \in \arg \max_s \left\{ \int u[y - D(y, a, h)] \phi[y|a, h, s] dy - v(s) \right\}
\]

debt-overhang may reduce labor market performances.
5 Conclusions

Simple models of incentive problems applied to human capital investments

Lots of interesting economics to explore