

# Credit and Insurance for Investments in Human Capital

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June, 2012

# 1 Disclaimer

*The views expressed here are those of the author and do not necessarily reflect those of the Federal Reserve Bank of St. Louis or the Federal Reserve System.*

## 2 Introduction

Extensive literature on credit constraints and human capital:

Micro:

structural and IV estimates.

Macro:

social mobility, inequality;

cross-country differences.

Most papers use simple, ad-hoc models of credit constraints.

## **This paper: Simplest Human Capital Investment Model**

*Credit limitations and lack of insurance*

distorted investments and consumption; lower welfare.

*Incentive problems*

*limited commitment* (complete and incomplete markets)

*moral hazard* (in school, in labor markets)

*costly state verification*

## **Main message:**

Useful to move beyond standard notions of credit constraints and look at endogenous constraints on financing.

cross section variation; response to economy-wide changes.

default highlights the importance of insurance and incentives.

## **Ongoing/Future work:**

Integrate endogenous labor market risk (e.g. unemployment, disability) with optimal investment in human capital.

### 3 Government Student Loans and Limited Commitment

#### GSL programs

lending is directly tied to investment.

upper loan limits

extended enforcement vis-a-vis private loans.

$$d_g \leq \min \{ \tau h, \bar{d} \}. \quad (1)$$

## Private Lending

punishment for default (credit bureaus, costly avoidance actions)

foreseen by rational lenders

A fraction  $0 < \tilde{\kappa} < 1$

$$d_p \leq \tilde{\kappa} R^{-1} a f(h). \quad (2)$$

## Overall credit:

Simple two-period model:

$$d = d_g + d_p \leq \min \{h, \bar{d}\} + \tilde{\kappa} R^{-1} a f(h). \quad (3)$$

Life-cycle model

$$d_p \leq \kappa_1 \Phi a h^\alpha + \kappa_2 d_g, \quad 0 \leq \kappa_1 \leq 1 \ \& \ \kappa_2 > -1. \quad (4)$$

## **Empirical Implications.**

- (1) Schooling is strongly positively correlated with ability over time.
- (2) The correlation between schooling and family income (conditional on ability and family background) has grown since the early 1980s.
- (3) There has been a sharp increase in the fraction of undergraduates borrowing the maximum amount from GSL programs since the 1990
- (4) There has been a dramatic rise in student borrowing from private lenders since the mid-1990s.

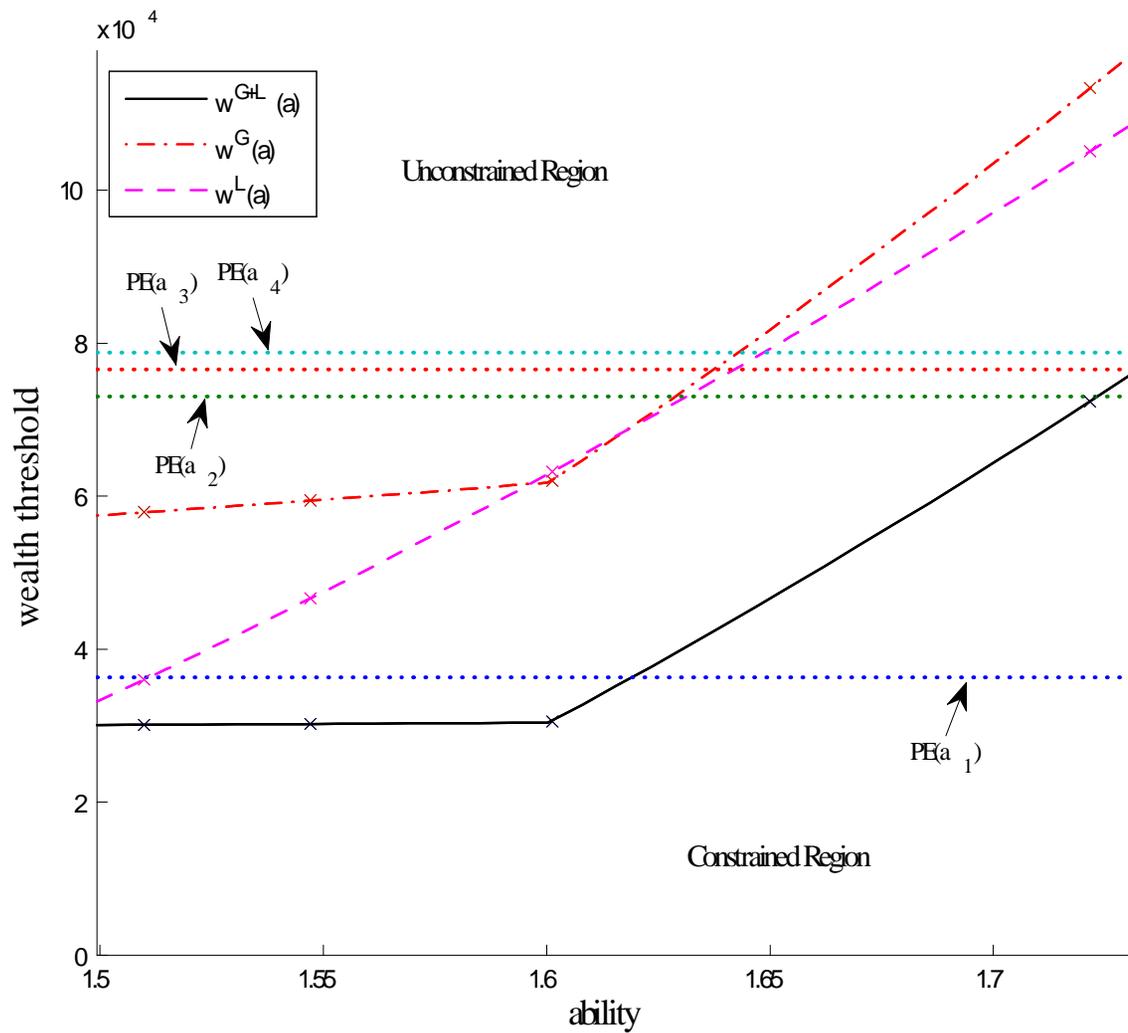
## **A Rise in the Costs of and Returns to Schooling**

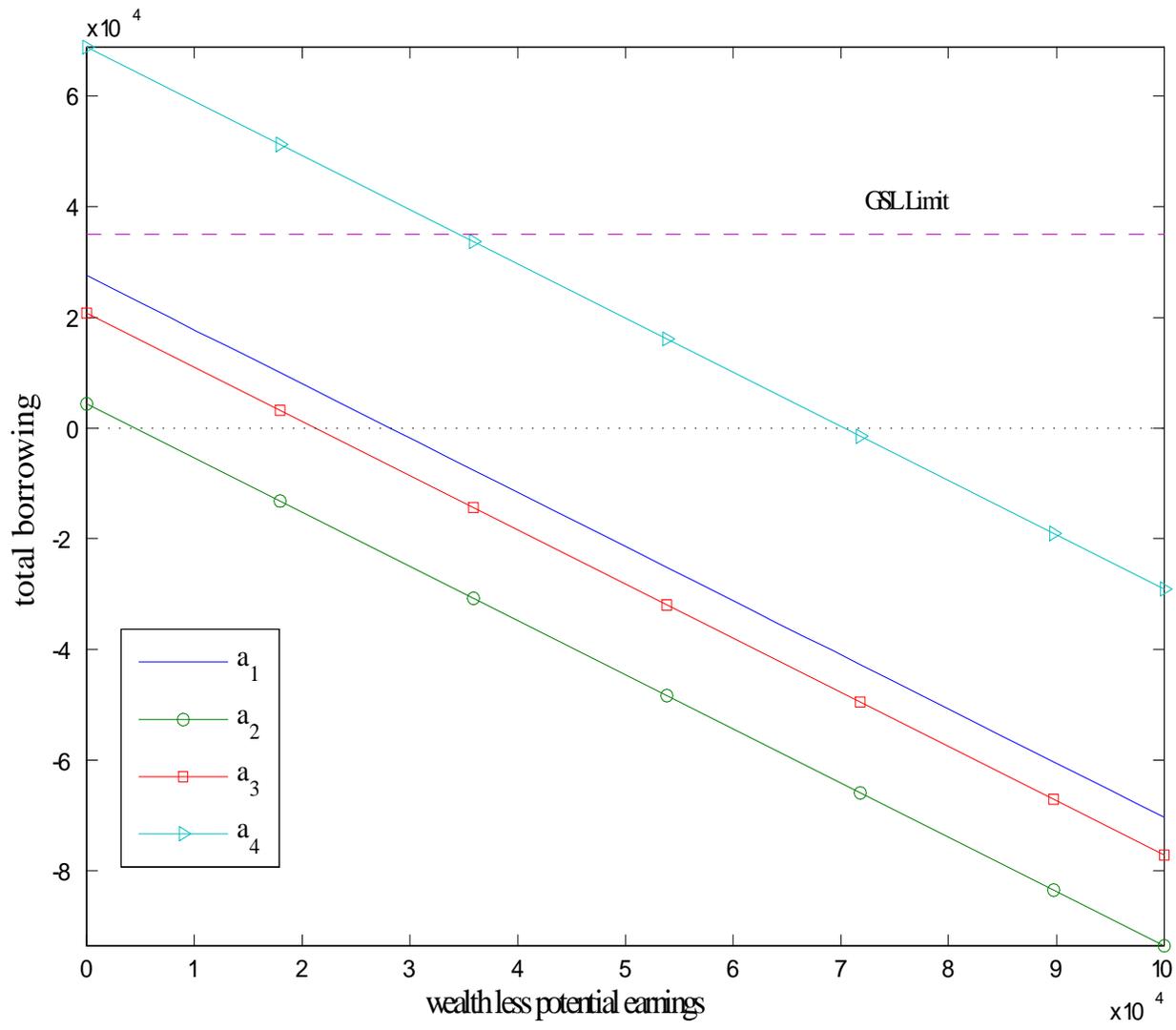
Aim: compare the U.S. economy in 1980s vs. 2000s.

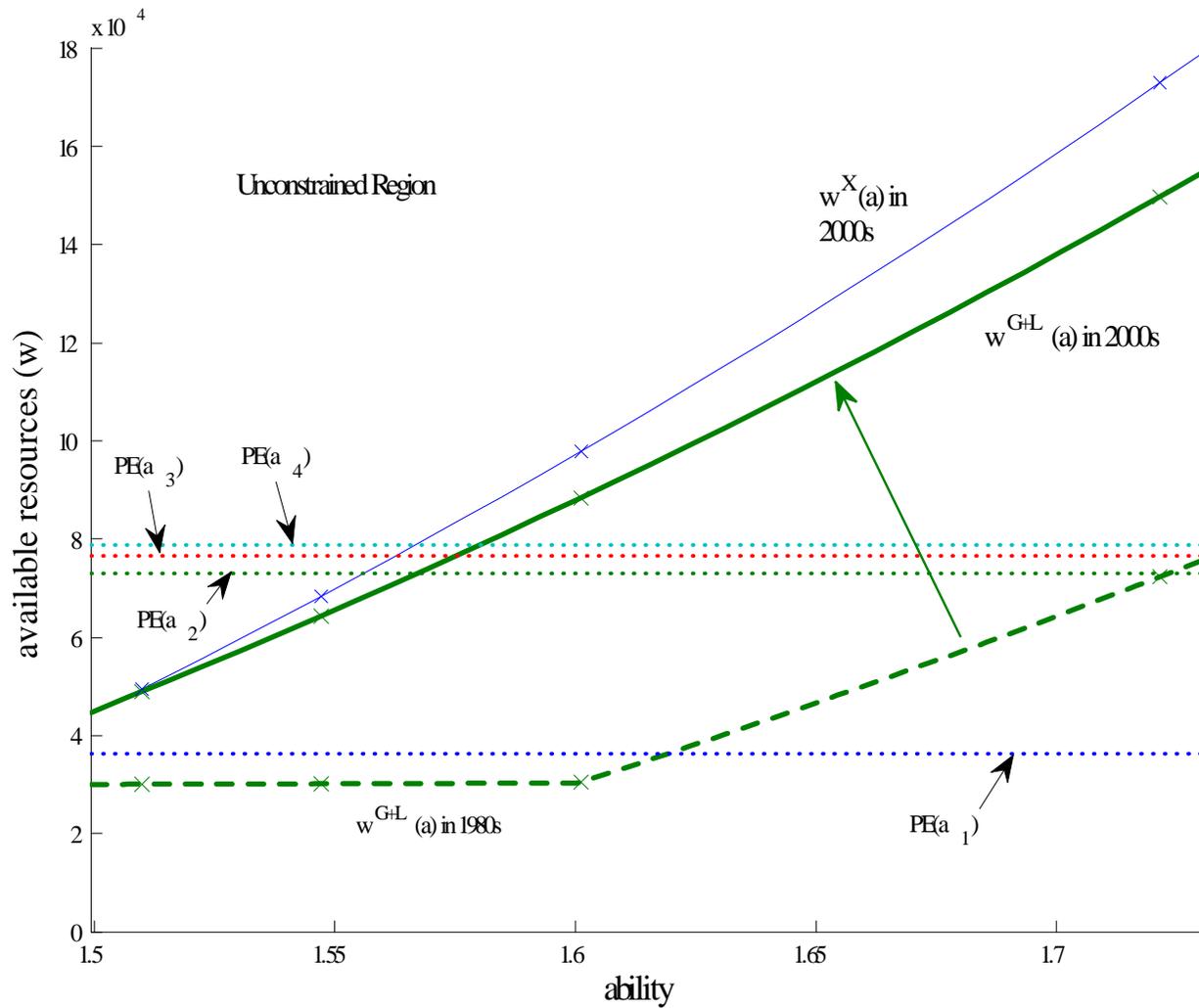
increase the cost of investment and the returns to human capital

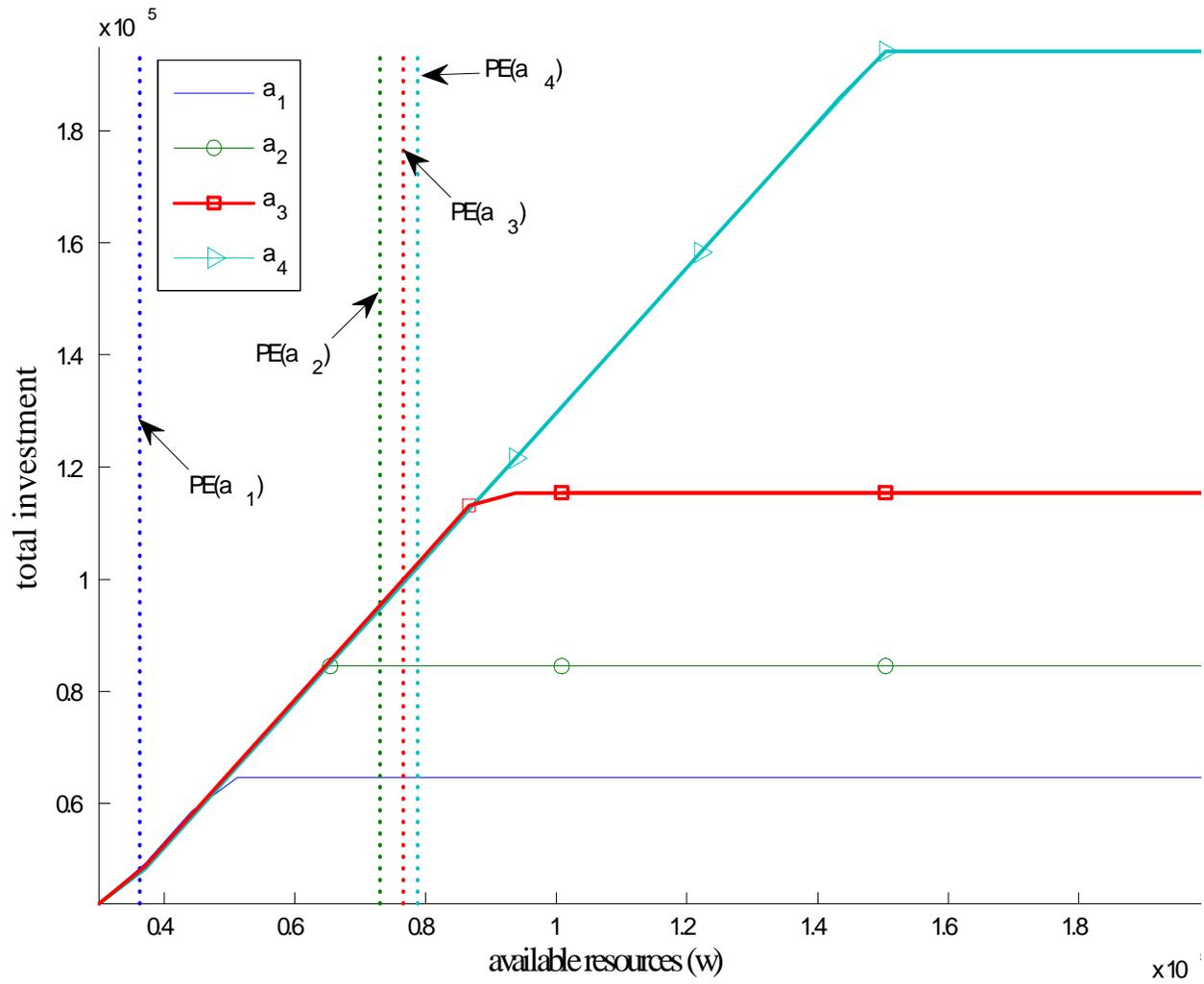
keep constant GSL borrowing limits

keep constant private enforcement









## Comparison with Exogenous Constraints Model (EX.C)

calibration as in the 1980s

comparable credit limits as with our model

### Results:

**EX.C model** compatible with higher impact of family wealth in the 2000s,  
but...

it also implies a negative ability-investment relationship

## 4 Uncertainty, Default and other Incentives

### A Simple Environment

stochastic second period price of human capital

$i = 1, \dots, N$ , realizations,  $p_i = \text{prob. } w_{1,i}; w_{1,1} < w_{1,2} < \dots < w_{1,N}$ .

preferences

$$U = u(c_0) + \beta \sum_{i=1}^N p_i u(c_{1,i}),$$

where  $c_{1,i}$  is the second period consumption in realization  $i$ .

$D_i$  (possibly negative) debt in second period

$$c_0 = W - h + \sum_{i=1}^N q_i D_i,$$
$$c_{1,i} = af(h) w_{1,i} - D_i, \quad i = 1, \dots, N.$$

Arrow prices:  $q_i = \beta p_i$ .

## Unrestricted optima

Let  $\bar{w}_1 \equiv \sum_{i=1}^N p_i w_{1,i}$  be the expected price of skill at  $t = 1$

*Optimal investment:*

$$\bar{w}_1 a f' [h^U (a)] = \beta^{-1},$$

neither preferences nor initial wealth  $W$  are factors.

*Optimal consumption:*

$$u' (c_0) = u' (c_{1,i}), \text{ for all } i = 1, \dots, N.$$

## Limited Commitment with Complete Markets

*Participation constraints:*  $u [w_{1,i} a f (h) - D_i] \geq V^D (w_{1i}, a, h)$ . This limits the set of assets/debts individuals can hold as well as their ability to insure against some future states.

Letting  $\lambda_i \geq 0$  denote the LM. (discounted) multipliers

$$u' (c_0) = (1 + \lambda_i) u' (c_{1,i}).$$

Assume:  $V^D (w_{1i}, a, h) = u [(1 - \tilde{\kappa}) w_{1i} a f (h)]$ .

Then: solvency constraints  $D_i \leq \tilde{\kappa} w_{1,i} a f (h)$  for all  $i = 1, \dots, N$ .

Optimal human capital investment  $h^{LC}(a, W)$  satisfies

$$\bar{w}_1 a f' [h^{LC}(a, W)] \left[ \frac{\sum_{i=1}^N p_i w_{1,i} \left( \frac{1 + \lambda_i \tilde{\kappa}}{1 + \lambda_i} \right)}{\bar{w}_1} \right] = \beta^{-1}.$$

...underinvestment.

.....but not really a model for default.

## Limited commitment with incomplete markets

Same temptation for default.

Non-contingent debt.

Threshold:  $\tilde{w}_1(D, a, h) \equiv \frac{D}{\tilde{\kappa} a f(h)}$ .

Probability of default,  $\Pr [w_{1,i} < \tilde{w}_1(D, a, h)]$ .

Credit:

$$Q(D, a, h) = \beta \left\{ D - \sum_{w_{1,i} < \tilde{w}_1} p_i [D - \tilde{\kappa} w_{1,i} a f(h)] \right\}.$$

A 'hard' borrowing constraint is given by  $\sup_D \{Q(D, a, h)\} < \infty$ .

*Consumption:* first order condition for  $D$ :

$$u'(c_0) = E[u'(c_{1,i}) | w_{1,i} \geq \tilde{w}_1].$$

*Human capital:* optimal  $h$  is

$$\bar{w}_1 a f'(h) \left[ \frac{\sum_{i=1}^N p_i u'(c_{1,i}) w_{1,i} - \tilde{\kappa} \sum_{w_{1,i} < \tilde{w}_1} p_i u'(c_{1,i}) w_{1,i}}{\bar{w}_1 u'(c_0) (1 - Q_h)} \right] = \beta^{-1},$$

where  $0 < Q_h < 1$  is the partial derivative (subgradient) of  $Q$  wrt  $h$ .

**Investment:** (relative to full insurance)

investment expand credit, which encourages investment.

some benefits of investment lost when default, discourages investment.

a precautionary motive, which may or may not encourage investment.

## Default:

can occur in equilibrium.

if it happens, it is for low realizations of  $w_{1,i}$

serves an insurance.

may enhance investments.

**Interest rates:**  $R(D, a, h) \equiv D/Q(D, a, h)$ .

premium for default.

reduced by ability and investment, increased by debt. ( $Q_a > 0$ ,  $Q_h > 0$ ,  $Q_{ah} > 0$ ;  $Q_D < 1$ ).

**Pros:**

model of default and interest rates

provides a more interesting policy trade-off

**Cons:**

Conceptual: Exogenous incompleteness.

Applicability: Abstracts from potential interesting incentive problems.

**Question:**

Quantitative: Can the model explain observed data?

## Moral Hazard (while investing)

Assume  $z$  is a continuous rv, support  $Z = [0, \infty)$ .

Earnings  $w_1(z) = w_1z$

Effort (while in school) is:

costly: if  $e_0 < e_1$ , then disutility  $v(e_1) > v(e_0)$ .

productive:  $e_1 > e_0$ , then  $\Phi(z|e_1) \leq \Phi(z|e_0)$  (first order dominance).

For now:  $e \in \{e_0, e_1\}$ .

## Optimal Contract

$$\max_{h,e,d,\{R(z)\}} u[W - h + d] - v(e) + \beta \int_Z u[w(z)af(h) - R(z)]\phi(z|e)dz$$

BEC of the lender (with the LM  $\lambda$ ):

$$[\lambda] : -d + \beta \int_Z [R(z) \phi(z|e_H)] dz \geq 0,$$

ICC (assuming  $e_H$  is optimal; LM  $\mu \geq 0$ )

$$\begin{aligned} [\mu] : & -v(e_H) + \beta \int_Z u[w(z)af(h) - R(z)]\phi(z|e_H)dz \\ & \geq -v(e_L) + \beta \int_Z u[w(z)af(h) - R(z)]\phi(z|e_L)dz. \end{aligned}$$

FOCs:

WRT  $\mathbf{R}(z)$ :

$$u' [c_0] = \left[ 1 + \mu \left( 1 - \frac{\phi(z|e_L)}{\phi(z|e_H)} \right) \right] u' [c_1(z)],$$

WRT  $\mathbf{h}$ :

$$u' [c_0] = \beta a f' (h) w_1 \left\{ \int_0^\infty z u' [c_1(z)] \phi(z|e_H) dz \right. \\ \left. + \mu \left[ \int_0^\infty z u' [c_1(z)] [\phi(z|e_H) - \phi(z|e_L)] dz \right] \right\}$$

**Key implication:** As long as  $e = e_H$  optimal investment is first best:

$$\beta^{-1} = a f'(h) w_1 \left[ \int_0^{\infty} z \phi(z|e_H) dz \right].$$

i.e. as long as the first best effort is implemented, then the first best level of investment is also implemented.

But, consumption must be 'distorted'.

## Impact of $W$ : income effects vs. ICC.

Implementing  $e = e_H$  may not be feasible for too low or too high  $W$  levels

If  $e = e_L$  and  $\mu = 0$  and

$$\begin{aligned} \text{full insurance: } & u' [c_0] = u' [c_1 (z)], \\ \text{underinvestment: } & \beta^{-1} = a f' (h) w_1 \left\{ \int_0^{\infty} z \phi(z | \mathbf{e}_L) dz \right\}, \end{aligned}$$

i.e. consumption is not distorted but investment is.

## **This model of moral hazard while investing:**

Either investment or consumption is distorted. But not both.

interesting interactions of wealth, ability and investment.

## Costly State Verification

Assume no incentive problems in inducing  $e_H$ .

However, cost  $\vartheta$  to verify a borrower's outcome.

Verification: threshold  $\bar{z}$

Verification ( $z < \bar{z}$ ): Full insurance  $c_1(z) = c_0$

$$R(z) = zw_1af(h) - [W + d - h].$$

No verification ( $z > \bar{z}$ ):, borrower repays  $\bar{R}$ .

Maximum repayment,

$$\bar{R} = \bar{z}w_1af(h) - [W + d - h] \quad \text{or} \quad \bar{z} = \frac{\bar{R} + W + d - h}{w_1af(h)}.$$

The BEC for the lender:

$$\beta \left\{ \begin{array}{l} w_1af(h) \int_0^{\bar{z}} z\phi(z|e_H)dz - (W + d + \vartheta - h) \Phi(\bar{z}|e_H) \\ + \bar{R} [1 - \Phi(\bar{z}|e_H)] \end{array} \right\} \geq d.$$

The Lagrangean boils down to

$$\begin{aligned}
 L = & \max_{\{h,d,\bar{z}\}} \min_{\lambda} u [W - h + d] \\
 & + \beta \left\{ \begin{array}{l} u [W - h + d] \Phi (\bar{z}|e_H) \\ + \int_{\bar{z}}^{\infty} u[(z - \bar{z}) w_1 a f(h) + W + d - h] \phi(z|e_H) dz \end{array} \right\} \\
 & + \lambda \left\{ \begin{array}{l} \beta [w_1 a f(h) [\int_0^{\bar{z}} z \phi(z|e_H) dz + \bar{z} [1 - \Phi (\bar{z}|e_H)]] - \vartheta \Phi (\bar{z}|e_H) - W + h] \\ -d(1 + \beta) \end{array} \right\}
 \end{aligned}$$

## Consumption:

with verification:

$$c_1(z) = c_0.$$

no verification:

$$c_1(z) = w_1 z a f(h) - \bar{R} > c_0.$$

## Underinvestment in Human Capital:

Can show from FOCs:

$$\beta^{-1} \leq w_1 a f'(h) \{ \Gamma E(z|e_H) + (1 - \Gamma) E[\min\{z, \bar{z}\} | e_H] \},$$

where  $\Gamma \equiv \frac{(1+\beta)E_{z>\bar{z}, e_H}[u'[c_1(z)]]}{u'(c_0) + \beta \left\{ \Phi(\bar{z}|e_H)u'[c_0] + [1 - \Phi(\bar{z}|e_H)]E_{z>\bar{z}, e_H}[u'[c_1(z)]] \right\}}$

We have  $0 < \Gamma < 1$  because  $c_1(z) > c_0$  for  $z > \bar{z}$ .

*Underinvestment unless  $\vartheta \rightarrow 0 \implies \bar{z} = \infty$  i.e. full insurance.*

## **Pros:**

endogenous incompleteness.

model of default and interest rates

provides interesting framework for policies on  $\vartheta$

## **Cons:**

full insurance in case of default!

## Moral Hazard with Costly State Verification

*Optimal Contract*

$$\max_{\bar{z}, h, e, d, \{\bar{R}, R(z)\}} u[W - h + d] - v(e) + \beta \int_Z u[w(z)af(h) - R(z)]\phi(z|e)dz$$

BEC of the lender (LM  $\lambda$ ):  $-d + \beta \int_Z [R(z) - \vartheta\chi(z)]\phi(z|e_H)dz \geq 0,$

the optimal 'auditing' or verification incentive compatibility:

$$R(z) \leq \bar{R} \text{ for } z < \bar{z}$$

ICC for  $e = e_H$  (LM  $\mu \geq 0$ )

$$\begin{aligned} & -v(e_H) + \beta \int_Z u[w(z)af(h) - R(z)]\phi(z|e_H)dz \\ & \geq -v(e_L) + \beta \int_Z u[w(z)af(h) - R(z)]\phi(z|e_L)dz. \end{aligned}$$

## Consumption:

with verification:

$$u' [c_0] = \left[ 1 + \mu \left( 1 - \frac{\phi(z|e_L)}{\phi(z|e_H)} \right) \right] u' [c_1 (z)]$$

no verification:

$$c_1 (z) = w_1 z a f (h) - \bar{R}$$

**Pros:**

endogenous incompleteness.

model of default and interest rates

provides interesting framework for policies on  $\vartheta$

no full insurance in case of default!

**Cons:**

abstracts from other incentive problems

**Question:**

Quantitative implications.

## Moral Hazard: effort in labor markets

second period utility:  $E [u (c_1)] - v (s)$ .

earnings distribution  $\phi [y|a, h, s]$

equilibrium exertion of effort: given  $D (y, a, h)$

$$s^* \in \arg \max_s \left\{ \int u [y - D (y, a, h)] \phi [y|a, h, s] dy - v (s) \right\}$$

debt-overhang may reduce labor market performances.

## **5 Conclusions**

Simple models of incentive problems applied to human capital investments

Lots of interesting economics to explore