Mismatch, Rematch, and Investment*

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November 2008, revised, February 2012

Abstract

This paper studies rigidities in sharing joint payoffs (non-transferability) as a source of excessive segregation in labor or education markets. The resulting distortions in ex-ante investments such as education acquisition link such mismatches to the possibility of simultaneous under-investment by the underprivileged and over-investment by the privileged. This creates an economic rationale for rematch policies like affirmative action, which have to be evaluated in terms of both incentives and the assignment quality. We compare a number of such policies that have empirical counterparts. Our results indicate that some of these policies can be beneficial on both equity and efficiency grounds.

Keywords: Matching, nontransferable utility, multidimensional attributes, affirmative action, segregation, education.

JEL: C, I, J.

1 Introduction

Some of the most important economic decisions we make – where to live, which profession to enter, whom to marry – depend for their consequences not only on our own characteristics or “types” (wealth, skill, or temperament),

* We are grateful for comments from John Moore, Glenn Loury, Andy Postlewaite and seminar participants at Amsterdam, Brown, Budapest, UC Davis, Essex, Frankfurt, Ottawa, Penn and ThReD. Gall thanks DFG for financial support (grant GA-). Legros thanks financial support of the Communauté Française de Belgique (ARC /), PAI Network P and the FNRS (crédit aux chercheurs).

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but also on those of the people with whom we live or work. These decisions matter not only in a static sense, for our own well-being or those of our partners, but also dynamically — the prospect of being able to select particular kinds of neighbors, associates or mates, or the environment those partners provide, affects the costs and benefits of investment. The impact of those investments may extend far beyond our immediate partners to the economy as a whole.

A natural question – one in which policy makers in rich and poor countries have taken a direct interest – is whether the market outcome of our "matching" decisions leads to outcomes that are socially desirable. Indeed, it has often been contended in public policy debates in the U.S., U.K., India and elsewhere that the market has failed to sort people desirably — there is too much segregation, whether by educational attainment, ethnic background or caste. Certain groups appear to be excluded from normal participation in economic life, and that in turn depresses their willingness to invest in human capital. If the market does "mismatch" people in this way, policy remedies might include "rematching" individuals into other partnerships via affirmative action, school integration or corporate diversity policies.

Much discussion about policies aimed at correcting mismatch tends to rely on motivations like equity, social cohesion or righting past wrongs, with an acknowledgement that there may be a cost in aggregate performance — the classic equity-efficiency trade-off. One reason for this focus may be that economic theory shows that some form of imperfection needs to be present if a policy intervention is to generate performance gains.[1] It remains an open question, however, whether policies that directly constrain matches between agents necessarily conflict with efficiency when there are imperfections.

This paper will be concerned with one important but understudied imperfection — rigidities in the distribution of surplus among matched partners. Though it is well-known that such "non-transferability" can distort matching patterns relative to the no-rigidity case, there has been little work characteriz-

[1] If the characteristics of matched partners (ability, gender, or race) are exogenous, then under the assumptions that (1) partners can make non-distortionary side payments to each other (transferable utility or TU), (2) there is symmetric information about characteristics, and (3) there are no widespread externalities, stable matching outcomes are social surplus maximizing: no other assignment of individuals can raise the economy's aggregate payoff. Even if characteristics (such as income or skill) are endogenous, the result of investments made either before matching or within matches, under the above assumptions re-matching the market outcome is unlikely to be desirable (Cole et al., Felli and Roberts).

ing those patterns, much less their implications for investment. Our analysis will show specifically that the market may deliver more segregation than it would without rigid surplus sharing, and will link that outcome to the possibility of simultaneous *o er-in est ent t t e top n un er-in est ent t t e otto* (OTUB) underprivileged individuals invest less than they would in an otherwise identical economy without rigidities, while privileged ones invest more. This creates an economic rationale for policies that "rematch" individuals into new partnerships (called "associational redistribution" in Durlauf, 1 6). Evaluating such policies must take account of incentives as (for instance everyone prefers higher types to lower types), this may lead to "excessive" segregation, at least from the viewpoint of ex-ante Pareto optimality, i.e., maximizing welfare from behind a veil of ignorance, before people know their types (as in Harsanyi, 1953; Holmström and Myerson, 1983).[3]

Moreover, since returns to investments in attributes made before the market depend on the anticipated matching possibilities resulting from investment, mismatch can also generate a dynamic inefficiency, distorting ex-ante investments such as education acquisition. When mismatch takes the form of excessive segregation in socio-economic background and returns to education are complimentary to background, the distortion may be in form of OTUB, with obvious implications for persistent inequality and socio-economic polarization. Though rematch policies cannot directly address the sources of NTU, they may provide an instrument for correcting inefficiency of the match as well as distortions in investment incentives, if properly designed.

Despite the importance of NTU in many parts of economics, its implications for the nature of market matches, the level and distribution of investment in such markets, and for the effects of re-matching policy have received scant attention, although the literature has looked at other potential sources of mismatch like incorrect beliefs and search frictions. In addition to the gap in theoretical understanding, the case of NTU as a fundamental driver of mismatch appears to be consistent with empirical observations: the removal of affirmative action policies that have been in place for a while often results in reversion to the pre-policy status quo, for instance in case of the end of high school desegregation.[4] NTU also provides a natural explanation for political opposition to affirmative action policies: it is difficult to compensate the unfavored group; otherwise the market would have already done so.

The setup we employ to analyze various forms of rematch is as follows. Agents have a binary background type reflecting whether they are *privileged*

[3] Affirmative action policies typically do not yield ex-post Pareto improvements unless accompanied by compensation, i.e., monetary transfers, which are by nature severely limited in a nontransferable utility framework. Nevertheless we will evaluate allocations in terms of aggregate surplus. This is a standard approach to evaluating mechanisms (or institutions) and may correspond, for instance, to how future parents would vote on educational policy. Equally important, the link between the design of rematch policies and aggregate performance measures such as GDP is of interest from a positive viewpoint.

[4] Orfield and Eaton ([illegible]) report evidence of increased segregation in the South of the U.S. in districts where court-ordered high school desegregation ended; see also Clotfelter et al. ([illegible]) and Lutz ([illegible]). Weinstein ([illegible]) finds increased residential segregation as a consequence of the mandated desegregation.

or not. Privilege confers a productivity benefit, in terms of (increased) market output, for instance due to superior access to resources. Agents can affect their labor market productivity (also a binary variable) by investing in education, which determines the probability of attaining a high achievement.[5] In the labor market, when achievements have been realized, agents match into firms whose output depends on both the members' achievements and their backgrounds (thus we are dealing with a multi-dimensional assignment problem). The production technology is such that some diversity (heterogeneity) within firms is more productive, and would be the outcome under unrestricted side payments. We model NTU in the simplest possible way: output is shared equally within firms.

Under non-transferability, the labor market segregates in educational achievement and background. This means the laissez-faire equilibrium outcome is inefficient from an aggregate surplus perspective. When agents' types enter the production function directly, individual returns from education investment depend positively on the productivity of the match in the labor market. This is the source of the OTUB result: the underprivileged find investing to be too costly or unremunerative, while the privileged receive inefficiently high rewards in the labor market.

Rematch policies that affect the labor market match can thus be used to influence investment behavior, as well as having a direct effect on assignment quality. Indeed an often-voiced concern about rematch policies is that they may harm the investment incentives of the group favored by the policy by guaranteeing its members minimal payoffs and that they may reduce the incentives of *unfavored* groups, whose members may obtain rents under laissez-faire. When there is over-investment by the privileged, at least the latter effect may become socially desirable.

Analyzing a plausible parametric case of the model in detail we evaluate two particular variants of rematch policies that are frequently used by policymakers: affirmative action, where preference is awarded to underprivileged individuals when comparing individuals of the same achievement level, and "busing", where assignment to teams replicates the population composi-

[5] Stochastic investment in attributes with a continuum of agents allows the use of the deterministic limit of the attribute distribution in the market. If the investment technology ensures that the distribution has full support, equilibrium investments under rational expectations are unique. This is quite convenient, since such settings are often plagued by problems of multiple equilibria (see Cole et al.).

tion in expectation, ignoring achievement. While both policies may generate higher aggregate surplus than laissez-faire, affirmative action always dominates busing in terms of aggregate surplus, investment, and income. Since both policies improve the sorting to a similar extent, this is mainly due to differential investment incentives under the two policies under affirmative action encouragement of the underprivileged outweighs discouragement of the privileged, resulting in higher aggregate investment than both laissez-faire and the first best. The opposite holds for busing guaranteeing low achievers a high achieving match with positive probability provides implicit insurance against low achievements, depressing incentives for education acquisition considerably. For the same reason policies that ignore background, but rematch individuals based on achievements are always dominated by laissez-faire or affirmative action in our setting, where some diversity in backgrounds is desirable. If one is primarily concerned with decreasing inequality, both of education investments and income, a busing policy dominates affirmative action, laissez-faire, and the first best if the underprivileged are a majority, and affirmative action dominates if the underprivileged are a minority.

Literature

The literature on school and neighborhood choice (see among others Benabou, 1 3, 1 6 Epple and Romano, 1 8) typically finds too much segregation in types. This may be due to market power (see, e.g., Board, 200) or widespread externalities (see also Durlauf, 1 6 Fernandez and Rogerson, 2001). When attributes are fixed, aggregate surplus may be raised by an adequate policy of bribing some individuals to migrate (see also de Bartolome, 1 0). Fernandez and Galí (1) compare matching market allocations of school choice with those generated by tournaments the latter may dominate in terms of aggregate surplus when capital market imperfections lead to non-transferability. They do not consider investments before the match. Peters and Siow (2002) and Booth and Coles (2010) present models where agents invest in attributes before matching in a marriage market under strict NTU. The former finds that allocations are constrained Pareto optimal (with the production technology they study, aggregate surplus is also maximized), and does not discuss policy. The latter compares different marriage institutions in terms of their impact on matching and investments. Gall et al. (2006) analyzes the impact of timing of investment on allocative efficiency.

Several recent studies consider investments before matching under asymmetric information (see e.g., Bidner, 2008; Hopkins, 2012; Hoppe et al., 2009), mainly focusing on wasteful signaling, while not considering rematch policies.

Rematch has been supported on efficiency grounds in the case where there is a problem of statistical discrimination: Coate and Loury (1993) provides one formalization of the argument that equilibria where under-investment is supported by "wrong" expectations may be eliminated by affirmative action policies (an "encouragement effect"), but importantly also points out a possible downside ("stigma effect"). Other imperfections, such as rationing the number of jobs available (Fryer and Loury, 2007), may also give an efficiency rationale for affirmative action or education subsidies. A related literature discusses the possibility that affirmative action may lead to mismatch in the sense that the beneficiaries of the policy end up being worse off than in the market outcome as admitting them to better schools may lower their expected grades and economic outcomes (Sander, 2004; Fryer and Loury, 2005; Arcidiacono et al., 2011). These studies focus on the static effects; issues of investment and dynamic incentives are not discussed. Finally, on comparing different rematch policies, Fryer et al. (2008) finds that a color blind policy (in our framework equivalent to an achievement based policy) sometimes is more desirable than a color sighted (our affirmative action and busing policies) in a world where agents have a binary choice for education. This finding is opposite to ours; a crucial difference to our study is the absence of mismatch in the labor market, illustrating why the consideration of NTU can be informative for the policy discussion.

The emphasis here is on characterizing stable matches and contrasting them with ones imposed by policy. Thus we shall not be concerned with the market outcome under search frictions (Shimer and Smith, 2000; Smith, 2006), nor with mechanisms employed to achieve either stable matches or ones with desirable welfare properties (e.g., Roth and Sotomayor, 1990). Matching policies in this paper might, of course, use such mechanisms.

The paper proceeds as follows. Section 2 lays out the model framework; first best and laissez-faire allocations are derived in Section 3. Section 4 compares them to policies of affirmative action. Section 5 provides some extensions, while Section 6 concludes. All proofs and calculations not in the text can be found in the appendix.

2 Model

The market is populated by a continuum of agents with unit measure. Though we refer to it as a "labor market," it can also be interpreted in other ways, for instance as a market for places in university. Agents may differ in their educational *achievement* $a \in \{h, \ell\}$ (for high and low) and their *background* $b \in \{p, u\}$ (for privileged and underprivileged). While individual background is given exogenously, achievement is a consequence of individual investments taken before the market. Achieving h with probability e requires an investment in education of e at individual cost $e^2/2$.

In the market an agent is fully characterized by an *attribute*, a pair ab. Matching into a firm $(ab, a'b')$, two agents with attributes ab and $a'b'$ generate surplus $z(ab, a'b')$ separable in achievements and background

$$z(ab, a'b') = f(a, a')g(b, b') \tag{1}$$

$$\text{where,} \quad f(h,h) = 2,\ f(h,\ell) = f(\ell,h) = 1,\ f(\ell,\ell) = 0, \tag{2}$$

$$g(p,p) = 1,\ g(p,u) = g(u,p) = \delta,\ g(u,u) = \delta/2, \tag{3}$$

with $\delta > 1/2$. Note that the "production" function $f(.)$ has constant returns to achievement $f(a,h) - f(a,\ell) = 1$ for any a.[6] Therefore the matching pattern is driven entirely by background effects if, for instance, $g(b,b') = 1$ for all b, b', $z(ab, a'b') = f(a,a')$, and all matching patterns yield the same aggregate surplus.

The condition $\delta > 1/2$ implies that agents with attribute hu are more productive than those with ℓp this assumption is for convenience and guarantees a complete order on attributes ab, in the sense that for any attribute ab, if $a'b' > a''b''$ then $z(ab, a'b') > z(ab, a''b'')$

$$\ell u < \ell p < hu < hp. \tag{4}$$

The "peer group effect" function $g(b,b')$ has strictly decreasing differences (that is, $2g(p,u) > g(p,p) + g(u,u)$) if, and only if, $\delta > 2/3$. The assumption that $g(u,u) = g(p,u)/2$ is only for convenience as long as $g(u,u) \geq g(p,u)/2$

[6] Our framework is compatible with more general surplus functions of the form $(a + a')^{\alpha}(b + b')^{\beta}$ with $\alpha \geq$ and $\beta \leq$. As long as the privileged agents advantage is great enough $p\ u > {}^{1+(\alpha-1)/\beta} -$ both NTU and TU equilibrium matching patterns remain unchanged though incentives and therefore policy effects change but the qualitative results carry over. Computations would be more cumbersome however.

the matching patterns under NTU and TU remain the same — see Section 5. This simplification allows us to focus on a tradeoff between two key parameters — the measure of the privileged π and the labor market disadvantage of the underprivileged δ, capturing for instance the difficulty of generating high return from a given output in the market (in form of access to financial markets, business and social networks).

2.1 Timing

The timing in the model economy is as follows.

1. Policies, if any, are put in place.

2. Agents of background b choose investment e_b. Given an investment e the probability of achievement h is e and of achievement ℓ is $1 - e$.

3. Achievement is realized and is publicly observed.

4. Agents form groups of size two in a matching market with no search frictions, though it may be constrained by policies.

5. Once groups are formed, output is realized and is shared between the agents.

2.2 Equilibrium

The matching market outcome (absent a policy intervention) is determined by a stable assignment of individuals into groups of size two given attributes ab, which are in turn determined by individuals' optimal choice of education acquisition e under rational expectations. A *labor market equilibrium* is therefore defined as a bijective matching function between individuals characterized by attributes ab, and a share of output for each agent within a group such that

- (Payoff Feasibility) Within a group (i, j), the sum of the shares at most exhausts the total output $z(a_i b_i, a_j b_j)$.

- (Stability) There do not exist two individuals who can be strictly better off by matching and choosing a feasible share of output given their equilibrium payoffs.

Existence of such an equilibrium is standard, see, e.g., Kaneko and Wooders (1 86). That is, a labor market equilibrium determines individual payoffs depending on attribute ab. Equilibrium payoffs will generally depend on the distribution of attributes, which is determined by education choices and the initial distribution of backgrounds. An *investment equilibrium* is defined as individual education choices $\{e_i\}$ such that

- (Individual Optimality) Every agent i s education choice e_i maximizes i s utility from the expected labor market equilibrium payoffs consistent with $\{e_i\}$.

The fact that attributes in the labor market are determined by stochastic achievements of a continuum of agents simplifies matters. Let individuals be indexed such that individual i is $i \in [0, 1]$, which is endowed with Lebesgue measure. W.l.o.g. assume that all agents $i \in [0, \pi)$ have background p and all agents in $i \in (\pi, 1]$ have background u. If the investment level of agents with background b is e_b, then, by a law of large numbers, the measures of the different attributes ℓu, ℓp, hu, and hp are respectively $(1-\pi)(1-e_u)$, $\pi(1-e_p)$, $(1-\pi)e_u$, and πe_p. Hence, given education choices e_b the distribution of attributes in the labor market is deterministic.

This implies that labor market equilibrium payoffs only depend on aggregates e_u and e_p. Therefore in any investment equilibrium all u individuals face the same optimization problem, and all p individuals face the same optimization problem. Hence, in all pure strategy investment equilibria all agents of the same background b choose the same education investment e_b.

Our analysis will describe the matching patterns in terms of attributes because there may be ‘unbalanced measures of different attributes, the equilibrium matches of a given attribute may specify different attributes. For instance, matches (hp, hu) and $(hp, \ell u)$ may be part of an equilibrium. This can be consistent with our definition of equilibrium matches only if the matches between attributes are measure-preserving.

2.3 Degree of Transferability

We will consider two extreme cases. As a benchmark, we use the equilibrium allocation with perfect transferability in this allocation investment choices and the equilibrium match maximize total surplus, as we will show.

Our main focus, however, is on laissez-faire and policy outcomes under non-transferability. To facilitate exposition we assume strictly nontransferable utility, so that only a single vector of payoffs is feasible in any firm each partner obtains exactly half the output. Equal sharing under strict NTU is for convenience what matters qualitatively is that every type would prefer to be matched with higher rather than lower types. Strict NTU can also be relaxed. All results in the paper are robust to allowing for some transferability by admitting for either a sufficiently small range of perfect transferability, or for sufficient curvature in the Pareto frontier within matched partnerships.[7]

3 Laissez-Faire, Mismatch and Incentives

To start our analysis we will characterize the equilibria under full transferability and laissez-faire (NTU), including the effects of each regime on the choice of investment by u and p agents.

3.1 Full Transferability

When there is full transferability within matches the Pareto frontier for a match $(ab, a'b')$ is obtained by sharing rules in the set $\{s \quad w(ab) \quad s, w(a'b') \quad z(ab, a'b') - s\}$. It is well known that under full transferability agents with the same attribute must obtain the same payoff.[8] Because of equal treatment there is no loss of generality in defining the equilibrium payoff of an attribute, denoted by $w(ab)$.

We now characterize the equilibrium. The structure of payoffs and the stability conditions lead to the following observations.

Fact 1. (i) $(hp, \ell u)$ matches cannot be part of a first best allocation.

(ii) Conditional on agents of a given background matching together, segregation in achievement maximizes aggregate surplus.

[7] Strict NTU even with monetary payoffs can be obtained in various ways e.g. as the outcome of ex-post bargaining as already mentioned or as the limiting outcome of a standard moral-hazard-in-teams model where the partners unobservable efforts become perfect complements.

[8] Otherwise if one agent obtains strictly less than another this violates stability since the payoff difference could be shared between the first agent and the partner of the second agent.

(iii) Conditional on high achievement agents matching together, segregation by background is surplus efficient if, and only if, $\delta < 2/3$.

(iv) A first best allocation exhausts all possible $(hu, \ell p)$ matches.

The first statement follows because in a $(hp, \ell u)$ firm hp agents lose more compared to their segregation payoffs than ℓu agents gain. (ii) holds whenever $f(a, a')$ has weakly increasing returns. For (iii) recall that $\delta < 2/3$ implies that $g(b, b')$ has strictly increasing differences. Therefore having a privileged partner is more valuable to a privileged than to an underprivileged agent. Observation (iv) is perhaps a little surprising even when both $f(a, a')$ and $g(b, b')$ have increasing differences, which tends to favor segregation, some integration of hu and ℓp is efficient.[9] The reason for this is that aggregates of achievement $f(a, a')$ and background $g(b, b')$ in a team are complements. When matching a privileged low achiever and an underprivileged high achiever, who were previously segregated, the increase in surplus $z(.)$ due to peer effects $g(.)$ is sufficient to offset the possible loss of surplus due to the change of inputs to production $f(.)$

$$\begin{aligned} & f(h,\ell)[g(u,p) - g(u,u)] - f(h,\ell)[g(p,p) - g(u,p)] \\ & > -[f(h,\ell) - f(\ell,\ell)]g(p,p) + [f(h,h) - f(h,\ell)]g(u,u). \end{aligned}$$

Figure 1 summarizes these observations and shows the possible equilibrium matching patterns under full transferability. Dotted arrows indicate matches subject to availability of agents after exhausting matches denoted by solid arrows.

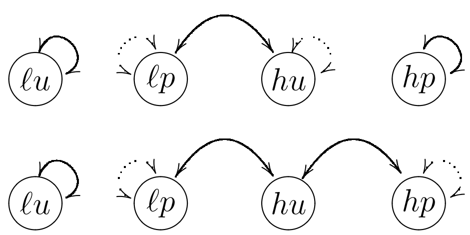


Figure 1 TU equilibrium matchings for $\delta < \frac{2}{3}$ (top) and $\delta > \frac{2}{3}$ (bottom).

[9] This extends to cases when both $f(a, a')$ and $g(b, b')$ have strictly increasing differences. Hence the condition to have segregation as the surplus maximizing allocation, i.e. supermodularity of the surplus function $z(ab, a'b')$ is substantially more demanding in a world with multidimensional attributes than in a one-dimensional world.

3.2 TU Wages

There are some reasons to suspect that diversity in backgrounds is indeed desirable (i.e., $\delta > 2/3$). For instance, when the privileged have preferential access to resources, distribution channels, or information, the benefit of a privileged background will diminish in the number of privileged agents already on the team. Furthermore, teams that are heterogeneous in backgrounds are able to cater to customers of different socioeconomic characteristics, for instance through language skills and knowledge of cultural norms. Finally, when teams perform problem-solving tasks, groups with diverse backgrounds tend to perform well, because members differ in their use of heuristics (Hong and Page, 2001). We assume that possible drawbacks of background diversity (for instance in form of transaction cost) in a team is outweighed by the benefits of higher potential revenue.

Suppose therefore that $\delta > 2/3$ (though we examine the case $\delta \in (1/2, 2/3)$ in the Appendix). Under TU all possible $(hu, \ell p)$ matches are exhausted, then all remaining (hp, hu) matches. All remaining attributes segregate. Therefore $w(\ell u) = 0$. Wages for other attributes will depend on relative scarcity, which in turn will depend on initial measure of privileged π and achievable surplus $z(ab, a\prime b\prime)$. The following statement summarizes the properties of TU equilibrium investment levels.

Fact 2. Suppose $\delta > 2/3$. Under full TU investment levels e_p and e_u increase in π. $\delta \leq e_p < 1$ for $\pi < 1$ and $e_p = 1$ for $\pi = 1$. $\delta/2 \leq e_u < \delta$ for $\pi < 1$ and $e_u = \delta$ for $\pi = 1$.

That is, investment in education increases in the measure of privileged. This is because for the underprivileged the payoff of hu agents determines e_u as $w(\ell u) = 0$ and increases in the measure of available privileged matches, and approaches δ as ℓp agents become abundant. The payoff of hp agents increases in the measure of surplus hp agents that will segregate, while the payoff of ℓp agents decreases as the measure of hu agents decreases in π.

If one thinks of the first best outcome as the matching pattern that maximizes total output, the following lemma states that the equilibrium of the TU environment indeed leads to a first best allocation. The proof proceeds by showing that the TU wages $w(ab)$ coincide with the social marginal benefit of investment by an individual of background b.

Lemma 1. *The equilibria of the TU environment lead to first best allocations:*

p te n s surplus efficiently en t e re l ze ttr utes, n n est ent le d s p ze e - nte tot l surplus net o n est ent costs.

3.3 NTU Market Equilibrium

Recall that in the laissez-faire environment agents split the surplus, each getting $z(ab, a'b')/2$ the Pareto frontier for a match $(ab, a'b')$ consists therefore of a single point.

The laissez-faire equilibrium allocation under strictly nontransferable utility has full segregation in attributes. This is because monotonicity of the function $z(\cdot)$ implies that $\max\{z(ab, ab), z(a'b', a'b')\} > z(ab, a'b')$. This in turn makes it impossible to have a positive measure of $(ab, a'b')$ firms, with $ab \; / \; a'b'$, in equilibrium because this would violate stability. Equilibrium pay s are therefore

$$w(hp) \quad 1, w(\ell p) \quad 0, w(hu) \quad \delta/2, w(\ell u) \quad 0.$$

Corresponding investment levels are

$$e_p^* \quad 1 \text{ and } e_u^* \quad \delta/2.$$

A comparison of laissez-faire market equilibrium investment levels e_b^* and the first best ones derived in Fact 2, visualized in Figure 2, yields the following proposition.

Proposition 1 (OTUB). *e pr e e o er- n est r* $\pi < 1$. *e un er- pr e e ne er o er- n est n un er- n est* $\pi > \frac{\delta}{2+\delta}$, *n e se t ere s ot o er- n est ent t t e top n un er- n est ent t t e otto o t e roun str ut on.*

The presence of simultaneous under-investment by the underprivileged and over-investment by the privileged is implied by two properties of the surplus function. First, diversity in backgrounds is beneficial holding constant the composition of achievements (this is implied by $\delta > 2/3$). The second property is complementarity of diversity and returns to investments (implied by separability of achievement and background in $z(.)$ and the fact that $g(u, p) > g(u, u)$).[10] Both properties guarantee that there will be over-investment at the top and under-investment at the bottom for π high enough.

[10] Desirabi ity of bac ground diversity is not a necessary condition for OT B in genera .

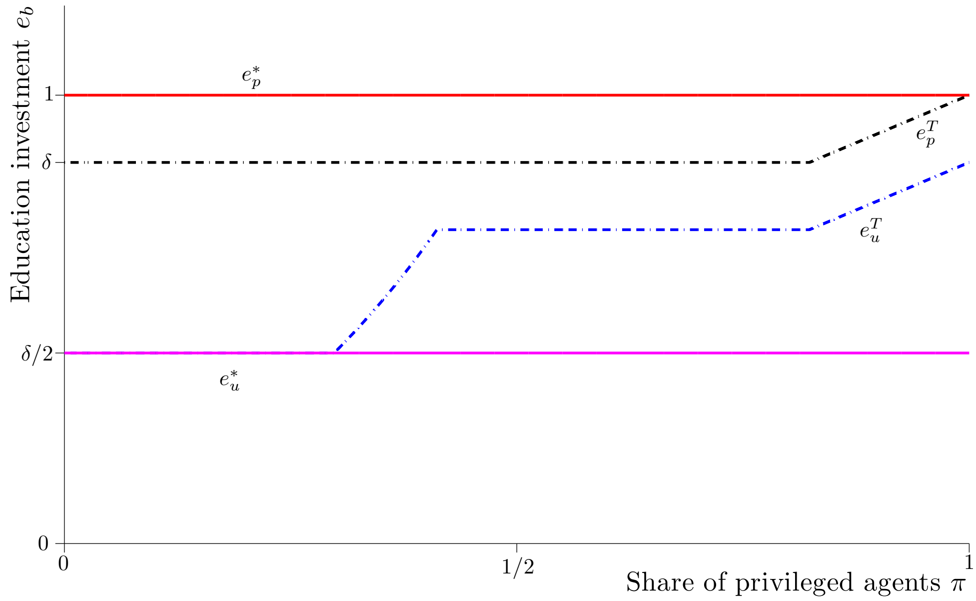


Figure 2 Education investments under laissez-faire and TU.

This observation extends to more general settings (details are available from the authors).

This result is interesting for several reasons. First, it states that excessive segregation as a consequence of market frictions may discourage the underprivileged, an effect that is often quoted as a rationale for rematch policies. Moreover, excessive segregation may encourage the privileged to invest beyond efficient levels. This would suggest that the discouragement effect that such policies arguably have on those not favored, i.e., the privileged, could be *desirable* from a total surplus point of view.

Second, the result connects well to empirical findings. Interpreting background as race, a black-white test score gap already in place at early ages (Heckman, 2008) would be amplified by background segregation. Recent evidence for this is provided in Card and Rothstein (2007) and Hanushek et al. (200). Interpreting background as gender, with females as underprivileged, links gender segregation in the workplace to female under- and male over-investment in education. (If females have a cost advantage in investment, then once the workplace is integrated, be it by policy or social change, females may have higher investment than males, as appears to be the case currently in the U.S. see Section 5.) And if privileged background corresponds to pref-

For instance OTB occurs in this setting a so when $< \delta <$ 3 for $\pi \in ($, $)$. It is necessary however that so e background integration occurs in the bench ark a ocation.

erential access to resources and markets, the pattern of investments appears similar to the observation in Banerjee and Munshi (2004) outsiders and insiders segregate, and the empirical evidence is consistent with under-invest by the former and over-investment by the latter.

Third, excessive segregation also has implications for inequality. Computing variance as a measure of inequality yields the following corollary.

Corollary 1. *E uc tion in est ents* e_b

4.1 Affirmative Action Policy

Affirmative action is defined as priority given to underprivileged background agents for positions at *a given* level of achievement.

Definition 1. Consider an equilibrium and a match $(ap, a'b)$. An *affirmative action policy* (denoted A policy) requires that an agent with attribute au must not strictly prefer to join $a'b$ to staying in his current assignment.

For instance, if there is a match $(hp, \ell p)$, then it must be the case that an agent hu does not strictly prefer to be in a match $(hu, \ell p)$ and that an agent ℓu does not prefer to be in a match $(hp, \ell u)$. That is, this rule gives precedence for an underprivileged candidate over a privileged competitor of the *same* achievement level. It is widely used (for instance the "positive equality bill" in the U.K., *Gleichstellung* in the German public service, or reservation of places for highly qualified minority students at the *grandes écoles* in France).

Note that some matching patterns will violate an A policy even if they are stable in the absence of this policy under nontransferable utility. For instance, consider a situation where attributes segregate, which is the equilibrium outcome under laissez-faire. Any match (hp, hp) clearly violates the policy, since a hu agent strictly gains by joining a hp agent, who strictly loses.

Lemma 2. *Under an A policy, low achievers do not match with high achievers, and all* (hp, hu) *matches are exhausted, that is the measure of such inter-attribute matches is* $\min\{(1-\pi)e_u, \pi e_p\}$.

Proof. While hp agents would prefer to segregate, since hu agents strictly prefer to match with a hp agent than with any other agent, (hp, hp) can arise only if there are no hu agents who are not matched with hp agents. Hence, all (hp, hu) matches must be formed, and there is a measure $\min\{(1-\pi)e_u, \pi e_p\}$ of such matches. The other high achievers segregate. There is indeterminacy for the matches of the low achievers, since any match between them give a zero output. □

The equilibrium matching pattern under an A policy is shown in Figure 3. Investment levels under an A policy depend on payoffs, which in turn depend on the likelihood an agent will be assigned to each attribute, that is, on relative scarcity of attributes in the market. The following statement

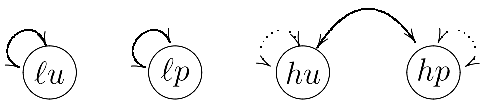


Figure 3 Equilibrium matching under an A policy.

sums up the properties of investments under an A policy details are in the appendix

F ct 3. Under an A policy $\pi e_p^A > (1-\pi)e_u^A$ if and only if $\pi > 1/2$, e_p^A and e_u^A increase in π and are given by

(i) $e_p^A \quad \delta$ and $e_u^A \quad \frac{\delta}{4}(1+\sqrt{1+8\frac{\pi}{1-\pi}})$ if $\pi < 1/2$,

(ii) $e_u^A \quad e_p^A \quad \delta$ if $\pi \quad 1/2$,

(iii) $e_p^A \quad \frac{1}{2}(1+\sqrt{1-4\frac{1-\pi}{\pi}\delta(1-\delta)})$ and $e_u^A \quad \delta$ otherwise.

$e_p^T \le e_p^A < e_p^*$ and $e_u^* \le e_u^T < e_u^A$ for $\pi \in (0,1)$.

That is, an A policy encourages the underprivileged and discourages the privileged compared to the laissez-faire outcome. Interestingly encouragement for the underprivileged is strong enough to generate investment e on the first best levels, i.e., there is overshooting for the underprivileged. In contrast, privileged agents investment levels are lower than under laissez-faire but still exceed the first best benchmark. Figure 4 compares investment levels under an A policy to both laissez-faire and benchmark levels. Total

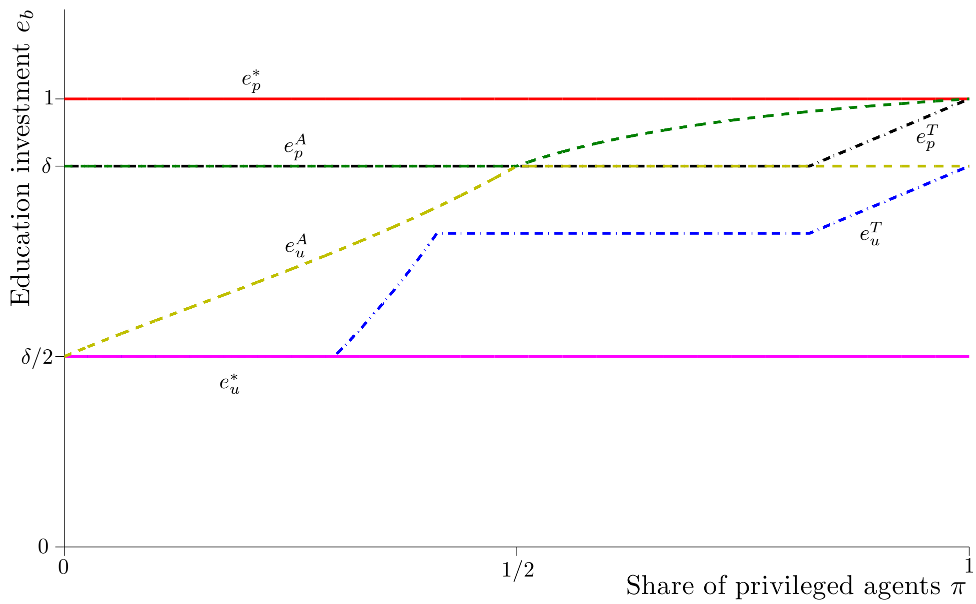


Figure 4 Education investments in the di erent regimes.

surplus can be expressed as

$$S = \pi\frac{(e_p)^2}{2} + \pi w(\ell p) + (1-\pi)\frac{(e_u)^2}{2} + (1-\pi)w(\ell u),$$

where $w(ab)$ denotes payoffs and e_b is the equilibrium investment. Since an A policy does not affect low achievers outcomes relative to laissez-faire, $w^A(\ell b) = w^*(\ell b)$, where we use the superscript A for an A policy and a star for laissez-faire. Therefore $S^A > S^*$ if, and only if

$$\pi\frac{(e_p^A)^2 - (e_p^*)^2}{2} + (1-\pi)\frac{(e_u^A)^2 - (e_u^*)^2}{2} > 0.$$

The left hand side can be decomposed into a static effect of correcting mismatch and a dynamic effect on investment incentives

$$\underbrace{\pi e_p^* w^A(hp) + (1-\pi)e_u^* w^A(hu) - [\pi e_p^* w^*(hp) + (1-\pi)e_u^* w^*(hu)]}_{\text{static output change by rematch given by payoff differences}}$$
$$+ \underbrace{\pi(e_p^A - e_p^*)w^A(hp)}_{\text{discouragement effect}} + \underbrace{(1-\pi)(e_u^A - e_u^*)w^A(hu)}_{\text{encouragement effect}}$$
$$- \underbrace{\frac{\pi[(e_p^A)^2 - (e_p^*)^2] + (1-\pi)[(e_u^A)^2 - (e_u^*)^2]}{2}}_{\text{investment cost change}} > 0,$$

While the static effect is always positive, the sign of the dynamic effect depends on the relative investment distortions under the two regimes. In aggregate, for an A policy to generate higher surplus than laissez-faire the encouragement effect on the underprivileged has to outweigh the discouragement effect on the privileged. The following proposition shows that this trade-off is linked to the diversity δ.

Proposition 2 (Affirmative Action Policy). *There is $\delta^*(\pi) \in [2/3, 2/\sqrt{7}]$ such that total surplus under an A policy is higher than under laissez-faire if and only if $\delta > \delta^*(\pi)$. $\delta^*(\cdot)$ attains unique maximum of $2/\sqrt{7}$ at $\pi = 1/2$.*

4.2 Background Integration

The goal of this policy is to remove segregation in backgrounds by giving underprivileged the option to match with a randomly drawn privileged, given the capacity constraint. This policy differs from an A policy in that it gives priority to u agents unconditional on achievement, and does not let u agents use information on achievement either.

Definition 2. A *busing policy* (denoted B policy) offers any u agent assignment to a p agent unconditional on achievement, using uniform rationing if necessary.

That is, this policy is best understood as one that departs from the laissez-faire outcome of full segregation and randomly reassigns agents to match the population measure π of privileged. This closely mirrors policies that are or have been used around the world. The most prominent are probably the use of "busing" in the U.S. to achieve school integration and reservation used in India to improve representation of schedule castes and tribes (other examples include the Employment Equality Act in South Africa, under which some industries such as construction and financial introduced employment or representation quotas, and the SAMEN law in the Netherlands, which has been repealed in 2003, however). Independence of the assignment rule on achievement means that both ℓ and h agents of background b have the same chance of being matched to a h agent of background b'. The following lemma and Figure 5 characterize the matching pattern under this policy.

Lemma 3. *Under a B policy a u agent obtains an hp match with probability $e_p \max\{\pi/(1-\pi), 1\}$ and an ℓp match with probability $(1-e_p)\max\{\pi/(1-\pi), 1\}$. If $\pi > 1/2$ ($\pi < 1/2$) a measure $(2\pi - 1)$ of privileged ($1 - 2\pi$) of underprivileged are [illegible] in achievements.*

Proof. u agents now have the outside option to match with a random p agent. Since hu agents prefer (hu, hu) to $(hu, \ell u)$ matches, u agents choose between a payoff of 0 for ℓu and $\delta/2$ for hu and random assignment to some p agent. Expected payoff from this is $e_p/2$ for ℓu and $\delta/2 + e_p\delta/2$ for hu agents. Since p are assigned randomly or segregate, an hp agent expects higher payoff than an ℓp agent, which implies $e_p > 0$. Therefore all u agents prefer random assignment to a p agent to their segregation payoff, which implies the statement. $\square$

Using Lemma 3 it is routine to compute the investment levels under a B policy

Fact 4. $e_u = \delta/2$ and $e_p = \frac{1-\pi}{\pi}\delta + \frac{2\pi-1}{\pi}$ if $\pi > 1/2$ and $e_p = \delta/2$ otherwise.

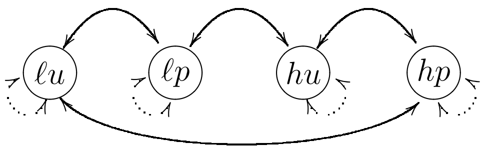


Figure 5 Equilibrium matching under an B policy.

Hence, a B policy has undesirable incentive effects. It does not encourage the underprivileged to invest more than under laissez-faire, while the privileged are discouraged substantially in fact there is undershooting in that the privileged agents invest below the efficient level when $\pi < (2-\delta)/\delta$. Figure 6 graphically compares investment levels under a B policy to both laissez-faire and benchmark levels.

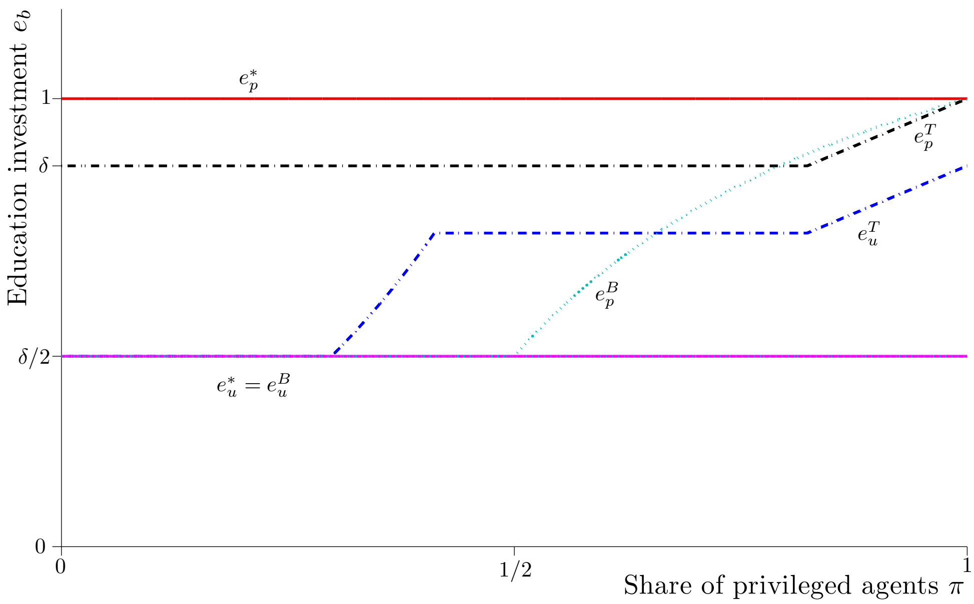


Figure 6 Education investments in the different regimes.

To examine whether these adverse incentive effects may be compensated by increased assignment quality let us turn to aggregate surplus. For $\pi \leq 1/2$ and $\delta^2 > 4/5$ total surplus under a B policy is

$$S^B \quad \frac{\delta^2}{8}(1+4\pi) > \pi\frac{1}{2} + (1-\pi)\frac{\delta^2}{8} \quad S^*,$$

Hence, if δ is sufficiently high, a busing policy generates higher total surplus than the laissez-faire allocation. Comparing total surplus under busing to total surplus under affirmative action, $S^B > S^A$ if

$$\frac{\delta^2}{8}(1+4\pi) > \pi\delta^2 + (1-\pi)\frac{e_u^A(\delta - e_u^A)}{2},$$

with $e_u^A = \delta/4(1+\sqrt{1+8\pi/(1-\pi)})$. Calculations reveal that $\pi > 3/4$ is necessary for this inequality to hold, a contradiction to our assumption that $\pi \leq 1/2$. Hence, $S^A > S^B$ for $\pi \leq 1/2$. This result extends to the case $\pi > 1/2$, treated in the appendix, as stated in the following proposition.

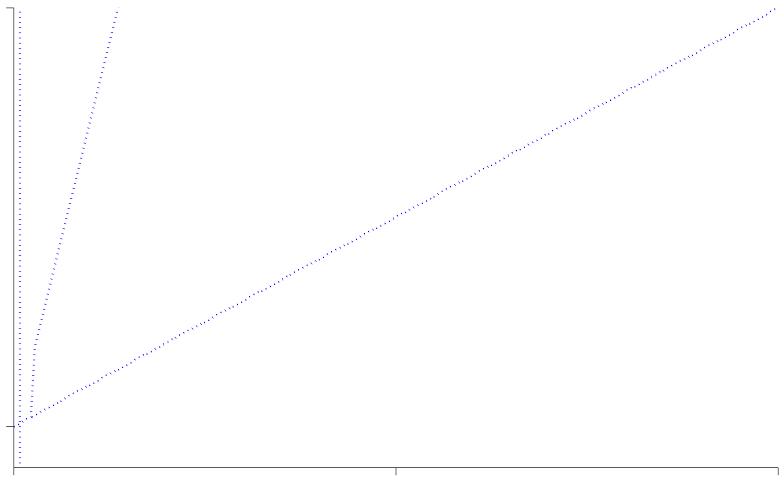
Proposition 3 (Busing Policy). *There is $\delta^B(\pi) \in [2(\sqrt{2}-1), 2/\sqrt{5}]$ such that total surplus under the busing policy is higher than under laissez-faire if and only if $\delta > \delta^B(\pi)$. δ^B attains its supremum of $2/\sqrt{5}$ for $\pi \leq 1/2$. Total surplus is always greater under an affirmative action policy than under a busing policy.*

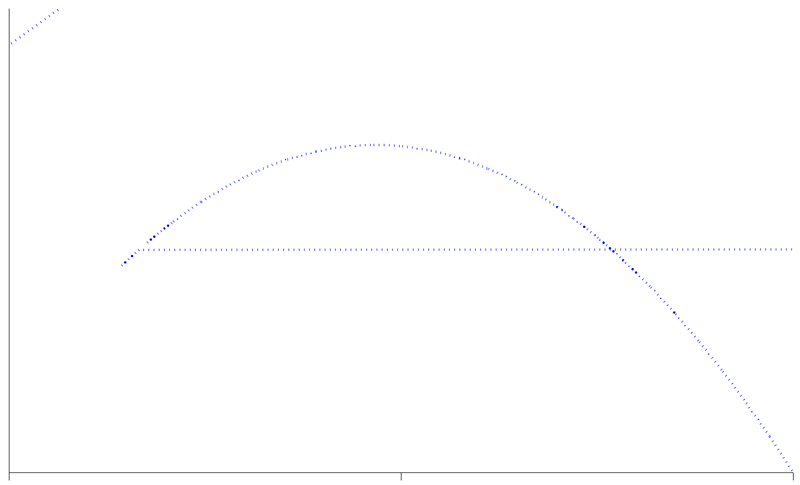
4.3 Discussion

To summarize, both rematch policies, affirmative action and busing, increase aggregate surplus compared to the laissez-faire outcome if diversity in teams is sufficiently desirable, i.e., if δ is large enough. Both polices improve the quality of sorting, but do not replicate the first best matching, and depress incentives of the privileged, which is desirable from a social point of view whenever there is over-investment at the top.

Yet A and B policies differ substantially in the aggregate investment level they induce; while an A policy encourages the underprivileged beyond the first best level and does not discourage the privileged below their first best level, a B policy does not encourage the underprivileged and tends to discourage the privileged below their first best level. Hence, an A policy leads to larger aggregate investment levels than a B policy or laissez-faire; moreover, aggregate investment under an A policy exceed the first best level. Because an A policy does not implement the first best matching, aggregate income may be higher in the first best allocation (see Figure 7).

The different policies also affect inequality of education acquisition and income quite differently; an A policy unambiguously reduces inequality compared to laissez-faire and to the first best benchmark, whereas a B policy may increase it for economies with a high share of privileged, see Figures 8 and 9.





e_u^B increase for low $\pi < 1/2$ but remain the same otherwise, resulting in an upwards shift of the lowest horizontal in Figures 3 and 5. The qualitative properties of the welfare comparisons remain unchanged, the threshold values $\delta^*(\pi)$ depend on β, however. For instance, a sufficient condition for an A policy to achieve higher surplus than laissez-faire is now $\delta^2 > (1+\beta^2)/2$.

5.1 Heterogenous Cost

Often background not only affects gains from matching through the function $g(b, b')$ but also the cost of investment itself. Assume therefore that an agent of background b who chooses education effort e incurs cost e^2/θ_b. The interesting case occurs when $\theta_p < \theta_u \leq 1$, i.e., the under-privileged have a cost advantage at investment, but a disadvantage at marketing that investment compared to the privileged. In this case investments are given by

$$e_b = \theta_b[w(hb) - w(\ell b)],$$

with $w(ab)$ denoting the market payoff of an agent with attribute ab. Therefore under laissez-faire $e_p = \theta_p$ and $e_u = \theta_u\beta$.

Indeed there is scope for investment distortions generated by excessive segregation and a version of Proposition 1 holds.

Fact 5. Both e_p and e_u increase in π, with $e_p^* \geq e_p^T$ and $e_u^* \leq e_u^T$. There is simultaneous over-investment by the privileged and under-investment by the underprivileged if $\beta\theta_u/(1+\beta\theta_u) < \pi < \delta\theta_u/(1-\theta_p-\delta\theta_u)$. $e_p^* > e_u^*$ if, and only if, $\theta_p > \theta_u\beta$, but $(e_p^T - e_u^T)$ decreases in π and $e_p^T > e_u^T$ for all $\pi \in [0, 1]$ if and only if $\theta_p > \delta\theta_u$.

Notice that it is possible that while the underprivileged invest less than the privileged under laissez-faire, they invest *more* in the surplus efficient outcome, which is characterized by integration of underprivileged high achievers. This can occur when the share of privileged is sufficiently high, or when the cost advantage compensates the underprivileged's disadvantage in the labor market. Returning to the interpretation of background as gender, this appears consistent with the move from a segregated to a more integrated labor market outcome over the last decades, accompanied by a reversal of educational inequality, at least measured in years of schooling. This change might have been brought about by policy, or by social change either ameliorating payoff rigidities, or increasing the benefits of gender diversity in the workplace δ, a possibility that we will explore in greater detail below.

5.2 Some Transferability

Suppose agents can transfer surplus within a firm up to some exogenous limit L, which may be interpreted as individuals liquidity, for instance. If $L < \min\{\beta - \delta/2\ \ 1 - \delta\}$, there is segregation in the laissez-faire equilibrium allocation and investments are given by $e_p^* \quad \theta_p$ and $e_u^* \quad \beta\theta_u$ as above.

Another plausible case arises when there is some transferability, possibly because p agents are privileged also in terms of the ability to make transfers within a match, possibly because of better access to credit or greater wealth. The following proposition states the properties of the laissez-faire allocation in such a situation, the details are in the appendix.

Proposition 4. *Let $L_u < 1-\delta$ n $L_p > \beta-\delta/2$. n t el issez- re outco e ll possi le $(hu, \ell p)$ t es re e uste , ll ℓu n hp ents se te. en p ents un er- n est r lo n nter e te π n u un er- n est r nter e te π.*

Note that the same outcome occurs when all agents are subject to the same liquidity constraint L and $\beta - \delta/2 < L < 1 - \delta$. The properties of the resulting matching pattern carry a "glass ceiling" flavor the underprivileged match with the privileged, but only in $(hu, \ell p)$ firms, not in (hu, hp) firms. For intermediate π this is reflected in wages and investments compared to the laissez-faire allocation with $L < \min\{\beta - \delta/2\ \ 1 - \delta\}$ (when the labor market fully segregates) underprivileged high achievers earn higher wages and choose higher education investments, though these still fall short of the TU benchmark. Moreover, there are parameters such that for intermediate π, $e_u > e_p$ for $\beta - \delta/2 < L < 1 - \delta$, but $e_u < e_p$ if $L < \beta - \delta/2 < 1 - \delta$. That is, a change of the labor market outcome toward more integration as a consequence of less pay rigidities or greater desirability of diversity in the work place can be accompanied by a reversal of educational inequality.[12] In particular the latter seems consistent with the changes in women s educational achievements and labor market participation over the last three or

[12] An increase in L ay be associated to better credit ar et conditions abor ar et deregu ation or better contract enforce ent due to i prove ents in the ega syste . An increase of δ ay be attributed to a change in the disadvantage of a ixed (u, p) partnership co pared to a privi eged partnership for instance due to transaction cost or socia stig a.

four decades, see e.g., Goldin et al. (2006).

6 Conclusion

Excessive segregation could be construed as "discrimination" our framework provides a fresh perspective on this, since "discrimination" arises here because of the failure of the price system — the rigidity in surplus allocation within firms — and not because of a taste for discrimination or self-fulfilling beliefs about the productive abilities associated with certain backgrounds.

By comparing two simple policies, one based on both background and achievement and the other based simply on a priority given to underprivileged, we show that these two policies may improve on the laissez-faire and can be ranked in terms of aggregate performance. However, their ranking in terms of inequality in achievement and earnings, is a function of the relative proportions of privileged and under-privileged, suggesting against a one-size-fit-all approach for correcting mismatches.

Because the set of policies we examine is clearly not exhaustive, our analysis provides a lower bound on the potential benefits of rematch policies. While of interest, the question of the "optimal policy" is best left to future research. This quest will require us, for instance, to depart from the assumption that all agents benefit from the policy, or it may require complex contingencies, which will raise the issue of its practical implementation. For these reasons we feel that our focus on policies that are actually used by policymakers around the world provides a first and convincing argument of the economic benefits of rematch policies when there are rigidities in surplus division within firms. What is clear however is that the optimal policy will not be to try to mimic the first-best matching outcome as we have noted in section 4.3, this policy not only weakens incentives for all agents but may also lead to a lower aggregate surplus than our A policy.

An important extension of the approach will be to a dynamic setting. Indeed, while our approach assumes fixed and exogenous proportions of privileged and under-privileged, these proportions may be the consequence of historical policies that discriminated against some ethnic backgrounds. A dynamic version of our model could provide an economic interpretation of affirmative action and positive discrimination aiming to "right past wrongs". For instance in such a dynamic setting the value of δ, one of the key parameters of our analysis, is likely to reflect the degree of diversity in equilibrium matches from previous periods.

Since, as found in section 4.3, rematch policies may induce a higher aggregate level of education than laissez-faire, the price of education is also likely to increase, generating a potential dampening effect for the policy from a static perspective. On the other hand, also aggregate income may be higher under a rematch policy than under laissez-faire, providing higher income to pay (or to borrow) for education. Beyond aggregate levels, the change in the inequality in access to education or in income levels due to rematch policies documented above may also affect growth. Nevertheless the relationships between aggregate levels, inequality of education or income, and growth are complex, and a full analysis is best left for further research.

Finally, as we emphasized in parts of the text, δ may capture not so much the actual production differences of different backgrounds, but the ability of the agents to market their output, which could be due to differences in access to the financial market, or to networks of established traders. For instance "old boy network" types of phenomena may lead to a low value of δ for women and lead not only to difficulties to generate integration between men and women of a given achievement level but also to depressed incentives for women to achieve this level. This suggests that matching policies in one market impact on the performance of matching policies in another market, posing the interesting question of the complementarity or substitutability of rematch policies on sequential markets.

A Mathematical Appendix: Proofs

Proof of Fact 1

For (i)

$$z(hp, hp) + z(\ell u, \ell u) \quad 2 > 2\delta \quad 2z(hb, \ell u),$$

implying that any contract for $(hp, \ell u)$ can be competed away by (hp, bp) since $z(\ell u, \ell u) \quad 0$. For (ii)

$$z(hp, hp) + z(\ell p, \ell p) \quad 2 \geq 2 \quad 2z(hp, \ell p) \text{ and}$$
$$z(hu, hu) + z(\ell u, \ell u) \quad \delta \geq \delta \quad 2z(hu, \ell u).$$

For (iii)

$$z(hp, hp) + z(hu, hu) \quad 2 + \delta > 4\delta \quad 2z(hp, hu)$$

if and only if $\delta < 2/3$.

For (iv)

$$z(hu, hu) + z(\ell p, \ell p) \quad \delta < 2\delta \quad 2z(hu, \ell p)$$

implying that segregation for $hu, \ell p$ is unstable since $(hu, \ell p)$ matches can offer strictly higher payoffs to both attributes. Since $hu, \ell p$ will not segregate, let us compare now the benefit of hu integrating with ℓp versus integrating with hp. By facts (i), (ii) and (iii), it is sufficient to compare the matching pattern $\{(hp, hp), (hu, \ell p), (\ell u, \ell u)\}$ to the matching pattern $\{(hp, hu), (\ell p, \ell u)\}$. In the first pattern, equal treatment implies that hp get y, since ℓp segregates and gets zero payoff and therefore in the match $(hu, \ell p)$, hu gets δ. Note that $1 + \delta > 2\delta$ while (hp, hu) can share at most 2δ in the second pattern. Since $\delta < 1$, the second pattern cannot be stable.

Proof of Fact 2

Depending on relative scarcity of hu, ℓp, and hp agents we distinguish five cases.

Case (1) $\pi(1 - e_p) < (1 - \pi)e_u < \pi$. Then some hp segregate and $w(hp) \quad 1$, hu match with hp and ℓp and obtain $w(hu) \quad 2\delta - 1$. Therefore $w(\ell p) \quad \delta - (2\delta - 1)$. This implies $e_u \quad (2\delta - 1)$ and $e_p \quad \delta$. The condition $\pi(1 - e_p) < (1 - \pi)e_u < \pi$ becomes

$$\frac{1 - \delta}{(2\delta - 1)} < \frac{1 - \pi}{\pi} < \frac{1}{(2\delta - 1)}.$$

Case (2) $\pi < (1 - \pi)e_u$. Then some hu segregate and $w(hu) \quad \delta/2$. $w(hp) \quad 2\delta - w(hu)$ and $w(\ell p) \quad \delta - w(hu)$. Therefore $e_u \quad \delta/2$ and $e_p \quad \delta$. The condition becomes

$$\frac{1 - \pi}{\pi} > \frac{2}{\delta}.$$

Case (3) $\pi(1-e_p) > (1-\pi)e_u$. Then hp segregate, so that $w(hp) \quad y$. ℓp oversupplied, therefore $w(\ell p) \quad 0$ and $w(hu) \quad \delta$. $e_p \quad 1$ and $e_u \quad \delta$. Hence, case (2) obtains if $0 > (1-\pi)\delta/\pi$, which is a contradiction to $\pi \in [0,1]$ and $\delta > 1/2$.

Case (4) $\pi(1-e_p) \quad (1-\pi)e_u < \pi$. Again hp segregate, so that $w(hp) \quad 1$. $w(\ell p) \quad \delta - w(hu)$ and $(2\delta - 1) \leq w(hu) \leq \delta$. $e_u \quad w(hu)$ and $e_p \quad (1-\delta) + w(hu)$. That is,

$$w(hu) \quad \pi\delta \text{ and } w(\ell p) \quad (1-\pi)\delta + \pi.$$

This case obtains if

$$0 \leq \frac{1-\pi}{\pi} \leq \frac{1-\delta}{2\delta - 1}.$$

Case (5) $\pi(1-e_p) < (1-\pi)e_u \quad \pi$. Then $w(\ell p) \quad \delta - w(hu)$ and $w(hp) \quad 2\delta - w(hu)$, and $\delta/2 \leq w(hu) \leq 2\delta - 1$. $e_p \quad \delta$ and $e_u \quad w(hu)$ $\pi/(1-\pi)$. For this case we need

$$\frac{2}{\delta} \geq \frac{1-\pi}{\pi} \geq \frac{1}{2\delta - 1}.$$

Summarizing, $e_p \quad \delta$ if $\pi \leq \frac{2\delta-1}{\delta}$ and $e_p \quad 1$ if $\pi \geq 1$, and $\delta < e_p < 1$ otherwise. $e_u \quad \frac{\delta}{2}$ if $\pi \leq \frac{\delta}{2+\delta}$, $\frac{\delta}{2} < e_u < 2\delta - 1$ if $\frac{\delta}{2+\delta} < \pi < \frac{2\delta-1}{2\delta}$, $e_u \quad 2\delta - 1$ if $\frac{(2\delta-1)}{2\delta} \leq \pi \leq \frac{2\delta-1}{\delta}$, $2\delta - 1 < e_u < \delta$ if $\frac{2\delta-1}{\delta} < \pi < 1$, and $e_u \quad \delta$ if $\pi \geq 1$.

TU Wages for $\delta < 2/3$

F ct 6. If $\delta < 2/3$ first best investments are

(i) $e_p \quad (1-\delta/2)$ if $\frac{1-\pi}{\pi} \geq 1$, $e_p \quad 1$ if $\frac{1-\pi}{\pi} < 0$, and $(1-\delta/2) < e_p < 1$ otherwise.

(ii) $e_u \quad \delta/2$ if $\frac{1-\pi}{\pi} \geq 1$, $e_u \quad \delta$ if $\frac{1-\pi}{\pi} < 0$, and $\delta/2 < e_u < \delta$ otherwise.

(iii) e_p decreases in π and e_u increases in π.

roo Since $\delta < 2/3$ the first best exhausts all $(hu, \ell p)$ matches, all remaining types segregate. Therefore

$$w(hp) \quad 1 \text{ and } w(\ell u) \quad 0.$$

Wages for other attributes depend relative scarcity. Three di erent cases may arise.

Case (1) $\pi(1-e_p) < (1-\pi)e_u$. hu are oversupplied, therefore $w(hu)$ $\delta/2$ and $w(\ell p)$ $\delta/2$. e_p $(1-\delta/2)$ and e_u $\delta/2$. Hence, case (1) if

$$\frac{1-\pi}{\pi} > 1.$$

Case (2) $\pi(1-e_p) > (1-\pi)e_u$. Now ℓp are oversupplied, therefore $w(\ell p)$ 0 and $w(hu)$ δ. e_p 1 and e_u δ. Hence, case (2) obtains if

$$\frac{1-\pi}{\pi} < 0.$$

Case (3) $\pi(1-e_p)$ $(1-\pi)e_u$. Then $w(\ell p)$ $\delta - w(hu)$ and $\delta/2 \leq w(hu) \leq \delta$. e_u $w(hu)$ and e_p $(1-\delta)+w(hu)$. That is,

$$w(hu) \quad \pi\delta \text{ and } w(\ell p) \quad (1-\pi)\delta.$$

This case obtains if

$$0 \leq \frac{1-\pi}{\pi} \leq 1.$$

These cases establish the statement above. □

Proof of Lemma 1

To establish static surplus efficiency suppose the contrary, i.e. there are agents with equilibrium payoffs $w(ab)+w(a'b') < z(ab,a'b')$. Then there are feasible wages $w'(ab)+w'(a'b')$ $z(ab,a'b')$, which both strictly prefer to their equilibrium payoffs, a contradiction to stability. Therefore matching is surplus efficient given investments.

The second part of the lemma on efficiency of investments requires some work. Let $\{ab\}$ denote a distribution of attributes in the economy, and $\mu(ab,a'b')$ the measure of $(ab,a'b')$ firms in a surplus efficient match given $\{ab\}$. Since $\mu(ab,a'b')$ only depends on aggregates πe_p, $\pi(1-e_p)$, $(1-\pi)e_u$, and $(1-\pi)(1-e_u)$ and investment cost is strictly convex, in an allocation maximizing total surplus all p agents invest the same level e_p, and all u agent invest e_u.

An investment profile (e_u, e_p) and the associated surplus efficient match $\mu(.)$ maximize total surplus ex ante if there is no (e'_u, e'_p) and an associated surplus efficient match $\mu(.)$ such that total surplus is higher.

Denote the change in total surplus Δ_b by increasing e_b to e'_b $e'_b+\epsilon$. If there are positive measures of (hp, hp) and (hp, hu) firms, it is given by

$$\begin{aligned}\Delta_p &= \epsilon[z(hp, hu) - z(\ell p, hu)] - \epsilon e_p - \epsilon^2/2 \text{ and} \\ \Delta_u &= \epsilon[z(hp, hu) - z(hp, hp)/2] - \epsilon e_u - \epsilon^2/2,\end{aligned}$$

reflecting the gains from turning an ℓp agent matched to an hu agent into an hp agent matched to an hu agent, and from turning an ℓu agent matched to an ℓu agent into an hu agent matched to an hp agent, who used to be matched to an hp agent.

That is, assuming that indeed $\pi > (1-\pi)e_u > \pi(1-e_p)$ the optimal investments are given by $e_p = z(hp, hp)/2$ and $e_u = z(hp, hu) - z(hp, hp)/2$. Recall that TU wages are given in this case by $w(hp) = z(hp, hp)/2 = 1$ and $w(\ell p) = z(hu, \ell p) - w(hu)$, and $w(hu) = z(hp, hu) - z(hp, hp)/2 = 2\delta - 1$ and $w(\ell u) = 0$. Hence, TU investments are $e_p^T = z(hp, hu) - z(hu, \ell p)$ and $e_u^T = z(hp, hu) - z(hp, hp)/2$. That is, TU investments are optimal with respect to marginal deviations.

Checking for larger deviations suppose only e_u increases by ϵ, such that the measure of (hu, hu) firms becomes positive after the increase. The change in total surplus is now

$$\Delta = \epsilon_1[z(hp, hu) - z(\ell p, hu)] + \epsilon_2[z(hu, hu)/2 - z(\ell u, \ell u)/2] - \epsilon e_p - \epsilon^2/2,$$

for $\epsilon_1 + \epsilon_2 = \epsilon$ such that the measure of (hp, hp) under e_u was $\epsilon_1/2$. Clearly, $\Delta < 0$ for $e_u = z(hp, hu) - z(\ell p, hu)$, since cost is convex and surplus has decreasing returns in an efficient matching. Suppose now that e_p decreases by ϵ large enough to have a positive measure of $(\ell p, \ell p)$ firms after the decrease (a decrease in e_u would have the same effect). The change in total surplus is

$$\Delta = -\epsilon_1[z(hp, hu) - z(\ell p, hu)] - \epsilon_2[z(hp, hp)/2 - z(\ell p, \ell p)/2] + \epsilon e_p - \epsilon^2/2,$$

which is negative for $e_p = z(hp, hu) - z(hu, \ell p)$ since cost is convex and surplus has decreasing returns in an efficient matching. Finally, an increase of e_p will not affect the condition $\pi > (1-\pi)e_u > \pi(1-e_p)$.

A similar argument holds for all five cases described in the proof of Fact 2.

Proof of Fact 3

Since low achievers match with low achievers $w(\ell p) = w(\ell u) = 0$. High achievers payoffs depend on relative scarcity, however.

Case 1 $(1-\pi)e_u \geq \pi e_p$. Then hu agents outnumber hp agents and $w(hp) = \delta$. The expected payoff of an hu agent is

$$Ew(hu) = \frac{\pi e_p}{(1-\pi)e_u}\delta + \left(1 - \frac{\pi e_p}{(1-\pi)e_u}\right)\frac{\delta}{2}.$$

Since $e_p = w(hp) - w(\ell p) = \delta w$, and $w(\ell u) = 0$ this becomes a quadratic equation in e_u. It has a solution $\delta/2 \leq e_u \leq \delta$ if $\pi \leq 1/2$, which is given by

$$e_u = \frac{\delta}{4}\left(1 + \sqrt{1 + 8\frac{\pi}{1-\pi}}\right).$$

Case 2 $(1-\pi)e_u < \pi e_p$. Then hp agents outnumber hu agents and $w(hu) = \delta$. The expected payoff of an hp agent is

$$Ew(hu) = \frac{(1-\pi)e_u}{\pi e_p}\delta + \left(1 - \frac{(1-\pi)e_u}{\pi e_p}\right).$$

Since $e_u = w(hu) - w(\ell u) = \delta$, and $w(\ell p) = 0$ this becomes a quadratic equation in e_p. It has a solution $\delta \leq e_p \leq 1$ if $\pi \geq 1/2$, which is given by

$$e_p = \frac{1}{2}\left(1 + \sqrt{1 - 4\frac{1-\pi}{\pi}\delta(1-\delta)}\right).$$

The last statement follows from comparing e^B to the first best and laissez-faire levels.

Proof of Proposition 2

Note that $e_p^* = 1$ and $e_u^* = \delta/2$. Given the expressions in the text, $S^A > S^*$ if and only if

$$\pi\frac{\delta^2 - 1}{2} + (1-\pi)\frac{\delta^2(1 + \sqrt{1 + 8\frac{\pi}{1-\pi}})^2 - 4\delta^2}{32} > 0 \text{ if } \pi \leq 1/2$$

$$\pi\frac{(1 + \sqrt{1 - 4\frac{1-\pi}{\pi}\delta(1-\delta)})^2 - 4}{8} + (1-\pi)\frac{3\delta^2}{8} > 0 \text{ if } \pi > 1/2.$$

For $\pi \leq 1/2$ calculations reveal that $S^A > S^*$ if and only if $\delta > \delta^*(\pi)$ with

$$\delta^*(\pi) \quad \frac{2}{3 - \frac{1}{4}\frac{1-\pi}{\pi}\left(\sqrt{1 + 8\frac{\pi}{1-\pi}} - 1\right)}$$

Taking the derivative reveals that $\delta^*(\pi)$ increases in π on $[0, 1/2]$, with $\delta^*(1/2) \quad 4/7$ and $\delta^*(0) \quad 2/3$. For $\pi \geq 1/2$ δ^* has to satisfy

$$\frac{1-\pi}{\pi}\delta^*(7\delta^* - 4) \quad 2\left(1 + \sqrt{1 - 4\frac{1-\pi}{\pi}\delta^*(1-\delta^*)}\right).$$

Solving numerically yields that there is a unique $\delta^*(\pi)$ in $[2/3, 1]$ for $\pi \in [1/2, 1]$. It decreases in π, $\delta^*(1/2) \quad 2/\sqrt{7}$, and $\delta^*(1) \quad 8/11$.

Proof of Fact 4

Suppose first that $\pi \geq 1/2$. Then

$$w(hu) \quad \frac{\delta}{2}(1 + e_p) \text{ and } w(\ell u) \quad \frac{\delta}{2}e_p.$$

Therefore $e_u \quad \delta/2$. p agents obtain a p match with probability $(2\pi - 1)/\pi$, in which case the policy allows them to segregate in achievement. Hence,

$$w(hp) \quad \frac{1-\pi}{\pi}\frac{\delta}{2}(1 + e_u) + \frac{2\pi - 1}{\pi} \text{ and } w(\ell p) \quad \frac{1-\pi}{\pi}\frac{\delta}{2}e_u.$$

Therefore

$$e_p \quad \frac{1-\pi}{\pi}\frac{\delta}{2} + \frac{2\pi - 1}{\pi}.$$

If $\pi < 1/2$ on the other hand,

$$w(hp) \quad \frac{\delta}{2}(1 + e_u) \text{ and } w(\ell p) \quad \frac{\delta}{2}e_u.$$

Therefore $e_p \quad \delta/2$. u agents obtain a p match with probability $\pi/(1-\pi)$, and otherwise the policy allows them to segregate in achievement. Hence,

$$w(hu) \quad \frac{\pi}{1-\pi}\frac{\delta}{2}(1 + e_p) + \frac{1-2\pi}{1-\pi}\frac{\delta}{2} \text{ and } w(\ell u) \quad \frac{\pi}{1-\pi}\frac{\delta}{2}e_p.$$

Then $e_u \quad \delta/2$, which holds since $z(hu, \ell p) \quad z(hu, hu)$.

Proof of Proposition 3

The case $\pi \leq 1/2$ has been dealt with in the text. Suppose therefore $\pi > 1/2$. Using the expression for e_p^B total surplus under busing is

$$S^B \quad \frac{3}{8}(1-\pi)\delta^2 + \frac{1}{2}\left(\frac{1-\pi}{\pi}\frac{\delta}{2} + \frac{2\pi-1}{\pi}\right)\left((1-\pi)\frac{3}{2}\delta + 2\pi - 1\right).$$

Since $S^* \quad \pi/2 + (1-\pi)\delta^2/8$, $S^B > S^*$ if

$\delta TTT\delta\delta TTu$

$$\delta > \frac{2}{3-\pi}\left(\sqrt{13\pi^2 - 6\pi + 1} - 2(2\pi - 1)\right) \quad \delta^B(\pi).$$

It is readily verified that $\delta^*(\pi)$ strictly decreases in π, $\delta^*(1/2) \quad \sqrt{4/5}$ and $\delta^*(1) \quad 2(\sqrt{2}-1)$.

Comparing total surplus under a busing policy to the one

Proof of Proposition 4

Since $L_p > \beta - \delta/2$ $(hu, \ell p)$ matches will form since $z(hu, hu) + z(\ell p, \ell p) < 2z(hu, \ell p)$ and wages $w(hu) > \beta$ are possible in an $(hu, \ell p)$ firm. (hu, hp) will not form since wages $w(hp) > 1$ are not possible in an (hu, hp) firm since $L_u < 1 - \delta$. Therefore hp and ℓu agents segregate, so that $w(hp) \quad 1$ and $w(\ell u) \quad 0$ in all cases. All possible $(hu, \ell p)$ matches form, however, and pay s will depend on relative scarcity.

(i) $(1-\pi)e_u > \pi(1-e_p)$, that is, hu agents are oversupplied and $w(hu) \quad \beta$ and $w(\ell p) \quad \delta-\beta$. Therefore $e_p \quad \theta_p(1+\beta-\delta) < \theta_p\delta \quad e_p^T$ and $e_u \quad \theta_u\beta \quad e_u^T$ if $\pi < \frac{\theta_u\beta}{\theta_u\beta+1-\theta_p(1-\delta+\beta)}$.

(ii) $(1-\pi)e_u < \pi(1-e_p)$, that is, ℓp agents are oversupplied and $w(hu) \quad \delta$ and $w(\ell p) \quad 0$. Therefore $e_p \quad \theta_p \quad e_p^T$ and $e_u \quad \theta_u\delta \quad e_u^T$ if $\pi > \delta\theta_u/(\delta\theta_u + 1 - \theta_p)$.

(iii) $(1-\pi)e_u \quad \pi(1-e_p)$, that is $w(hu) \quad \delta - w(\ell p) \quad e_u/\theta_u$ and $e_p \quad \theta_p(1 - w(\ell p))$. This yields $w(\ell p) \quad \frac{(1-\pi)\delta\theta_u - \pi(1-\theta_p)}{(1-\pi)\theta_u + \pi\theta_p}$, so that $e_p \quad e_p^T$ and $e_u \quad e_u^T$ for $\pi \geq \frac{1+\theta_u-(1-\delta)\theta_p}{1+(1+\delta)\theta_p-\theta_u}$ and $e_p < e_p^T$ otherwise. Since $e_u^T \quad \theta_u(2\delta-1)$ for $\frac{(2\delta-1)\theta_u}{(2\delta-1)\theta_u+(1-\delta\theta_p)} < \pi < \frac{1+\theta_u-(1-\delta)\theta_p}{1+(1+\delta)\theta_p-\theta_u}$ and $e_u^T > \theta_u\beta$ whenever $\pi > \frac{\beta\theta_u}{\beta\theta_u+1}$ we have that $e_u < e_u^T$ if $\frac{\beta\theta_u}{\beta\theta_u+1} < \pi < \frac{1+\theta_u-(1-\delta)\theta_p}{1+(1+\delta)\theta_p-\theta_u}$.

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