One-sided Commitment and College Enrollment

B. Ravikumar, Yaping Shan and Yuzhe Zhang

St Louis Fed, University of Iowa and Texas A&M

June 2012
Financing college education

Student loan has been steadily rising, is more than credit card debt now.
GROWTH OF STUDENT LOAN VOLUME (IN CONSTANT DOLLARS), PERCENT GROWTH SINCE 1990

Student Loan Volume:
Cumulative Growth Rate = 282%
Average Annual Growth Rate = 8%

Sources: U.S. Department of Education, Office of Postsecondary Education and FY2009 President’s Budget.
Financing college education

- Student loan has been steadily rising, is more than credit card debt now.

- Financial resources of the family have become more important in the college enrollment decision.

- Skills and future earnings serve as poor collateral.
Financing college education

- Student loan has been steadily rising, is more than credit card debt now.

- Financial resources of the family have become more important in the college enrollment decision.

- Skills and future earnings serve as poor collateral.

Endogenous borrowing constraints

- Ability-enrollment correlation.
Part 1: Principal-agent relationship

- Borrowing over the life cycle.
- One-sided commitment.

Part 2: College enrollment

- Role of life-cycle consumption smoothing.
- Role of one-sided commitment.
An agent (or a consumer, or a student) lives for $T$ periods. Preferences are

$$\sum_{t=0}^{T} \beta^t u(c_t).$$

His initial wealth is $W$.

Earnings profile $w_t$ has a hump shape, that is, there is a $T^*$ such that $w_t$ increases with $t$ before $T^*$ and decreases after $T^*$.

There is a risk-neutral principal whose discount factor is also $\beta$. 
At time 0, the agent can sign a contract with the principal. The principal takes the wealth and income of the agent, and in exchange, the agent receives a consumption path \( \{ c_t; t = 0, ..., T \} \).

The agent can choose to leave the contract.

Default implies
- Autarky
- Fraction \( \gamma \) of his labor income seized every period.

Participation constraint

\[
\sum_{s=t}^{T} \beta^s u(c_s) \geq \sum_{s=t}^{T} \beta^s u((1 - \gamma)w_s), \forall t.
\]
Constrained efficient allocation

\[
\max \{c_t; t \geq 0\} \sum_{t=0}^{T} \beta^t u(c_t),
\]

subject to \[
\sum_{t=0}^{T} \beta^t c_t = \sum_{t=0}^{T} \beta^t w_t + W
\]
\[
\sum_{s=t}^{T} \beta^s u(c_s) \geq \sum_{s=t}^{T} \beta^s u((1 - \gamma)w_s), \forall t.
\]
It is useful to view the participation constraints in terms of flows.

Normalized outside option

\[
V_t = \frac{\sum_{s=t}^{T} \beta^s u ((1 - \gamma) w_s)}{\sum_{s=t}^{T} \beta^s}.
\]
Normalized outside option \( \underline{V}(\cdot) \)
First-order condition

\[(1 + \beta^{-t} \lambda_0 + \beta^{1-t} \lambda_1 + \ldots + \lambda_t) u'(c_t) = \Phi,\]

where \(\Phi\) is the multiplier for the budget constraint and \(\lambda_t\) is for the participation constraint (PC) in period \(t\).

\[c_t \begin{cases} 
  = c_{t-1}, & \text{if PC is slack at } t; \\
  > c_{t-1}, & \text{if PC is binding at } t.
\end{cases}\]
Minimum consumption $c_t$

- Let $c_t$ be the agent’s consumption if the participation constraint binds.
- It is the minimum consumption required to prevent default.
- Efficient allocation:

$$c_t = \max\{c_{t-1}, c_t\}.$$
The minimum consumption is \((1 - \gamma)w_t\) until period \(T_1\) and \(u^{-1}(\mathcal{V}_t)\) afterward.
The participation constraint is initially slack, then binds and finally becomes slack for $t \geq T_1$. 

Minimum consumption $c_t$
Minimum consumption before $T_1$

If the participation constraint binds at both $t$ and $t + 1$, then

$$c_t = (1 - \gamma)w_t.$$ 

$$\sum_{s=t}^{T} \beta^s u(c_s) = \sum_{s=t}^{T} \beta^s u((1 - \gamma)w_s)$$

$$\sum_{s=t+1}^{T} \beta^s u(c_s) = \sum_{s=t+1}^{T} \beta^s u((1 - \gamma)w_s)$$

imply that

$$u(c_t) = u((1 - \gamma)w_t).$$
Implementation with endogenous borrowing constraint

- How to decentralize the constrained efficient allocation?

- Let the agent optimally borrow and save at the interest rate
  \[ r = \frac{1}{\beta} - 1 \]
  - Subject to a sequence of borrowing constraints.
Problem P:

\[
\max_{c_t; t \geq 0} \sum_{t=0}^{T} \beta^t u(c_t),
\]
subject to

\[
c_t + \beta B_{t+1} = B_t + w_t
\]

\[
B_t \geq B_t, \forall t,
\]

\[
B_0 = W,
\]

where $B_t$ is the endogenous borrowing constraint.
How to find the sequence $B_t$?

Construct $B_t$ such that an agent with wealth $B_t$ in problem $P$ achieves the same utility as in autarky.

This construction satisfies the participation constraint in every period.

**Proposition**

The borrowing constraint is initially slack, then binds and finally becomes slack for $t \geq T_1$. 
Borrowing constraint

- If \( t < T_1 \) and \( B_t = \underline{B}_t \), then the agent’s participation constraint binds.

- His consumption path is
  \[
  (1 - \gamma)w_t, (1 - \gamma)w_{t+1}, \ldots, (1 - \gamma)w_{T_1-1}, (1 - \gamma)w_{T_1}, (1 - \gamma)w_{T_1}, \ldots
  \]

\[
-B_t = \sum_{s=t}^{T} \beta^{s-t} (w_s - c_s)
\]

\[
= \sum_{s=t}^{T} \beta^{s-t} \gamma w_t + \sum_{s=T_1}^{T} \beta^{s-t} (1 - \gamma) (w_s - w_{T_1}),
\]
Borrowing constraint

\[-B_t = \sum_{s=t}^{T} \beta^{s-t}(w_s - c_s)\]

\[= \sum_{s=t}^{T} \beta^{s-t} \gamma w_t + \sum_{s=T_1}^{T} \beta^{s-t} (1 - \gamma) (w_s - w_{T_1}),\]

There are two components of income to be borrowed against:

1. penalty that can be collected after default
2. cost savings from consumption smoothing in \([T_1, T]\).
age \times 10^4

\begin{align*}
  w_t \\
  (1-\gamma)w_t
\end{align*}
Area(A) - Area(B) > 0

\[ u^{-1}(V(t)) \]

\[ (1 - \gamma) w_t \]

\[ (1 - \gamma) w_{T_1} \]
Remarks on Borrowing constraint

- If income path is higher, the amount of borrowing is also higher.
- When $\gamma = 1$, the agent can borrow against all income. Autarky is undesirable. This is equivalent to full commitment.
- Even if $\gamma = 0$, the agent can still borrow, due to the cost savings from consumption smoothing in $[T_1, T]$. 
Earnings Uncertainty

- Participation constraint depends on the realization of the income shock.
- Constrained efficient consumption depends on the history of shocks.
- The optimal contract provides insurance.
Endogenous Earnings

- Constrained efficient allocation could include human capital capital accumulation.

- Allocation in the optimal contract affects outside option.
Agents are heterogeneous in ability $a$ and initial wealth $W$.

Income depends on ability, education level and age:

- High school income: $w_t(H)a$.
- College income: $w_t(C)f(a)$.

College tuition is $\tau$.

An agent spends one period in college.

Agent’s income is zero during college.
Age vs. Income Comparison

- **High-school Income**
- **College Income**

Income values are shown on a logarithmic scale (x10^4).
Assumption

\[\frac{f(a)}{a}\] increases in \(a\).

- The agent compares two paths (one for college and one for high school) and chooses the one with a higher utility.
- The agent’s initial wealth is \(W - \tau\) if he chooses college, and \(W\) if he does not.
The agent’s utility relies only on the sum of discounted earnings and initial wealth.

The agent compares the total discounted earnings under college path and high school path.

He chooses college if and only if

\[
\sum_{t=0}^{T} \beta^t w_t(H)a \leq \sum_{t=1}^{T} \beta^t w_t(C)f(a) - \tau
\]
**Proposition**

There exists a threshold $\tilde{a}_{fc}$ such that agent with ability $a \geq \tilde{a}_{fc}$ enrolls in college and agent with ability $a < \tilde{a}_{fc}$ chooses to enter the labor market as a high school graduate.

- Wealth does not enter the comparison.
One-sided commitment

Benefit and cost of college

- +: discounted income is higher
- +: college graduate may borrow more.
- -: tuition payment.
- -: consumption path is more distorted.

Unlike full commitment, comparison of discounted income alone is not sufficient.
Age

High-school Consumption

College Consumption

$\times 10^4$
Wealth enters the comparison under one-sided commitment.

Rich agents are more likely to attend college.

**Proposition**

If an agent with wealth $W$ is indifferent between college and high school, then an agent with wealth $W_1 > W$ strictly prefers college.
High ability students are more likely to attend college, analogous to the full-commitment allocation.

**Proposition**

If $W \leq \tau$, then there exists a threshold $\tilde{a}(W)$ such that agent with ability $a \geq \tilde{a}(W)$ enrolls in college and agent with ability $a < \tilde{a}(W)$ chooses to enter the labor market as a high school graduate.
Commitment friction distorts the college-enrollment decision.

**Proposition**

*College enrollment under full commitment is greater than that under one-sided commitment, i.e., \( \tilde{a}(W) > \tilde{a}_{fc} \).*
Earnings Uncertainty

- It is efficient to have the repayment of student loan contingent upon the history of earnings shocks.
- The optimal contract provides insurance.
- Default might be an element of the optimal contract.
National Student Loan Cohort Default Rates
Dropouts

- Negative signal in the first two years regarding future earnings.
- Dropout as a result of accumulated debt.
Adverse Selection - unobservable ability

- Information problems in addition to commitment problems.
- Low-ability agents could mimic high-ability agents, borrow resources for college and enroll in college.
- The optimal contract has to screen out the low-ability student by asking the agent with a low-income realization to repay the loan as well.
- Although the payment reduces insurance, it deters the low-ability agents from enrolling in college.