Networks and Markets
Lecture 1
Social Structure and Economic Activity

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Prologue

Social embeddedness: individual action is constrained by social structures and ‘non-economic’ institutions. Polanyi (1944), The Great Transformation, argued that in pre-modern societies, economic activity takes place primarily within social structures such as the family, kinship, and the political hierarchy.

He argued that the essence of a modern society is the emergence of non-social and ‘anonymous’ markets as the centres of economic activity. These ideas were widely adopted in the social sciences.

Prologue

Indeed, this adoption was so pervasive that in 1985, Mark Granovetter wrote his celebrated paper, ‘Economic action and social structure: the problem of embeddedness’.

He argued that actors do not behave as anonymous agents outside a social context, nor do they mechanically follow roles defined by their social situation. Instead, individuals’ attempts at purposive action are embedded within a system of social relations.

He also addressed the classical economic dichotomy between markets and firms and argued for networks as a third mode of organization.

Introduction

Over the last three decades, there has been a parallel development in economics; probably as important. The scope of the subject is now seen to be much broader than before.

Introduction

• Paul Samuelson (1958) edition: economics is about the world of ‘prices, wages, interest rates, stocks and bonds, banks and credit, taxes and expenditure’.

• Greg Mankiw (2004) : ‘there is no mystery about what an ‘economy’ is. An economy is just a group of people interacting with one another as they go about their lives’.

But we must be careful here … for, at the start, we read:

• Marshall (1890), ‘Economics is the study of people in the ordinary business of life’.

Textbook Economics

Traditional models assume individuals are anonymous and interaction is centralized/ random.

The cornerstone of economic theory

• General equilibrium
• Imperfect competition
• Contracts and transaction costs
• Game theory

reflect these assumptions.

Introduction

Fascinating and important empirical phenomena.

• The diffusion of ideas and products
• Contagion/resilience in society, economy, politics
• Structure and dynamics of WWW, Facebook, Twitter
• Variations in crime and school performance
• Patterns of collaboration among firms
• Local clustering and spatial patterns of inequality

Traditional models inadequate to address them.

Introduction

The defining feature of these phenomena is

• large set of individuals
• any individual interacts only with a small subsets
• these small sets are overlapping and span the population.

Tension between the large and the small is fundamental to understanding social and economic behaviour.

Networks provide a conceptual framework.
Choice and Structure

In his celebrated essay, Granovetter (1985) argued for a middle path between the extremes of social determinism and the anonymous individual entirely independent of the social structure.

Economists have made significant progress in understanding how social structure and human agency shape behaviour and collective outcomes.

Introduction

Over the last two decades, a broad and ambitious research programme on networks has developed in economics. This programme combines markets (prices and competition) and complex patterns of connections between individuals.

My aim is to

• Introduce the conceptual approach
• Discuss new research
• Draw attention to exciting open problems

Lectures Outline

• Lecture 1: Introduction
• Lecture 2: How social structure shapes behaviour?
• Lecture 3: What are the origins of market power?
• Lecture 4: How choice shapes social and economic structure?

Networks and Markets

Lecture 2: Social structure and Behavior

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Introduction

- Individuals are located on nodes of a network. They choose actions and their rewards depend on these actions along with the actions of others on the network. The key point:
  - The effect of player 1’s action on player 2’s payoff depends on where the two players are located in a network.

Examples: 1. Value of learning a language depends on how many friends and colleagues learn the same language. 2. Value of acquiring information on market prices depend on how much information friends acquire. 3. Firms collaborate but also compete in markets. 4. Efforts feed off each other: it pays to work hard on a project if colleagues also work hard.

Two conceptual issues

1. A single action or link specific actions? Single action is reasonable is some contexts: such as consumer search about product prices. While in other contexts link specific action is more natural – e.g., effort in research projects, individuals or firms have the choice of putting different amounts of resources in different projects. The single action formulation is simpler and most applications to date assume this.
2. Direct or indirect effects of actions? Both will be studied.

Games on fixed networks

- Suppose each player \(i\) takes an action \(s_i\) in \(S\), where \(X\) is a compact subset of \([0, 1]\). The payoff (utility or reward) to player \(i\) under the profile of actions \(s = (s_1, ..., s_n)\) is given by \(\Pi_i: S^n \times \mathcal{G} \rightarrow \mathbb{R}\).

- In the games where the action set \(S\) is continuous, it will be assumed that \(S\) is also convex. Define \(s_{-i}\) as the profile of all strategies other than player \(i\).
Concepts

1. **Networks in the payoff function**: A network has a number of different attributes—such as neighborhood size, average path length, degree distribution, centrality—and it is clear that these attributes will play more or less important roles depending on the particular context under study.

2. **Neighbors and non-neighbors**. The simplest way to model this is to classify other players into two categories, neighbors and non-neighbors, and to treat all members in each group alike. The effects of actions of neighbors are then termed local effects, while the actions of non-neighbors are termed global effects.

Interactions

1. **Pure local effects**: Define the function \( \phi_k : S^k \to \mathcal{R} \). In this case:

\[
\Pi_i(s|g) = \phi_{n_i(g)}(s_i, \{ s_j \}_{j \in N_i(g)})
\]

2. **Remarks**: same payoffs of players with same degree; Two, the payoff function is anonymous. If \( \{ s'_j \}_{j \in N_i(g)} \) is a permutation of actions in \( \{ s_j \}_{j \in N_i(g)} \) then

\[
\phi_{n_i(g)}(s_i, \{ s'_j \}_{j \in N_i(g)}) = \phi_{n_i(g)}(s_i, \{ s_j \}_{j \in N_i(g)}).
\]

3. **Pure global effects**: the actions of all players have the same effects on payoffs.

\[
\Pi_i(s|g) = \phi_{n-1}(s_i, s_{-i}).
\]

Exteralities

1. **Definition**

A game with pure local effects satisfies positive externality if for each \( \phi_k \), and for \( s, s' \in S^k \), if \( s \geq s' \) then \( \phi_k(s_i, s) \geq \phi_k(s_i, s') \). Similarly, the game exhibits negative externality if for each \( \phi_k \), and for \( s, s' \in S^k \), if \( s \geq s' \) then \( \phi_k(s_i, s) \leq \phi_k(s_i, s') \).

2. **Definition**

A game with pure local effects exhibits strategic complements if for all \( \phi_k \), \( s_i > s'_i \), \( s, s' \in S^k \), if \( s \geq s' \) then

\[
\phi_k(s_i, s) - \phi_k(s'_i, s) \geq \phi_k(s_i, s') - \phi_k(s'_i, s').
\]

The inequality is reversed for the strategic substitutes.
Public goods in networks: Bramoulle and Kranton 2007, JET

- There are \( n \) players. Suppose each player chooses a search intensity \( s_i \in S \), where \( S \) is a compact and convex interval in \( \mathbb{R}_+ \). The payoffs to a player \( i \), in a network \( g \), faced with a profile of efforts \( s = \{s_1, s_2, \ldots, s_n\} \), are given by:

\[
\Pi_i(s|g) = f(s_i + \sum_{j \in N_i(g)} s_j) - cs_i
\]  

where \( c > 0 \) is the marginal cost of effort. It is assumed that \( f(0) = 0, f' > 0 \) and \( f'' < 0 \).

- A game of pure local effects. It is also a game of positive externality and strategic substitutes.

Criminal networks: Ballester et al. 2006, Econometrica

- The role of interaction effects in shaping the level of criminal activity has been a recurring theme in different literatures such as social psychology and economics. Consider a \( n \) player game with linear quadratic payoffs. The payoffs to player \( i \) faced under strategy profile \( s \), are given by:

\[
\Pi_i(s) = \alpha s_i + \frac{1}{2} \rho s_i^2 + \sum_{j \neq i} \gamma g_{ij}s_i s_j
\]  

- Assume that \( \alpha > 0, \rho < 0 \) and \( \gamma > 0 \). So this is a game of pure local effects, positive externalities and complements.

Networks and markets: Goyal and Moraga 2001, Rand

- Firms increasingly choose to collaborate in research with other firms. This research collaboration takes a variety of forms and is aimed both at lowering costs of production as well as improving product quality and introducing entirely new products.

- Suppose demand is linear and given by \( Q = 1 - p \) and that the initial marginal cost of production in a firm is \( \bar{c} \) and assume that \( n\bar{c} < 1 \). Each firm \( i \) chooses a level of research effort given by \( s_i \in S = [0, \bar{c}] \). Collaboration between firms is at a bilateral level and it allows for firms to share research efforts which lower costs of production. The marginal costs of production of a firm \( i \), in a network \( g \), facing a profile of efforts \( s \), are given by:

\[
c_i(s|g) = \bar{c} - (s_i + \sum_{j \in N_i(g)} s_j).
\]  

Networks and Markets, continued

- The cost of efforts is given by \( Z(s_i) = \alpha s_i^2/2 \), where \( \alpha > 0 \). Given costs \( c = \{c_1, c_2, \ldots, c_n\} \), firms choose quantities \( \{q_i\}_{i \in N} \), with \( Q = \sum_{i \in N} q_i \). Solve for market quantity equilibrium given any cost profile. Then payoffs of firm \( i \), located in network \( g \), faced with a research profile \( s \) is:

\[
\Pi_i(s|g) = \left[1 - \bar{c} + s_i[1 - n\bar{c}] + \sum_{j \in N_i(g)} s_j[1 - n\bar{c}] - \sum_{j \in N_i(g) \cup \{i\}} s_j[1 - n\bar{c}] \right] - \frac{\alpha s_i^2}{2}.
\]

- This game exhibits local & global effects. Positive externality across neighbors and negative externality across non-neighbors actions. Actions of neighbors are complements, while the actions of non-neighbors are substitutes.
Local public goods

▶ Existence of Nash equilibrium: The action set is compact, the payoffs are continuous in actions of all players are concave in own action, it follows from standard theorem that there is Nash equilibrium in pure strategies.

▶ Networks and equilibrium: A useful first step is a general property concerning aggregate level of effort – own plus the neighborhood – enjoyed by any individual. Let $\hat{s}$ be such that $f'(\hat{s}) = c$ and define $s_{N(g)} = \sum_{j \in N_i} s_j$. From the concavity of $f(\cdot)$, it then follows that if $s_{N(g)} \geq \hat{s}$ then marginal returns to effort are lower than the marginal cost and so optimal effort is 0, while if $s_{N(g)} < \hat{s}$, then marginal returns from effort to player are strictly larger than marginal costs $c$ and so optimal effort is positive and in fact given by $\hat{s} - s_{N(g)}$.

Nature of equilibrium

▶ Specialized equilibria: profile where some players choose positive action while others choose 0 action. such profiles illustrate free riding in a specially acute form.

▶ Point 1: In the empty network – there is a unique equilibrium in which every player chooses $\hat{s}$: no free riding. It turns out that this is the only network in which no free riding is possible. How do we prove this?

Maximal independent sets

▶ An independent set of a network $g$ is a set of players $I \subseteq N$ such that for any $i, j \in I$, $g_{ij} \neq 1$. A maximal independent set is an independent set that is not contained in any other independent set. Does there exist a maximal independent set in every network? The answer to this is yes. Proof by construction:

▶ First number the players 1, 2, ..., $n$. start by placing player 1 in $I$. If player 2 $\notin N_1(g)$, then include her in the independent set $I$, if not then include her in the complement set $I^c$. Next consider player 3: if player 3 $\notin N_1(g) \cup N_2(g)$, then include her in $I$, while if she is not then include her in $I^c$. Move next to player 4 and proceed likewise until you reach player $n$.

Proposition

1. A profile of actions $s^* = \{s_1, s_2, ..., s_n\}$ is a Nash equilibrium if and only if for every player $i$ either (1). $s_{N_i(g)}^* \geq \hat{s}$ or (2). $s_{N_i(g)}^* \leq \hat{s}$ and $s_i^* = \hat{s} - s_{N_i(g)}$.
2. There are two types of players: those who receive aggregate effort in excess of $\hat{s}$ from their neighbors and exert no effort on their own, and two, players who receive less than $\hat{s}$ aggregate effort from their neighbors and contribute exactly the difference between what they receive and $\hat{s}$.
Examples of networks

▶ **Examples:** In the empty network there exists a unique maximal independent set and this is the set of all players $N$. In the complete network on the other hand, there are $n$ distinct maximal sets, each of which contain a single player. In the star network, there are two maximal independent sets, one, which contains the central player, and two, which contains all the peripheral players.

▶ Now assign the action $\hat{s}$ to every member of a maximal independent set and assign action 0 to every player who is not a member of the maximal independent set. This configuration constitutes an equilibrium in view of the characterization provided. Such an equilibrium is by construction a non-trivial *specialized* equilibrium.

Proposition: Specialization and free riding

1. In any non-empty network, a maximal independent set must be a strict subset of the set of players $N$.

2. There exists a specialized equilibrium in every network. In the empty network the unique equilibrium is specialized and every player chooses $\hat{s}$, so there is no free riding. In any non-empty network there exists a specialized equilibrium with free riding.

Network advantages

▶ Specialized equilibria point to unequal effort. Does this translate into unequal payoffs? Is there any systematic relation between network position and payoffs?

▶ **Example:** Star network. The two equilibria are both specialized and have clearly unequal payoffs.

▶ This shows that it is difficult to say anything definite on relation between degree and payoff. Why is this? How can we work around this? Come back to this in games with incomplete network knowledge.
Proposition: Welfare in Networks

1. Aggregate welfare from a strategy profile $s$ in network $g$ is defined as:

$$W(s|g) = \sum_{i \in N} [f(s_i + s_{N_i}(g)) - c_s].$$

Given a network $g$, a strategy profile profile $s$ is efficient if there is no other action profile $s'$ such that $W(s'|g) > W(s|g)$.

2. Proposition: Every equilibrium in a non-empty network is inefficient.

3. This result is a direct consequence of individual actions having positive externality on others.

Proof

Fix some non-empty network $g$, and let $s^*$ be an equilibrium with $s_i > 0$, for some $i \in N$, and suppose that for some $i$ and $j$, $g_{ij} = 1$. We know that if $s_i > 0$, then $s_{N_i}(g) + s_i = \hat{s}$ and this implies that $f'(s_i + s_{N_i}(g)) - c = 0$. Now consider the partial derivative of social welfare with respect to $s_i$ evaluated at $s^*_i$:

$$\frac{\partial W(s^*|g)}{\partial s_i} = \sum_{j \in \{i\} \cup N_j(g)} f'(s^*_j + \sum_{k \in N_j(g)} s^*_k) - c = \sum_{j \in N_i(g)} f'(s^*_j + \sum_{k \in N_j(g)} s^*_k)$$

which is strictly positive since $f'(\cdot) > 0$. So welfare can be strictly increased by increasing $s_i$. Thus the equilibrium is inefficient.

Effects of adding links

- Start with two stars each with 3 peripheral players. Fix an equilibrium in which the two centers exert action $\hat{s}$ while the peripheral players all choose 0.
- **First**, add a link between a center and a spoke of the other star. The old action profile still constitutes an equilibrium. It then follows that aggregate welfare increases on the adding of a link.
- **Second**, add a link between the centers. The two centers do not constitute a maximal independent set any more. Equilibrium must change. The best equilibrium is one in which the center of one star and spokes of the other star choose $\hat{s}$, and all other players choose 0. In this new equilibrium, welfare strictly decreases if $2c\hat{s} > f(4\hat{s}) - f(\hat{s})$.
- Thus effects of adding links depend on subtle details of the network.

Star network with free riders

![Star network with free riders diagram]
Quick summary

1. Role of networks in sustaining specialized equilibria: creating significant free riding and payoff inequality.
2. Network advantages: Multiple equilibria exist with contrasting relationships.
3. Adding links: increases welfare in some standard networks but not for other simple networks!

Strategic complements: criminal networks

- We start for simplicity with the following payoffs:

\[ \Pi_i(s|g) = \alpha s_i - \frac{1}{2} \beta s_i^2 + \lambda \sum_{j=1}^{n} g_{ij} s_i s_j \]  

(7)

where \( \alpha, \beta, \lambda \) are all positive.

- Differentiating payoffs (7) with respect to \( s_i \) we get:

\[ \frac{\partial \Pi_i(s)}{\partial s_i} = \alpha - \beta s_i + \lambda \sum_{j=1}^{n} g_{ij} s_j = 0. \]  

(8)

- Proposition: The game \( \Gamma \) has a unique interior Nash equilibrium \( s^*(\Gamma) = \{s^*_1, s^*_2, ... s^*_n\} \), which is given by

\[ s^*(\Gamma) = \frac{\alpha}{\beta} C_b(g, \lambda) \]  

(9)

where \( C_b(g, \lambda) \) is the vector of Bonacich centrality of nodes.

Examples

- **Example 1**: Consider a network with 3 players, 1, 2 and 3. Suppose links take on values of 0 and 1, and let the network consist of two links, \( g_{12} = g_{23} = 1 \). This network can be represented in an adjacency matrix \( G \) as follows.

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Table 2.1

- Simple computations now reveal that \( G^2 \) is given by:

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Table 2.2

Bonacich Centrality

- Consider the adjacency matrix \( G \) of network \( g \); in this matrix an entry in a square corresponding to a pair \( \{i,j\} \) signifies the presence or absence of a link.

- Let \( G^k \) be the \( k^{th} \) power of the matrix. The 0 power matrix \( G^0 = I \), the \( n \times n \) identity matrix. In \( G^k \), an entry \( g_{ik}^k \) measures the ‘number’ of walks of length \( k \) that exist between players \( i \) and \( j \) in network \( g \). The following example illustrates this idea.

- Let \( G^k \) be the \( k^{th} \) power of the matrix. The 0 power matrix \( G^0 = I \), the \( n \times n \) identity matrix. In \( G^k \), an entry \( g_{ik}^k \) measures the ‘number’ of walks of length \( k \) that exist between players \( i \) and \( j \) in network \( g \). The following example illustrates this idea.
Understanding Bonacich centrality

- Thus there is 1 walk of length 2 between 1 and 1 and between 3 and 3, but 2 walks of length 2 between 2 and 2. There are no other walks of length 2 in this network.
- Let $a \geq 0$ be a scalar and let $I$ be the identity matrix. Define the matrix $M(g, a)$ as follows:
  \[
  M(g, a) = [I - aG]^{-1} = \sum_{k=0}^{\infty} a^k G^k.
  \] (10)
- This expression is well defined so long as $a$ is sufficiently small. The entry $m_{i,j}(g, a) = \sum_{k=0}^{\infty} a^k g_{i,j}^k$ counts the total number of walks in $g$ from $i$ to $j$, where walks of length $k$ are weighted by factor $a^k$.

Understanding Bonacich Centrality

- To see this note that (12) can be rewritten as follows:
  \[
  C_b(i; g, a) = m_{i,i}(g, a) + \sum_{j \neq i} m_{i,j}(g, a).
  \] (13)
- Since $G^0 = I$, it follows that $m_{i,i}(g, a) \geq 1$ and so for every player $i$ in any network, $C_b(i; g, a) \geq 1$. It is exactly equal to 1 in case $a = 0$.
- The Bonacich centrality of a node can also be expressed as a function of the centrality of its neighbors. Let $\lambda(a)$ be the (largest) eigen value of the adjacency matrix $G$. The Bonacich centrality of a node can then be defined as:
  \[
  C_b(i; g, a) = \frac{1}{\lambda} \sum_{j \in N} g_{i,j} C_b(j; g, a).
  \] (14)
- Given parameter $a$, the Bonacich centrality vector is defined as
  \[
  C_b(g, a) = [I - aG]^{-1} \cdot 1
  \] (11)
- where 1 is the (column) vector of 1’s. In particular, the Bonacich centrality of player $i$ is given by:
  \[
  C_b(i; g, a) = \sum_{j=1}^{n} m_{i,j}(g, a).
  \] (12)
- This measure of centrality counts the total number of (suitably weighted) walks of different lengths starting from $i$ in network $g$.

Understanding Bonacich Centrality

- Star network with 3 players, the Bonacich centrality of peripheral nodes is 0.500 while that of central player is 0.707. The ratio of Bonacich centralities between central and peripheral player is $\sqrt{n-1}$, an increasing and a concave function of $n$.
- The vector of Bonacich centralities is given by
  \[
  C_b(g, a) = [I - aG]^{-1} \cdot 1,
  \] where $a$ is a scalar. The Bonacich centrality of a player $i$ is:
  \[
  C_b(i; g, a) = \sum_{j=1}^{n} m_{i,j}(g, a).
  \] (15)
- where $m_{i,j}(g, a)$ is the total number of weighted walks of all lengths between players $i$ and $j$, in network $g$. Define $\lambda^* = \lambda/\beta$ and for any $y \in \mathcal{R}^n$, let $\tilde{y} = y_1 + y_2 + ... + y_n$. 

Understanding Bonacich Centrality
Proposition: Nash equilibrium and Bonacich centrality

- The game $\Gamma$ has a unique interior Nash equilibrium $s^*(\Gamma) = \{s_1^*, s_2^*, \ldots, s_n^*\}$, which is given by
  \[
  s^*(\Gamma) = \frac{\alpha}{\beta} C_b(g, \lambda^*)
  \]  
  (16)

- Simple algebra now yields the following expression on equilibrium efforts:
  \[
  s_i^*(\Gamma) = \frac{C_b(i; g, \lambda^*)}{\bar{C}_b(g, \lambda^*)} \bar{s}^*(\rho)
  \]  
  (17)

  where $\bar{s}^* = \sum s_i^*$ and $\bar{C}_b(g, \lambda) = \sum C(i; g, \lambda)$.

Proof

- The necessary and sufficient condition for the matrix $[\beta I - \lambda G]^{-1}$ to be well defined and non-negative follow from Debreu and Herstein (1953). This condition suffices for the existence of a unique interior equilibrium.

- Next consider the characterization of equilibrium. From the first order conditions, it follows that an interior $s_i^*$ satisfies:
  \[
  \beta s_i - \lambda \sum_{j=1}^{n} g_{ij} s_j = \alpha.
  \]  
  (18)

- Using matrix $\Gamma$ we can rewrite this as follows:
  \[
  [\beta I - \lambda G] s^* = \alpha.1.
  \]  
  (19)

- The matrix $[\beta I - \lambda G]$ is generically non-singular and so there is a unique generic solution in $\mathbb{R}^n$.

Proof, continued

- Now exploit $U.s = \bar{s}.1$ to rewrite the above as:
  \[
  \beta[I - \lambda^* G]s^* = [\alpha].1.
  \]  
  (20)

- Inverting the matrix and using the definition of $C_b(.)$ we can write this as:
  \[
  \beta s^* = [\alpha] C_b(g, \lambda^*)
  \]  
  (21)

- Simple algebra then yields:
  \[
  s^* = \frac{\alpha}{\beta} C_b(g, \lambda^*).
  \]  
  (22)

- This completes the proof.

Remarks

- This expression captures the key insight: equilibrium efforts are proportional to Bonacich centrality.

- Recall, that in a star network the central node has a higher centrality than the peripheral nodes, and this result implies therefore that the central player will exert higher criminal efforts as well.

- By contrast, in a cycle or a complete network all players have the same centrality and so their efforts will be equal as well.
Effects of changing networks

The interest now turns to effects of adding links to a network. Let \( g' \) be denser than \( g \):

Suppose \( \beta \) is large and \( g' \) be denser than \( g \). Then the equilibrium in the denser network exhibits higher aggregate effort.

Intuition: the network changes via the addition of a link. Adding a link between \( i \) and \( j \), adds a new complementarity effect for players \( i \) and \( j \). This in turn raises their incentives for higher efforts and, via local complementarities, the incentives of their neighbors, and so on. This leads to an increase in efforts for everyone.

Summary

1. Introduce a framework that combines games and networks.
2. games with strategic substitutes or complements; games with full information on game and networks
3. Illustrate the use of concepts – maximal independent sets, Bonacich centrality.
4. Looking forward:
   4.1 Incomplete network knowledge
   4.2 link specific actions
   4.3 combination of strategic substitutes and complements
   4.4 choice on links: network formation

3. Readings

- Articles
  Goyal, S., A. Konovalov, J.L Moraga (2008), Hybrid R&D. *J. European Econ Association*
- Recent Work:

Networks and Markets

Lecture 3: The origins of market power

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Supply, service and trading chains are prominent in agriculture, in transport and communication networks, in international trade, and in finance. Goods and services pass through individuals or firms located on these chains. The routing of economic activity, the earnings of individuals and the efficiency of the system depend on the prices set by intermediaries.

Questions
1. How does network of intermediation shape market power, prices and efficiency?
2. How do networks form? Are networks resilient or fragile?
3. How should entry and mergers be assessed, and firms regulated in networked markets?

Collaborations and Papers

Collaborators
1. Syngjoo Choi (UCL)
2. Andrea Galeotti (Essex).

Papers:

Supply chains: coffee

Start with a small farmer in a developing country. Farmer chooses intermediaries (go from cherries to beans). Intermediaries sell beans to an exporting trading firm. The exporter sells to dealers/brokers. The dealers/brokers sell to roasters (like Nestle). Nestle then sells to large supermarkets and local stores. These supermarkets then sell to consumers.
Supply chains: issues

- Long chains are common in agricultural sector.
- Major concern: market power of intermediaries.
- 1990’s: felt state agencies discourage innovation/competition.
- Recent decades: large scale liberalization.
- Effects of liberalization are mixed.
- What are the determinants of competition and pricing in networked markets?

Theory: Benchmark Model

- There is a source $S$ and a destination $D$.
- There are $n$ traders located in a network $g$ that connects $S$ and $D$.
- Traders simultaneously post prices: cost of a ‘path’ between $s$ and $b$ is sum of prices of traders on the path.
- The surplus between $b$ and $s$ is 1.
- Pick cheapest path if it is less than 1 (randomize).
- Seller and buyer split residual surplus equally.
Theory: Benchmark Model

- Given $g$ and $p$, let $Q^* = \{q \in Q : c(q, p) = c(p), c(p) \leq 1\}$ be the set of feasible least cost paths.
- Given $g$ and $p$, the payoff to intermediary $i \in N$ is:
  \[
  \Pi_i(p) = \begin{cases} 
  0 & \text{if } i \notin q, \forall q \in Q^* \\
  \frac{\eta_i^*}{|Q^*|} p_i & \text{if } i \in q, q \in Q^*,
  \end{cases}
  \]
  where $\eta_i^*$ is the number of paths in $Q^*$ that contain intermediary $i$.
- We study pure strategy Nash equilibrium.
- $c^*(p)$ denotes the lowest cost under $p$.

Theory: Relation with Bid-Ask model

- All traders post asks-bids.
- Seller accepts highest positive price; buyer chooses lowest positive price (less than surplus).
- Results:
  - Equilibrium payoffs in posted prices can be implemented in equilibrium of bid-ask model.
  - There exist networks with equilibrium in bid-ask model that cannot be supported in posted price model.
  - Characterization of networks – trees, multipartite networks – in which two pricing models are equivalent.

Theorem: Equilibrium characterization

A trader $i$ is essential under $p$ if $i \in q, \forall q \in Q^*$.

A. **Existence:** In every network there exists an efficient equilibrium.

B. **Characterization:** An equilibrium $p^*$ is either inefficient ($c(p^*) > 1$), traders extract all the surplus ($c(p^*) = 1$), or they earn nothing ($c(p^*) = 0$). Moreover,

1. $c(p^*) = 0$ is an equilibrium iff no trader is essential.
2. $c(p^*) = 1$ is an equilibrium iff (i) every $i \in q, q \in Q^*$, with $p_i^* > 0$ is essential, and (ii) for every $i$, $i \notin q, \forall q \in Q^*$, if $i \in q'$ then $c_{-i}(q', p^*) \geq 1$.
3. $c(p^*) > 1$ is an equilibrium, iff $c_{-i}(q, p^*) \geq 1$ for every trader $i \in q$ and $\forall q \in Q$.

Theorem: remarks

- The argument for existence of equilibrium is constructive.
- In every efficient equilibrium surplus division is extremal.
- Critical traders create market power for intermediaries: as they are essential.
- Criticality dictates that all surplus accrues to intermediaries, but the theory is permissive about how it is distributed among them.
Theory: open issues

- Multiplicity of equilibria. Consider the ring network with 6 traders. All three equilibria described by Theorem ?? are possible.
- Motivates equilibrium refinement: trembling hand perfection, strictness, strong Nash equilibrium, elimination of weakly dominated strategies, and perturbed Nash demand games.
- Some refinements are too strong while others are not effective, e.g., a wide range of equilibrium may be sustained under trembling hand perfection, elimination of weakly dominated strategies, and perturbed bargaining.
- To gain a deeper understanding of the relation between networks and market power, we conduct an experiment.

Experiment: Design

- We consider four different networks.
  1. Ring networks (size = 4, 6, 10): competition stable, coordination grows.
  2. Ring 6 with hubs and spokes: critical traders plus competition.
- We ran two sessions per network treatment; in total, there are 8 sessions.
- 60 rounds in each session;
- Random matching and random assignment of network positions.

Experiment: Rings and Ring with Hubs

Experiment: Treatments and sessions

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring $n = 4$</td>
<td>16 / 240</td>
<td>16 / 240</td>
<td>32 / 480</td>
</tr>
<tr>
<td>Ring $n = 6$</td>
<td>18 / 180</td>
<td>24 / 240</td>
<td>42 / 420</td>
</tr>
<tr>
<td>Ring $n = 10$</td>
<td>20 / 120</td>
<td>20 / 120</td>
<td>40 / 240</td>
</tr>
<tr>
<td>Ring with hubs</td>
<td>18 / 180</td>
<td>24 / 240</td>
<td>42 / 420</td>
</tr>
</tbody>
</table>
Finding 1: Efficiency is remarkably high in all networks

<table>
<thead>
<tr>
<th>Network</th>
<th>minimum distance of buyer-sell pair</th>
<th>(Network, Network)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All (≥2)</td>
<td>1.00 (480) 1.00 (480)</td>
<td>(480) (480)</td>
</tr>
<tr>
<td>Ring 4</td>
<td>1.00 (420) 1.00 (289) 1.00 (131)</td>
<td>(420) (289) (131)</td>
</tr>
<tr>
<td>Ring 6</td>
<td>1.00 (240) 1.00 (49) 1.00 (87) 1.00 (69) 1.00 (35)</td>
<td>(240) (49) (87) (69) (35)</td>
</tr>
<tr>
<td>Ring 10</td>
<td>0.95 (420) 1.00 (126) 0.94 (155) 0.90 (109) 0.90 (30)</td>
<td>(420) (126) (155) (109) (30)</td>
</tr>
<tr>
<td>Ring with Hubs and Spokes</td>
<td>0.95 (420) 1.00 (126) 0.94 (155) 0.90 (109) 0.90 (30)</td>
<td>(420) (126) (155) (109) (30)</td>
</tr>
</tbody>
</table>

Note. The number of group observations is reported in parentheses.

Finding 2: Distribution of surplus is extremal

<table>
<thead>
<tr>
<th>Network</th>
<th>No. Paths = 1</th>
<th>No. Paths = 2</th>
<th>No. Paths = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring with Hubs and Spokes</td>
<td>No. Cr = 1</td>
<td>No. Cr = 2</td>
<td>No. Cr = 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. The number in a cell is the average fraction of costs charged by critical traders. The number of observations is reported in parentheses. #Cr denotes the number of critical intermediaries, #Paths denotes the number of paths connecting buyer and seller, d(q) denotes the length of path q beween buyer and seller.

Finding 3: Criticality yields large payoffs

<table>
<thead>
<tr>
<th>Network</th>
<th>(#Cr, #Paths, d(q), d(q'))</th>
<th>Rounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1–20</td>
</tr>
<tr>
<td>Ring with Hubs and Spokes</td>
<td>(1, 2, 3, 5)</td>
<td>0.56 (20) 0.68 (26) 0.72 (25)</td>
</tr>
<tr>
<td></td>
<td>(1, 2, 4, 4)</td>
<td>0.48 (16) 0.56 (13) 0.67 (10)</td>
</tr>
<tr>
<td></td>
<td>(2, 2, 4, 6)</td>
<td>0.73 (16) 0.77 (19) 0.80 (24)</td>
</tr>
<tr>
<td></td>
<td>(2, 2, 5, 5)</td>
<td>0.65 (8) 0.67 (8) 0.74 (11)</td>
</tr>
</tbody>
</table>

Notes. The number in a cell is the average fraction of costs charged by critical traders. The number of observations is reported in parentheses. #Cr denotes the number of critical intermediaries, #Paths denotes the number of paths connecting buyer and seller, d(q) denotes the length of path q beween buyer and seller.
### Summary: basic model

1. **Efficiency:** Theory says that efficient equilibrium exists, but inefficient equilibria are possible. *Experiments show that subjects select efficient outcomes.*

2. **Division of surplus:** Theory says division is extremal. *Experiment outcomes consistent with theory.*

3. **Network structure:** Theory is permissive. *Experimental outcomes that node criticality is both necessary and sufficient for market power.*

### Extensions

**Double marginalization and market power**

- Suppose demand is falling in price charged by intermediaries.
- We prove existence and characterize equilibrium.
  - Active Intermediaries set equal price
  - Price/efficiency falling in number of intermediaries.
- Experiments: length of chains have large impact on prices/efficiency.

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### Competition on paths

<table>
<thead>
<tr>
<th>Network</th>
<th>(d(q), d(q'))</th>
<th></th>
<th>cost1 - cost2</th>
<th>Freq. on a shorter path</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring 4</td>
<td>(2, 2)</td>
<td>3.99</td>
<td>-</td>
<td>--</td>
</tr>
<tr>
<td>Ring 4</td>
<td>(2, 4)</td>
<td>4.45</td>
<td>0.65</td>
<td>--</td>
</tr>
<tr>
<td>Ring 4</td>
<td>(3, 3)</td>
<td>4.01</td>
<td>-</td>
<td>--</td>
</tr>
<tr>
<td>Ring 10</td>
<td>(2, 8)</td>
<td>15.20</td>
<td>0.64</td>
<td>--</td>
</tr>
<tr>
<td>Ring 10</td>
<td>(3, 7)</td>
<td>5.30</td>
<td>0.68</td>
<td>--</td>
</tr>
<tr>
<td>Ring 10</td>
<td>(4, 6)</td>
<td>6.82</td>
<td>0.68</td>
<td>--</td>
</tr>
<tr>
<td>Ring 10</td>
<td>(5, 5)</td>
<td>5.01</td>
<td>-</td>
<td>--</td>
</tr>
</tbody>
</table>

Notes. We report the sample median of absolute differences of two competing paths, using the sample of last 20 rounds. The number in the last column is the frequency of trading on a shorter path.
Distance and efficiency

Minimum distance between buyer and seller

Frequency of efficient outcome

- Ring networks
- Line networks

Literature

- **Empirical work**: foundation for relation between ‘central’ traders and market power. 

- **Industrial organization**: model of pricing in general networks with asymmetric demand.

- **Network theory**: role of criticality in shaping behavior.

- **Experiments**: trading, dictator games, Nash bargaining.

Networks and Markets
Lecture 4
Economics of Network Formation

Sanjeev Goyal
University of Cambridge
2014 Summer School
Cambridge-INET and HCEO, Chicago

Choice and Structure

The key conceptual innovation is the idea that

**social and economic networks are shaped by individual choice**

This idea, taken in tandem with a number of concepts from the social and information sciences, creates a powerful new research methodology that has significantly expanded the scope of economics.
Formation of networks is a complicated process and subject to a variety of technological, economic and social forces.

The distinctive feature of an economic approach is human agency: preferences, costs/benefits and externalities.


General Framework:
- Bala and Goyal (2000), *Econometrica*
- Jackson and Wolinsky (1996), *Journal of Economic Theory*

Network formation: economic approach

- The distinctive feature of an economic approach is human agency: preferences, costs/benefits and externalities.


General Framework:

Outline

1. Introduction: structure of networks
2. Strategic foundations
3. Unilateral linking   
   Application: communication networks
4. Pairwise linking   
   Application: research alliances
5. Quick summary
6. Related themes
7. General theory: open problems

Strategic network games: antecedents

- Communication networks in cooperative games, Myerson (1979)
- Random/statistical linking due to Price (1972) and Erdos and Renyi (1960’s).

- Very active field of study. For a survey, see Goyal (2007)
Strategic foundations of networks

Key features of linking:

1. **Linking is a deliberate decision:**
   Examples: Scientists decide on whether to collaborate
   Firms choose to form an alliance;
   I decide on hyperlink with your homepage.

2. **Externality/spillover:** Link between 1 and 2 affects payoffs of 3 as well as her rewards from new links.
   Examples: capacity constraints in co-authorship;
   firm A and B collaborate affects firm C.

Combine 1 & 2: **Games of Network Formation.**

Unilateral linking

- Examples: Hyper links between pages, gifts, citations, peer to peer networks, phone calls...

- Unilateral linking is methodologically very convenient; it permits a thorough study of key questions:
  -- what is an equilibrium network,
  -- are equilibrium networks unequal
  -- are they socially efficient
  -- what are the dynamics of network formation.

The structure of networks

- A network describes a collection of nodes and the links between them.
- Once you begin to study networks it is difficult not to see them everywhere.
- Examples include: Internet, World wide web, airline networks, friendships, research alliances, co-authorships, trade and exchange.

Key issues in modelling:

1. Payoffs: linking generates rewards and entails costs.
   We define these formally.
2. Power: who decides on the link, one person, two persons, all players etc.
3. Information: what do I know -- about other players and about the network -- when I form a link?

We start with the simplest case: a player decides on whether to link with others. No transfers or bargaining. Full Information about rewards and costs of linking and about the network.
The Law of the Few: empirics

The classical early work of Lazarsfeld, Berelson, and Gaudet (1948) and Katz and Lazarsfeld (1955) investigates the impact of personal contacts and mass media on voting and consumer choice. They found

- Personal contacts a dominant role in sharing information.
- 20% of sample of 4,000 source of information for the rest.
- Minor differences between influencers and others.


The Law of the Few

- A summary of these empirical findings: in social groups, a majority of individuals get most of their information from a very small subset of the group, viz., the influencers.
- There are minor differences between the observable economic and demographic characteristics of the influencers and the others.
- Key question: can the law of the few be understood as a consequence of strategic interaction among identical individuals?

Law of the Few: empirics

- Zhang, Ackerman and Adamic (2007) study the Java Forum, an online community of users who ask and respond to queries concerning Java.
  - They identify 14,000 users
  - 55% users only ask questions,
  - 12% both ask and answer queries,
  - 13% only provide answers.

The Law of the Few: theory

Galeotti and Goyal (2010, AER): Individuals acquire information and form links with others to access their information.

- Information is valuable. It can be bought at cost $c>0$.
- Individuals can form bilateral links with others to access their information. A link costs $k>0$.
- Information bought by different individuals is a substitute.
- Payoff function for person:
  $$f(a_i + \sum a_j) - ca_i - k(\# \text{ links})$$
- Function $f$ is increasing and concave. Define $f'(x)=c$. 
Law of the Few: theory

THEOREM (Galeotti and Goyal, 2010)

• Every equilibrium exhibits law of the few.
• The network has a core-periphery architecture
• The players in the core acquire information personally
• Peripheral players buy no information but form links and get information from core players.
• Fraction of core players is small in large society.

Law of the Few: intuitions

Intuition:
1. Value of information increasing but concave: marginal cost=marginal return at 1.
2. Information acquired by individuals is substitutable. If Mr. A acquires information on his own and from Mr. B then must access 1.
3. Links are costly: Mr. A links with Mr. B then so must everyone else. Information acquirers fully linked. Aggregate information is 1.
4. Upper bound: Mr. A forms a link with Mr. B only if there is minimum information. Upper bound to number of information acquirers.
5. Core-periphery network: aggregate information is 1, so everyone who does not buy information must be linked to everyone who does.

Implications for welfare?

• Links are costly: for any fixed total information available it is best to minimize links. Star with hub acquiring all information is efficient network.
• In equilibrium, the hub sets marginal cost=marginal benefits: systematic under-provision of efforts.
Law of the Few: Experiment

- Van Leeuwen, Offerman and Schram (2013), conduct an experiment on the repeated version of the model.

- The design has a 2x2 format. The first set of experiments considers the effects of group size: 4 versus 8. The main finding is that the (stage game) Nash equilibrium is played infrequently: 20% of the cases.

- The authors then suppose that a fraction of linking costs are transferred to the agent who receives a link. In particular, the costs of linking are 20, of which 12 transferred to the receiver.

Summary: In basic game, hub player earns less than spokes. Subjects do not play star network. Inequity aversion? In new game with transfers, hub player earns more than spokes. Transfers facilitate the emergence of stable star and induce large investments, which approximate first best outcomes.

Pair-wise linking

- A link requires the agreement of both parties. E.g., friendship, co-authorships, trade agreements, research alliances, buyer seller relations.

- Need for new solution concepts involving both non-cooperative and cooperative elements of game theory.

- Several developments in the theory and many applications...

Pair-wise links

- Basic idea: individuals propose links with others.

- A link between I and 2 is created if BOTH of them want to link.

- Myerson (1991) link announcement game.

- Solution concept: Nash equilibrium too weak as linking involves coordination between players.

- Supplement Nash Equilibrium

- Jackson and Wolinsky (1996): pairwise stable networks
Networks of Collaboration: Empirics

• Leading firms in hi-tech industries rely on a combination of in-house and collaborations. Biotech, pharmaceuticals, IT.

• Hagedoorn (2004) shows
  1. Firms in non-exclusive & extensive relations.
  2. Research alliances have grown over time
  3. Especially prominent in high technology sectors.
  4. Core-periphery network architecture.

Firms, networks and markets: Theory

Networks of firm collaboration (Goyal and Joshi (2003, GEB), Goyal and Moraga (2001, Rand), Goyal, Konovalov and Moraga (2006, JEEA)).

• Firms bilaterally choose research links.
• Partners share technological information which lower costs of production. More links firms lead to lower costs, which leads to larger market share.
• However each link involves a fixed cost C.
• Key feature: 1. link decided bilaterally. 2. Networks and markets

Firms, networks and markets

• How do we solve such games? Nash equilibrium is too weak as there is a coordination problem in bilateral link formation, firm A offers no link since it expects no one else to form any links.
• To avoid this problem: refine Nash equilibrium with the requirement that no two players should have an incentive to form an additional link.
• Call this solution concept Pairwise Nash Equilibrium

Firms, networks and markets

• Key incentive issues:
  1. starting from a network, should a firm form an additional link? This depends on a comparison of additional profits and costs of a link: whether returns are concave or convex in own links?
  2. How does others linking alter incentives of A and B: whether links are strategic complements or substitutes?
• In the classical Cournot model of oligopoly payoffs are convex in own links and links are strategic substitutes.
• What are the Pairwise Nash Equilibrium networks?
Model of Network Formation

- **Players**: \(N=\{1,2,3,\ldots,n\}\) firms
- **Strategies**: Each firm announces intention to form 0-1 links with others. A link is formed if both firms want it.
- **Payoffs**: A link costs \(F>0\) to each firm and lower their costs of production by \(c\).
- **Links formed**: Define a network, which defines a vector of firm costs. The gains from a link depend on market competition.

**Strong competition**: unique lowest cost firm makes profits

**Moderate competition**: lower costs imply higher profits.

Networks of collaboration: theory

**THEOREM** (Goyal and Joshi 2003)

- Suppose \(F>0\) and small.
  - With strong competition, empty network is unique pair-wise equilibrium.
  - With moderate competition, complete network is unique pair-wise equilibrium.

Intuition: in non-empty network, there is always a firm which forms link but makes no profits. Better to delete all links! If two firms form links, gain at expense of other firms. Always form links.

**General Message**: Two-way influence... markets shape networks and networks define market performance.

**Networks of collaboration: theory**

Goyal and Joshi (2003), Deorian (2006)

**THEOREM**

- With \(F\) large, a pairwise equilibrium network has the dominant group architecture.

**THEOREM**

- With \(F\) large, if firms can make transfers for link formation then the star network and variants of the star are pairwise equilibrium networks.

**General Message**: In a classical oligopoly model, strategic linking leads to unequal connections, small average distance, high clustering in firm alliance networks.
Network advantages

- Does network degree and location confer advantages?

*Firm networks:* unequal degrees arise in equilibrium and more links translate into higher net profits.

General message: *strategic networking can create large inequalities across players who were ex-ante identical.*

Goyal and Joshi (2003).

Good and Bad networks

- Key idea: links are motivated by individual incentives. Individual linking generates externalities and spill-overs. Tension between equilibrium and socially optimal networks.

General message: *linking creates negative profit effects for other firms, there may be too many links in equilibrium.*

Collaboration networks: summary

- Generates sharp predictions on equilibrium networks: unequal degrees, small average distance, arise naturally. Good match with empirics.
- Strategic networking has powerful effects on payoff inequality as well as aggregate social welfare.
- Suggests role of policy – taxes and subsidies – to reorient network formation.

Collaboration Networks: experiment

Ziegelmeyer and Pantz (2008):
- This paper experimentally tests the network formation model.

First experiment: designed as a straight test of theory.
1. The data almost perfectly match the predictions when link formation costs are extreme. *Subjects choose empty or complete networks.*
2. Intermediate link formation costs: the prediction is asymmetric networks. *But subjects rarely form asymmetric networks. When they do, observed and predicted quantities are not in accordance with predicted outcome.*

Quick Summary

Presented two standard models of network formation.
- Start with empirical phenomena.
- Develop models of linking: costs/benefits and externalities.
- Sharp predictions ... on architecture, distribution and welfare.

Test the theory in an ideal environment: in laboratory
- Experiments point to successes
  -- emergence of stars, symmetric/asymmetric firm networks
- Highlight bounded rationality and concerns about fairness.

Networks arise at the intersection of individual incentives and social norms.

Collaboration Networks: experiment

Second experiment: reduce the complexity of the setting by transforming the original two-stage game into a one-stage game. Formation of inter-firm networks directly determines firms’ payoffs.

- *Observed networks coincide with the predicted ones:*

indicating that experimental subjects’ limited capacity to foresee the outcomes of the market stage may be driving the earlier discrepancies.
Choice and Structure: Themes

Great success is an extensive set of applications.

- Viral marketing: Galeotti and Goyal (2010), Campbell (2013).
- Peer effects: Cabrales, Calvo-Armengol and Zenou (2013).

- Buyer-seller: Kranton and Minehart (2001), Manea (2011)
- Cybersecurity/crime: Baccara and Bar-Issac (2008), Goyal and Vigier (2011)

Related themes

A. Weighted links

Existing work focuses on binary link setting: while most applications involve strength or depth of link...

E.g., bandwidth, time allocated to different social relations, strength of weak ties hypothesis (Granovetter, 1973).

- Formation of networks of weighted links is an important but very poorly understood process.


B. Mechanism design

Consider a network formation game with players, linking strategies and payoffs.

Define a network as efficient if it maximizes aggregate payoffs.

**Question:** Is there an allocation function which respects plausible criteria – such as component wise budget balance, fairness etc -- and implements efficient networks?

Negative results due to Jackson and Wolinsky (1996); Dutta and Muttuswami (2001), significant follow up work e.g., Bloch & Jackson 2006.
Related themes
C. Choosing partners and playing games

Very large literature on local interaction and games in economics... survey by Young (1998), Goyal (2007).

In many settings, individuals choose partners & behavior. Richer games: players choose links AND an action.


Related themes
D. Economic applications


• Trade agreements: countries enter into free trade agreements to lower tariffs and facilitate trade. Goyal and Joshi, 2006; Furusawa and Konishi 2008, Zissimos, 2008.

• Financial networks: bank links to share risk, Elliott, Golub, Jackson, 2014.

Related themes
E. Strategic network design

General problem: Set of players N=(1,2,...n) face a set of nodes K=(1,2...k).

Application 1: Players choose links between the nodes and compete for traffic, e.g., airline networks.

Application 2: one player chooses links to improve functionality, while second player seeks to lower performance. E.g., police and criminal networks, hackers and computer networks.
Related themes
E. 1. Airlines


- **Monopoly problem:** single player chooses routes to operate between k cities. Hub-spoke network is optimal due to economics of traffic.

- **Duopoly problems:** with aggressive competition, single active hub-spoke networks, with moderate competition, multiple hub-spoke networks active.

- **Entry deterence:** a dominant carrier uses hub-spoke network to keep out entrants in the regional local routes.

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Equilibrium airline networks

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Related themes
E. 2. Network Resilience

- Robust networks: designing networks faced with intelligent adversaries.

- Key idea 1: connections improve functionality but also make nodes more vulnerable to indirect infection.

- Key idea 2: suppose designer can protect a few nodes: protected nodes serve as firewalls, and block infection spread.

- Star network is robust in the face of intelligent attack and limited defence budgets.

- Baccara & Bar-Isaac 2008; Goyal & Vigier 2014.
General theory: open problems

A. Dynamic network formation: network advantages suggest the pressure to pre-empt others in the creation of links.

B. Network formation with large number of players: key role of incomplete information about players and about networks.

C. Networks and markets: traditionally economists focused on markets and ignored social structures. Recent work focuses on networks and ignores markets. A need to integrate networks and markets.

Closing the Circle

In the last two decades:

1. Economists have studied the role of social structure in understanding traditional issues – such as innovation, wages, market structure employment, inequality. A key conceptual innovation has been the idea that social networks themselves are an outcome of rational individual activity.

This is complemented by a broader longer scale movement:

2. Economists have broadened the scope of their enquiries and begun to address questions -- on crime, identity, gender, politics, the origin of networks -- that were viewed as lying beyond the pale. The theme of this summer school!