Optimal Parenting Styles: Evidence from a Dynamic Game with Multiple Equilibria

Marco Cosconati

Bank of Italy & IZA

FINET Workshop November 2012
Introduction
Motivation

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- It is controversial if leaving discretion to children is a better approach to parenting than setting strict limits.

It has been recently suggested that the “Tiger” parenting model, as opposed to “Western” parenting, is the main source of academic success of Asian children with respect to their peers. Addressing this debate has potentially important implications for public policies that ease parents’ monitoring costs by restricting children’s recreational activities.
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- Addressing this debate has potentially important implications for public policies that ease parents’ monitoring cost by restricting children’s recreational activities.
The strategic interaction between parents and children has received limited attention in economics.

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Parenting in Economics

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- The purpose of this paper is to address the "parenting-debate" by analyzing the development of adolescent skills as an equilibrium outcome of the \textit{repeated} interaction between parents and children
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The purpose of this paper is to address the "parenting-debate" by analyzing the development of adolescent skills as an equilibrium outcome of the repeated interaction between parents and children.

I extended my previous work by estimating a "fully" dynamic model whose equilibria hold under standard conditions adopted in the principal-agent model literature and abstracting from permanent asymmetric information.

Results indicate that the dynamic aspect is important to understand the quantitative impact of alternative parenting strategies.
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Introduction
Statistics on Parenting Styles

<table>
<thead>
<tr>
<th>Table: Curfew Limit by Age</th>
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<td>12-13</td>
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<td>Parents</td>
</tr>
<tr>
<td>Jointly/Child</td>
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<td>N</td>
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1984 cohort

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Marco Cosconati (Bank of Italy & IZA)
Introduction

Statistics on Parenting Styles

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<th></th>
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1984 cohort

Table: Friends Limit by Age

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**Table: TV Limit by Age**

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1984 cohort
Introduction
Statistics on Parental Limits

Table: Curfew by Race

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1984 cohort, age: 12-13
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Statistics on Parental Limits

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1984 cohort, age:12-13

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1984 cohort, age:12-13
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Statistics on Parental Limits

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*all cohorts, pooled data*
Introduction

Statistics on Parental Limits

**Table: Average PIAT and Limits**

<table>
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_all cohorts, pooled data_

**Table: Average GPA and Limits**

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_all cohorts, pooled data_

**Table: Average CAT-ASVAB and Limits**

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_all cohorts, pooled data_
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The game
Information structure and order of moves

- There are two forward looking players: the parent and the child
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Information structure and order of moves

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- Complete information, finite horizon (T periods)
The game

Information structure and order of moves

- There are two forward looking players: the parent and the child
- Complete information, finite horizon (T periods)
- The conflict b/w players stems from a mismatch in preferences, i.e. there are no behavioral differences which can advocated to justify parental intervention

The order of moves on the stage game is as follows:

- Conditional on the stock of human capital, $K_t$ and the beliefs about parents choose a parenting style $\rho_t \in \mathbb{R}^n$
The game

Information structure and order of moves

- There are two forward looking players: the parent and the child
- Complete information, finite horizon ($T$ periods)
- The conflict b/w players stems from a mismatch in preferences, i.e. there are no behavioral differences which can advocated to justify parental intervention

The order of moves on the stage game is as follows:
- Conditional on the stock of human capital, $K_t$ and the beliefs about parents choose a parenting style $\rho_t \in \mathbb{R}^n$
- The child chooses an effort level $a_t \in [0, 1]$
The game

Information structure and order of moves

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- Child’s new human capital $K_{t+1}$, becomes public
- The stage game is repeated
Primitives
Preferences

- Child cares about leisure and his adult human capital

\[ u_t = \begin{cases} 
  u(l_t) & \text{if } t = 1, 2, \ldots, T \\
  \Xi(K_{T+1}) & \text{when the game is over}
\end{cases} \]

with $u$ and $\Xi$ increasing

- Parent cares suffers from the monitoring cost and cares about the child’s adult human capital

\[ w_t = \begin{cases} 
  -c(\rho_t) & \text{if } t = 1, 2, \ldots, T \\
  \Pi(K_{T+1}) & \text{when the game is over}
\end{cases} \]

with $c$ and $\Pi$ increasing
Primitives
The Evolution of Skills

- Cognitive skills evolve stochastically according to a distribution $F(K_{t+1} | a_t, K_t)$ such that
  - $K_{t+1} \in [k, \bar{k}]$ for any $K_t$ and $a_t$
  - $F(K_{t+1} | a'', K_t) \text{ FOSD } F(K_{t+1} | a', K_t)$ for any $a'' > a'$, $K_t$
  - $F(K_{t+1} | a_t, K'') \text{ FOSD } F(K_{t+1} | a_t, K')$ for any $K'' > K'$, $a_t$

- I capture evolution in noncognitive skills through the changes in the discount factor $\delta(K_t)$. Endogenous formation of time preferences. A possible parametrization is:

$$\delta(K_t) = \frac{\exp(K_t)}{1 + \exp(K_t)}$$

- The model captures:
  - The cross and self-productivity of skills:
    $$\uparrow \delta(K_t) \Rightarrow \uparrow a_t \Rightarrow \uparrow K_{t+1} \Rightarrow \delta(K_{t+1})$$
The Child’s Constraint

- The child solves his maximization problem under the constraint $a \geq \tau$
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- The child solves his maximization problem under the constraint $a \geq \tau$
- $\tau \in [0, 1]$ is a random variable with conditional distribution $G(\tau|\rho)$
- Let $R = (\rho_1, \ldots, \rho_N)$ be ordered according to FOSD order, i.e.

$$\rho'' > \rho' \iff G(\tau|\rho'') \geq G(\tau|\rho')$$

- e.g. $\rho''$ is stricter than $\rho'$ if and only if $\rho'' > \rho'$
- A parenting style can be interpreted as a set of rules the parent imposes on the child’s recreational activities
The Child’s Constraint

- The child solves his maximization problem under the constraint $a \geq \tau$
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  e.g. $\rho''$ is stricter than $\rho'$ if and only if $\rho'' > \rho'$
- A parenting style can be interpreted as a set of rules the parent imposes on the child’s recreational activities
- The stochastic nature of the map between $\rho$ and $\tau$ reflects the existence of an imperfect monitoring problem
- Higher the variance of $\tau$ conditional on $\rho$ more pervasive the moral hazard problem
Strategies and Equilibria

- The parent’s history is $H_{t,p} = (K^t, a^{t-1}, \rho^{t-1}, \tau^t)$
- The child’s history is $H_{t,c} = (K^t, a^{t-1}, \rho^t, \tau^t)$ where $x^t = (x_1, \ldots, x_t)$
- The parent’s strategy is a map $\phi^p : H_{t,p} \to \mathbb{R}$
- The child’s strategy is a map $\phi^c : H_{t,c} \to [0, 1]$
- An optimal strategy profile for the child, $\vec{\phi}^c = \langle \phi^c_t(H^t) \rangle_{t=1}^T$, is given by a sequence of strategies such that:

$$\vec{\phi}^c \in \arg\max E \left[ \sum_{t=1}^{T} v(\phi^c_t(H^t,c)) | \vec{\phi}^p \right]$$ (1)

- An optimal strategy profile for the parent, $\vec{\phi}^p = \langle \phi^p_t(H^t,p) \rangle_{t=1}^T$, is given by a sequence of strategies such that:

$$\vec{\phi}^p \in \arg\max E \left[ \sum_{t=1}^{T} w(\phi^p_t(H^t,c)) | \vec{\phi}^c \right]$$ (2)
Recursive Representation

- I focus on Markov Strategies in which $H^{t,c} = (K_t, \tau_t)$, $H^{t,p} = K_t$
- The problem solved by the child is

$$V(K) = \left\langle \max_{a \geq \tau} \left[ u(1 - a) + \delta(K) \int_{\tau}^{K} V'(K') dF(K' | K, a) \right] \right\rangle$$

- $V' = \Xi$ if $t = T$
- $V'$ incorporates the parent’s strategy at $t + 1$ if $t \leq T - 1$
Recursive Representation

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- Let $a(\tau, K)$ denote the child’s best response function
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- $V' = \Xi$ if $t = T$
- $V'$ incorporates the parent’s strategy at $t + 1$ if $t \leq T - 1$
- Let $a(\tau, K)$ denote the child’s best response function
- The parent takes it as given and solves

$$W(k) = \left\langle \max_{\rho} \left[ -c(\rho) + \beta \int \limits_{0}^{1} \int \limits_{k} W'(K')dF(K' | K, a(\tau, K))dG(\tau | \rho) \right] \right\rangle$$

- Let $\rho(K)$ denote the best response correspondence of the parent
Monotone Comparative Statics

Let \( f : X \times \Theta \to \mathbb{R} \) with \( X \subseteq \mathbb{R}^n \), and \( \Theta \subseteq \mathbb{R}^m \).

We say that \( f \) has the single crossing property in \((x, \theta)\), if and only if for any \( x'' > x', \theta'' > \theta' \)

\[
f(x'', \theta'') - f(x', \theta'') \geq (>)0 \Rightarrow f(x'', \theta') - f(x', \theta') \geq (>)0
\]

Equivalently we say that \( f(x, \theta'') \) dominates \( f(x, \theta') \) according to the single crossing order \( f(x, \theta') \succeq f(x, \theta') \)
Monotone Comparative Statics

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  - We say that $f$ has the single crossing property in $(x, \theta)$, if and only if for any $x'' > x', \theta'' > \theta'$
    $$f(x'', \theta'') - f(x', \theta'') \geq (>)0 \Rightarrow f(x'', \theta') - f(x', \theta') \geq (>)0$$
  - Equivalently we say that $f(x, \theta'')$ dominates $f(x, \theta')$ according to the single crossing order $f(x, \theta') \succeq f(x, \theta'')$
  - We say that $f$ has increasing differences in $(x, \theta)$ if and only if $\Delta(\theta) = f(x'', \theta) - f(x', \theta)$ is increasing in $\theta$
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Let $f : X \times \Theta \rightarrow \mathbb{R}$ with $X \subseteq \mathbb{R}^n$, and $\Theta \subseteq \mathbb{R}^m$.

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- If $f$ has ID in $(x, \theta)$ then $f$ has the SCP in $(x, \theta)$
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- If \( f \) has ID in \((x, \theta)\) then \( f \) has the SCP in \((x, \theta)\)

- (Milgrom & Shannon): \( \arg\max_x f(x, \theta) \) is increasing in \( \theta \) if and only if \( f \) has the SCP
Under which conditions is $\rho(K)$ decreasing in $K$ for any $t$?
Characterizing Equilibria: Punishing Strategy Equilibria

- Under which conditions is $\rho(K)$ decreasing in $K$ for any $t$?
- In order to apply the standard Milgrom-Shannon theorem I need to show that the objective function of the child has the single-crossing property in $(K, \rho)$
Under which conditions is $\rho(K)$ decreasing in $K$ for any $t$?

In order to apply the standard Milgrom-Shannon theorem I need to show that the objective function of the child has the single-crossing property in $(K, \rho)$

This is complicated by the fact that the parent’s payoff function involve non-primitive objects - $a(\tau, K)$ and the value functions
Best Responses: Graphical Analysis

**Figure:** Properties of Best Responses

\[ e(\tau, k_H) \]

\[ e(\tau, k_L) \]
Best Responses: Graphical Analysis

Figure: Properties of Best Responses
Properties of the Best Response Function

Lemma

- If \( a(\tau, K) \) is increasing in \( K \) (I), then \( a(\tau, K') \succeq_{IDO} a(\tau, K'') \) for any \( K'' > K' \).

- If the objective function of the child is single-peaked (U) and (I) holds, \( \Delta(K) = a(\tau'', K) - a(\tau', K) \) is decreasing in \( K \).

An intuitive approach:

- If for any \( t \) (U) and (I) hold
Properties of the Best Response Function

Lemma

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An intuitive approach:

1. If for any \( t \) (U) and (I) hold
2. The parent’s payoff function preserves the DD property of \( \rho_t(K_t) \) then MS is applicable \( \Rightarrow \rho_t(K_t) \) is decreasing in \( K_t \) for any \( t \)
Sufficient Conditions

Lemma

If

1. \( F(K|k,a) = F_1(K|k) + F_2(K|a) \)
2. \( F_2 \) has the CDFC property, i.e. is convex in \( a \)
3. \( u \) is concave

then \( \rho_t(K_t) \) is decreasing in \( K_t \).

Main ingredients

- 1) exploits the fact that the sum of supermodular functions is supermodular by approximating an increasing function as a sum of steps function
- An increasing and concave (convex) transformation of a function with DD(ID) has the DD(ID) ( (2) and (3) )
- A theorem by Vives & Van Zandt preserves the DD property under integration
Data and Sample

The NLSY97 contains information on:

- the person setting the limit on the 3 activities (first three survey rounds for children born in 1983-1984)
- time spent watching TV and doing homework (first survey round for all cohorts)
- GPA achieved at the end of each academic (only for high school) year and PIAT test scores (first rounds all cohorts and all other rounds only 1984 cohorts)
Patterns
Homework

Table: Proportion of Children Spending Some Time Doing Homework in a Typical Week

<table>
<thead>
<tr>
<th>Age</th>
<th>Black Children</th>
<th>Hispanic Children</th>
<th>White Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>88.51 (296)</td>
<td>85.42 (240)</td>
<td>92.81 (612)</td>
</tr>
<tr>
<td>13</td>
<td>86.47 (414)</td>
<td>85.17 (344)</td>
<td>91.5 (871)</td>
</tr>
<tr>
<td>14</td>
<td>82.25 (462)</td>
<td>83.14 (344)</td>
<td>90.22 (941)</td>
</tr>
<tr>
<td>15</td>
<td>82.71 (133)</td>
<td>82.68 (127)</td>
<td>83.71 (264)</td>
</tr>
</tbody>
</table>

Number of observations in parenthesis

Table: Time Spent Doing Homework in a Typical Week

<table>
<thead>
<tr>
<th>Age</th>
<th>Black Children</th>
<th>Hispanic Children</th>
<th>White Children</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>S.D</td>
</tr>
<tr>
<td>12-16</td>
<td>4.34</td>
<td>3.75</td>
<td>3.71</td>
</tr>
<tr>
<td>12</td>
<td>4.53</td>
<td>4</td>
<td>3.23</td>
</tr>
<tr>
<td>13</td>
<td>5.10</td>
<td>4.5</td>
<td>3.53</td>
</tr>
<tr>
<td>14</td>
<td>5.29</td>
<td>4.5</td>
<td>2.29</td>
</tr>
<tr>
<td>15</td>
<td>5.79</td>
<td>5</td>
<td>3.65</td>
</tr>
</tbody>
</table>

Number of observations in parenthesis
Means and Medians are calculated on non-zero observations
Units: hours per week
Patterns TV watching

Table: Time Spent Watching TV in a Typical Week

<table>
<thead>
<tr>
<th>Age</th>
<th>Black Children</th>
<th>Hispanic Children</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>S.D</td>
</tr>
<tr>
<td>12-15</td>
<td>24.49</td>
<td>22</td>
<td>14.84</td>
</tr>
<tr>
<td>12</td>
<td>23.39</td>
<td>21</td>
<td>14.19</td>
</tr>
<tr>
<td>13</td>
<td>24.38</td>
<td>21</td>
<td>15.5</td>
</tr>
<tr>
<td>15</td>
<td>22.53</td>
<td>18</td>
<td>15.34</td>
</tr>
</tbody>
</table>

Number of observations in parenthesis
Means and Medians are calculated on non-zero observations
Units: hours per week
Multiplicity of equilibria

- Under (I) and (U) all the SPNE of the game have the (PS) property: there exists a matrix of cutoffs

\[ c = (c_1, c_2, \ldots, c_T) \]

\[ (n-1) \times T \]

such that

- \( \rho_t(k_{t-1}) = \rho_i \iff c_{i-1} \leq k_{t-1} \leq c_i \)
Multiplicity of equilibria

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\[
\begin{bmatrix}
    c_1 \\
    c_2 \\
    \vdots \\
    c_T \\
\end{bmatrix}
= \begin{bmatrix}
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    c_2 \\
    \vdots \\
    c_T \\
\end{bmatrix} \quad \text{such that}
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Multiplicty of equilibria

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  \vdots \\
  c
  \end{pmatrix}_{(n-1) \times T} = (c_1, c_2, \ldots, c_T)
  \]
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  - Given a set of cutoffs \( \mathbf{c} \), \( a(K, \tau) \) is unique
  - Given \( a(K, \tau) \) the matrix \( \mathbf{c} \) is not unique due to possible corner solutions and because of the dynamic aspect of the problem solved by the child
Figure: Multiple Equilibria
Two Step Estimation

- It is well known that multiple equilibria pose problem for the identification of the structural parameters due to the incompleteness problem.
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There is no “general” method to deal with multiple equilibria.
Two Step Estimation

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- I adopt a 2-step estimator analogous to the one proposed by Moro (IER 2003) which avoids equilibrium selection.
Two Step Estimation

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- There is no “general” method to deal with multiple equilibria
- I adopt a 2-step estimator analogous to the one proposed by Moro (IER 2003) which avoids equilibrium selection
  - First step: solve the child’s problem numerically and estimate $F$, $G \delta$, $u$ together with $c$ (child’s response is a function) through SML using measurement error
  - Second step: given $c$ recover $w_t$ using a GMM estimator based on the equilibrium conditions
First Step
Measures and Choices

The NLSY97 contains

- 2 measures of children’s cognitive skills: GPA and PIAT math test scores
  - I use a linear factor model assuming that they are proxies for the state variable of the model

\[
GPA_t = n_0 + n_1 k_t + \epsilon_t^G \\
PIAT_t = m_0 + m_1 k_t + \epsilon_t^P
\]

with \( \epsilon_t^G \) and \( \epsilon_t^P \) being normally distributed
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- 2 measures of time allocation: time spent watching TV and doing homework which are part of the choice set of the child
  - I adopt an hurdle model for the measurement equation to deal with both the intensive and the extensive margin:

\[
g(e^o) = \begin{cases} 
\rho e & \text{if } e^o = 0 \\
\frac{1-\rho e}{1-G(0|e)} g(e^o|e) & \text{if } e^o > 0
\end{cases}
\]

where
- \(e^o\) denotes the observed effort (time spent doing homework), while \(e\) the optimal effort implied by the model
- \(G\) is the CDF of a normal \(g(\cdot|e)\) with mean \(\lambda_{0,e} + \lambda_{1,e} e\) and variance \(\sigma_e^2\)
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  - An analogous model is adopted to model TV viewership

- I use the responses on the limits to construct three binary indicators (1 if the parent decides alone, 0 otherwise). Each response is allowed to be misclassified with positive probability
  \[ \Pr(\rho^o(j) = 1|\rho(j) = 1) = E_j + (1 - E_j)\Pr(\rho(j) = 1) \]
To map the model to the data I

- I model parent-child interaction from grade 6 to grade 12
First Step
Sample and Data-Model Map

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- I model parent-child interaction from grade 6 to grade 12
- To map the model to the data I divide the academic year into two semesters: fall and winter, which gives 14 “periods”
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In the sample I include

- Children who have no siblings living in the household at the time of the interview (treat in and out moves as exogenous)
To map the model to the data I

- I model parent-child interaction from grade 6 to grade 12
- To map the model to the data I divide the academic year into two semesters: fall and winter, which gives 14 “periods”

In the sample I include

- Children who have no siblings living in the household at the time of the interview (treat in and out moves as exogenous)
- High school graduates (do not model dropping out decisions) with a normal grade progression
Several issues if one wants to estimate the model by SML:

- the model contains an unobservable state variable ($\tau$)
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- there are missing variables for my measures of human capital
- computing conditional probabilities requires to integrate out all the possible realizations of the state variables
- to overcome the computational burden I implement the method developed by Keane and Wolpin (IER 2001) which
  - only requires to simulate “outcome” histories, i.e. only needs unconditional simulations
  - uses the densities of the measurement/classification errors to reconcile the predictions of the model with the observations in the data
First Step
Data Limitations, Heterogeneity and Identification

- Because “effort” is observed only once, permanent unobserved heterogeneity in “mental abilities” cannot be identified using the information on the child’s skills (GPA and PIAT).
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Key identification arguments

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**Key identification arguments**

Conditional on a matrix $c$

- the observed variation in effort as a function of $k$ is informative about the parameters entering $\delta(k)$
- the parameters entering the payoff functions are identified by unconditional observed variations of time allocation choices
- Given a stock of human capital, the variations in the effort conditional on an observed parenting style pins down the parameters entering $G(\tau|\rho)$

The matrixes $c$ are identified by the observed variations in the parenting style choices conditional on the human capital of the child (analogous to an ordered logit with heterogeneous cutoffs)
The goal of the second step is to recover the parameters into $\Pi$ and the costs $\kappa_i$.

The estimated matrix $\hat{c}$ provides, for each parental type at most $T \times (n - 1)$ moments where

$$
\int W(K) dF(K|a(\hat{c}_i, \rho(\hat{c}_i)), \hat{c}_i) = \kappa_{i-1} - \kappa_i > 0
$$

where in the last period $W = \Pi$. Let $\#\Pi$ denote the number of parameters entering into $\Pi$. Exact point-identification is achieved if and only if $\#\Pi = T = 14$. Overidentification is the typical case.
Second Step

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First Step

Parametrizations

Preferences

\[ \nu(\alpha_t) = \begin{cases} 
(1 - e_t)^\eta & \text{if } t < T + 1 \\
\Xi(k_T) & \text{if } t = T + 1 \text{ (the game is over)}
\end{cases} \]

with \( \eta \in (0, 1) \)

\[ \Xi(k_T) = \frac{1}{\xi} \left\{ 1 - \exp \left[ -\xi \left( \frac{k_T^{1-\sigma} - 1}{1 - \sigma} \right) \right] \right\} \]

with \( \sigma > 0 \) and \( \xi > 0 \)

\[ \delta(k) = \frac{\vartheta_0}{\vartheta_0 + \exp(-\vartheta_1 k)} \]

with \( \vartheta_0 > 0 \)
First Step
Parametrizations

Technologies

- \( G(\tau|\lambda) = \text{Beta}[d(\rho_t), 1] \) with \( \tau \in [0, 1], \lambda_1 > 0 \)
- I parametrize \( d(\rho_t) \) as follows

\[
d(\rho_t) = \frac{\lambda_0 \rho_t}{1 + \lambda_0 \rho_t}
\]

there exists a positive probability of misclassification

- The human capital production is the sum of two power density production functions

\[
F(K|e, k) = \frac{K^{a(e)}}{2} + \frac{K^{b(k)}}{2} \quad \text{with} \quad K \in [0, 1]
\]

with

\[
a(e) = \frac{\theta_0 + e^{\theta_1}}{1 + \theta_0 + e^{\theta_1}}
\]

\[
b(k) = \frac{\exp(\theta_2 k)}{1 + \exp(\theta_2 k)}
\]

with \( \theta_0 > 0, \theta_1 \in (0, 1) \) and \( \theta_2 > 0 \).
## Table: Estimates

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### Model Fit

#### Homework Doing

**Table: % of Children Studying**

<table>
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<th>age</th>
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<th>Model</th>
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<tbody>
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<td>15</td>
<td>87.14</td>
<td>89.24</td>
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**Table: Average Study Time**

<table>
<thead>
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<tr>
<td>15</td>
<td>4.57</td>
<td>4.81</td>
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Model Fit
Limits

Table: Proportion of Parents Setting the Curfew

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<td>48.27</td>
<td>52.72</td>
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Table: Proportion of Parents Setting Limits on TV shows

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<td>15.32</td>
</tr>
<tr>
<td>16</td>
<td>11.16</td>
<td>13.21</td>
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A Policy Question

Governments intervention in disciplining children is subject to debate:

In the early days of the Labour government there was much discussion in the media about where the boundary lay between interferences and appropriate involvement. There are some who view the family as a private institutions and believe that, except in the extreme cases, it should be largely free from state interferences. On the other hand there are those who take the position that government does have a role in enhancing support for the family - Coleman and Roker

Provided that the enforcement cost is low, why not eliminating parental monitoring cost?

Marco Cosconati (Bank of Italy & IZA)
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Counterfactuals

Tiger Mother

- No matter what: you stay home, no TV, no friends
- The ATE of such a policy on PIAT test scores would be small: an increase of about 3% in the PIAT test scores and of 10% in the GPA
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- However the are important distributional effects

**Table: Distributional Effect on GPA**

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
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<tbody>
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<td>17.6%</td>
<td>12.4%</td>
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<td>4.2%</td>
<td>2.9%</td>
<td>0.5%</td>
<td>-4.7%</td>
<td>-7.2%</td>
<td>-11.8%</td>
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**Table: Distributional Effect on PIAT**

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<tr>
<th></th>
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<th>G3</th>
<th>G4</th>
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<th>G6</th>
<th>G7</th>
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<th>G10</th>
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<td>10.1%</td>
<td>8.3%</td>
<td>5.1%</td>
<td>2.3%</td>
<td>1.9%</td>
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Counterfactuals
Complete Freedom

- Reassigning property rights: parent as a concierge
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<td>−3.9%</td>
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</tbody>
</table>
Road Ahead

- Welfare Analysis
Road Ahead

- Welfare Analysis
- Reshuffling: optimal assignment problem

Longer Term Research Agenda:
- Monetary Incentives
Road Ahead

- Welfare Analysis
- Reshuffling: optimal assignment problem

Longer Term Research Agenda:
- Monetary Incentives
- Multiple Siblings
Road Ahead

- Welfare Analysis
- Reshuffling: optimal assignment problem

Longer Term Research Agenda:
- Monetary Incentives
- Multiple Siblings
- Asymmetric Information