The Money Value of a Man

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Common View: Most valuable asset that most people hold is their own human capital.

Questions:

What are the properties of the value of an individual’s human capital?

What are the properties of the associated returns?
Value and Return Concepts:

\[ v_j \equiv E_j \left[ \sum_{k=j+1}^{J} m_{j,k} e_k \right] \]

\[ R_{j+1}^h \equiv \frac{v_j + 1 + e_j + 1}{v_j} \]

What we do: (1) justify this notion of value and (2) analyze \((v_j, R_{j}^h)\) using US data.
Motivation:

1. Portfolio advice.

2. International portfolio diversification puzzle

3. Welfare Gains of moving to a smoother consumption plan

4. Estimating preference parameters
Preview of our Findings

1. Value

- Far below the value of discounting future earnings at the risk-free rate
- Stock component is smaller than the bond component
- Large negative orthogonal component early in life

2. Returns

- Mean returns very large early in life and decline with age
- Small positive correlation with stock return
Related Literature:

1. Value of Human Capital
   - Farr (1853), Weisbrod (1961), Becker (1975), ... discount earnings at a deterministic rate. Not useful for analyzing returns.

2. Return to Human Capital
   - Campbell (1996), Baxter and Jermann (1997), ... use aggregate earnings data
   - Mincerian Returns literature focuses on a different notion of a return.

3. Portfolio Allocation
   - focus is on understanding what impacts portfolio composition
Justification for the Notion of Value:

Problem P1

\[ \max U(c, n) \quad \text{where} \quad c = (c_1, \ldots, c_J) \quad \text{and} \quad n = (n_1, \ldots, n_J) \]

1. \( c_j + \sum_{i \in I} a^i_{j+1} = e_j + \sum_{i \in I} a^i_j R^i_j \)

2. \( e_j = G_j(y^j, n^j, z^j), \quad 0 \leq n_j \leq 1 \quad \text{and} \quad a^i_{j+1} \geq 0 \)

Important: \( U \) is concave

Unimportant: Restrictions on \( G \), number of assets, …
Problem P2

\[ \max U(c, n) \]

1. \[ c_j + \sum_{i \in I} a_{j+1}^i + s_j + v_j + p_j n_j = s_j(v_j + d_j) + \sum_{i \in I} a_j^i R_j^i \]

2. \[ 0 \leq n_j \leq 1 \] and \[ a_{J+1}^i \geq 0 \]

Personalized prices: \( (p_j, v_j) \)
Definitions:

1. $m_{j,k}(z^k) = \frac{dU(c^*,n^*)/dc_k(z^k)}{dU(c^*,n^*)/dc_j(z^j)} \frac{1}{P(z^k|z^j)}$ - stochastic discount factor

2. $v_j(z^j) = E[\sum_{k=j+1}^{J} m_{j,k}d_k|z^j]$ - value of human capital

3. $d_j(z^j) = e^*_j(z^j) + p_j(z^j)n^*_j(z^j)$ - dividends

4. $p_j(z^j) = \frac{dU(c^*,n^*)/dn_j(z^j)}{dU(c^*,n^*)/dc_j(z^j)}$ - price of leisure
Theorem: If \((c^*, n^*, e^*, y^*, a^*)\) solves P1 and \(c^*\) is strictly positive, then \((c^*, n^*, a^*, s^*)\) solves P2, where \(s^*_j = 1\), when the agent takes \((v_j, d_j, p_j)\) as given.

Intuition: P2 is a concave program. Necessary conditions to P1 are sufficient for P2.

Extensions: value earnings and leisure components separately or allow financials frictions
Naive Value: \( v_j^{naive} = E\left[\sum_{k=j+1}^{J} \frac{1}{(1+r)^{k-j}} d_k | z^j \right] \)

Our Notion of Value: \( v_j = E\left[\sum_{k=j+1}^{J} m_{j,k} d_k | z^j \right] \)

When will the two notions differ?

\[
v_j = \sum_{k=j+1}^{J} E[m_{j,k} | z^j] E[d_k | z^j] + \sum_{k=j+1}^{J} \text{cov}(m_{j,k}, d_k | z^j)
\]

When: \( E[m_{j,k} | z^j] < 1/(1 + r)^{k-j} \) agent is on a corner

When: \( \text{cov}(m_{j,k}, d_k | z^j) < 0 \)
Simple Example

\[
\max E\left[ \sum_{j=1}^{J} \beta^{j-1} u(c_j) \right] \text{ subject to } \]
\[
(1) \quad c_j + a_{j+1} \leq a_j (1 + r) + e_j, \quad (2) \quad c_j \geq 0, \quad a_{J+1} \geq 0
\]

\[u(c) = \frac{c^{1-\rho}}{(1-\rho)} - \text{CRRA}\]

\[e_j = \prod_{k=1}^{j} z_k \text{ and } \ln z_k \sim N(\mu, \sigma^2) \text{ i.i.d.}\]

\[(1 + r) > 0 - \text{one risk-free asset}\]
**Theorem:** If \( 1 + r = \frac{1}{\beta} \exp(\rho \mu - \frac{\rho^2 \sigma^2}{2}) \) and initial assets are zero, then \((c_j, a_{j+1}) = (e_j, 0), \forall j\) solves the decision problem. Furthermore, at this solution:

(i) the value of human capital is \( v_j(z^j) = f_j e_j \),

\[
f_j = \sum_{k=j+1}^{J} \beta^{k-j} \exp((k-j)[(-\rho +1)\mu + (-\rho +1)^2 \frac{\sigma^2}{2}])\).

(ii) the return to human capital satisfies:

(a) \( R_{j+1}^h = \left(\frac{1+f_{j+1}}{f_j}\right) z_{j+1} \)

(b) \( E[R_{j+1}^h | z^j] = \frac{1}{\beta} \exp(\mu \rho + \frac{\sigma^2}{2}(1 - (1 - \rho)^2)) \)
Quantitative Analysis:

Model Periods: age 20 – 65

Flat Earnings Profile: $e_1 = 1$ and $\mu = -\sigma^2/2$ and $\sigma \in [0, .3]$ 

Risk Aversion Parameter: $\rho \in \{1, 2, 4\}$

Interest Rate: $1.01 = 1 + r = \frac{1}{\beta} \exp(\rho \mu - \frac{\rho^2 \sigma^2}{2})$
Figure 1: Human capital values and returns: simple example

Notes:

\[ U((1 + \Omega(\alpha))c) = U((1 - \alpha)c + \alpha c^{smooth}) \]

\[ \Omega'(0) = \left[ \sum_{j=1}^{J} \sum_{z_j} \frac{dU(c)}{dc_j(z_j)} (c_j^{smooth}(z^j) - c_j(z^j)) \right] \]

\[ \Omega'(0) = \frac{E[\sum_{j=1}^{J} m_{1,j} c_j^{smooth}|z^1]}{v^e_1(z^1) + e_1(z^1) + \sum_{i \in I} a^i_1(z^1) R^i(z^1)} - 1 \]
Benchmark Model:

\[ U(c_1, ..., c_J) = W(c_1, F(U(c_2, ..., c_J)); j) \] - Epstein-Zin utility

\[ W(a, b; j) = [(1 - \beta)a^{1-\rho} + \beta\psi_{j+1}b^{1-\rho}]^{1/(1-\rho)} \]

\[ F(x) = (E[x^{1-\alpha}])^{1/(1-\alpha)} \]

Two assets: bond and stock

Exogenous earnings: \( e_j = G_j(z^j) \)

Extra restrictions: \( a^s_{j+1} \geq 0 \) and \( a^s_{j+1} \leq p(a^s_{j+1} + a^b_{j+1}) \)
Empirical Framework I:

\[ \log e_{i,j,t} = u^1_t + u^2_{i,j,t} \]

\[ u^2_{i,j,t} = \alpha^i + \kappa_j + \zeta_{i,j,t} + \nu_{i,j,t} \]

\[ \zeta_{i,j,t+1} = \rho \zeta_{i,j,t} + \eta_{i,j,t+1} \text{ and } \zeta_{i,0,t} = 0 \]

\[ \alpha \sim N(0, \sigma^2_{\alpha}), \eta \sim N(0, \sigma^2_{\eta}(\Delta u^1_t)), \nu \sim N(0, \sigma^2_{\nu}(\Delta u^1_t)) \]
Empirical Framework II: \( y_t = (u_t^1, P_t)' \) where \( \log R_t^s = \Delta P_t \)

\[
y_t = v + \sum_{i=1}^{p} A_i y_{t-i} + \varepsilon_t
\]

If \( y_t \sim I(1) \) can rewrite as VECM \( (w_t \equiv \beta'y_t + \mu) \)

\[
\begin{pmatrix}
\Delta y_t \\
w_t
\end{pmatrix} = \begin{pmatrix}
\gamma \\
\beta' \gamma
\end{pmatrix} + \sum_{i=1}^{p-1} \begin{pmatrix}
\Gamma_i \\
\beta' \Gamma_i
\end{pmatrix} \Delta y_{t-i} + \begin{pmatrix}
\alpha \\
1 + \beta' \alpha
\end{pmatrix} w_{t-1} + \begin{pmatrix}
\varepsilon_t \\
\beta' \varepsilon_t
\end{pmatrix}
\]

\[
\begin{pmatrix}
\Delta y_t \\
w_t
\end{pmatrix} = \begin{pmatrix}
\gamma \\
\beta' \gamma
\end{pmatrix} + \begin{pmatrix}
\Gamma \\
\beta' \Gamma
\end{pmatrix} \begin{pmatrix}
\alpha \\
1 + \beta' \alpha
\end{pmatrix} \begin{pmatrix}
\Delta y_{t-1} \\
w_{t-1}
\end{pmatrix} + \begin{pmatrix}
\varepsilon_t \\
\beta' \varepsilon_t
\end{pmatrix}
\]
Data Sources:

Idiosyncratic Earnings:

- PSID 1967-1996 (three samples: FULL, HS and COL)
- Male heads age 22-60 w/ annual earnings > 1,000

Aggregate Component:

- PSID 1967-96 and CPS 1967-2009 (three samples)

Asset Returns:

Ken French’s Data Archive: (1) stock return is NYSE/AMEX/NASDAQ return, (2) bond return is based on 6-month TBill
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Full Sample</th>
<th>College Sub-sample</th>
<th>High School Sub-sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.957</td>
<td>0.959</td>
<td>0.902</td>
</tr>
<tr>
<td>$\sigma^2_\alpha$</td>
<td>0.110</td>
<td>0.092</td>
<td>0.121</td>
</tr>
<tr>
<td>av. $\sigma^2_\eta$</td>
<td>0.033</td>
<td>0.033</td>
<td>0.039</td>
</tr>
<tr>
<td>$\sigma^2_\eta (L) - \sigma^2_\eta (H)$</td>
<td>0.020</td>
<td>0.012</td>
<td>0.025</td>
</tr>
<tr>
<td>av. $\sigma^2_\varepsilon$</td>
<td>0.150</td>
<td>0.151</td>
<td>0.147</td>
</tr>
<tr>
<td>Linear time trend in $\sigma^2_\eta$</td>
<td>0.0011</td>
<td>0.0014</td>
<td>0.0013</td>
</tr>
</tbody>
</table>
Table 3: Steady-State Statistics for the Aggregate Process

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>No Cointegration</th>
<th>With Cointegration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\log R^b_t)$</td>
<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$E(\log R^s_t)$</td>
<td>0.041</td>
<td>0.045</td>
<td>0.047</td>
</tr>
<tr>
<td>$E(\Delta u^1_t)$</td>
<td>-0.002</td>
<td>-0.004</td>
<td>-0.003</td>
</tr>
<tr>
<td>$sd(\Delta u^1_t)$</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>$sd(\log R^s_t)$</td>
<td>0.187</td>
<td>0.187</td>
<td>0.188</td>
</tr>
<tr>
<td>$corr(\Delta u^1_t, \log R^s_t)$</td>
<td>0.184</td>
<td>0.177</td>
<td>0.172</td>
</tr>
<tr>
<td>$corr(\Delta u^1_t, \Delta u^1_{t-1})$</td>
<td>0.425</td>
<td>0.441</td>
<td>0.420</td>
</tr>
<tr>
<td>$corr(\log R^s_t, \log R^s_{t-1})$</td>
<td>0.057</td>
<td>0.055</td>
<td>0.039</td>
</tr>
<tr>
<td>$corr(\Delta u^1_t \log R^s_{t-1})$</td>
<td>0.372</td>
<td>0.398</td>
<td>0.390</td>
</tr>
<tr>
<td>$corr(\log R^s_t, \Delta u^1_{t-1})$</td>
<td>-0.292</td>
<td>-0.270</td>
<td>-0.292</td>
</tr>
</tbody>
</table>
Table 3: Steady-State Statistics for the Aggregate Process

<table>
<thead>
<tr>
<th></th>
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<th>College Sub-sample Without Cointegration</th>
<th>College Sub-sample With Cointegration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\log R^b_t)$</td>
<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$E(\log R^s_t)$</td>
<td>0.041</td>
<td>0.040</td>
<td>0.045</td>
</tr>
<tr>
<td>$E(\Delta u^1_t)$</td>
<td>0.000</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>$\text{sd}(\Delta u^1_t)$</td>
<td>0.023</td>
<td>0.023</td>
<td>0.023</td>
</tr>
<tr>
<td>$\text{sd}(\log R^s_t)$</td>
<td>0.187</td>
<td>0.187</td>
<td>0.186</td>
</tr>
<tr>
<td>$\text{corr}(\Delta u^1_t, \log R^s_t)$</td>
<td>0.248</td>
<td>0.251</td>
<td>0.243</td>
</tr>
<tr>
<td>$\text{corr}(\Delta u^1_t, \Delta u^1_{t-1})$</td>
<td>0.346</td>
<td>0.341</td>
<td>0.342</td>
</tr>
<tr>
<td>$\text{corr}(\log R^s_t, \log R^s_{t-1})$</td>
<td>0.057</td>
<td>0.084</td>
<td>0.050</td>
</tr>
<tr>
<td>$\text{corr}(\Delta u^1_t \log R^s_{t-1})$</td>
<td>0.377</td>
<td>0.387</td>
<td>0.367</td>
</tr>
<tr>
<td>$\text{corr}(\log R^s_t, \Delta u^1_{t-1})$</td>
<td>-0.225</td>
<td>-0.235</td>
<td>-0.229</td>
</tr>
</tbody>
</table>
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<th>With Cointegration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\log R_{t}^{b})$</td>
<td>0.012</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$E(\log R_{t}^{s})$</td>
<td>0.041</td>
<td>0.045</td>
<td>0.055</td>
</tr>
<tr>
<td>$E(\Delta u_{t}^{1})$</td>
<td>-0.007</td>
<td>-0.010</td>
<td>-0.008</td>
</tr>
<tr>
<td>$sd(\Delta u_{t}^{1})$</td>
<td>0.030</td>
<td>0.030</td>
<td>0.030</td>
</tr>
<tr>
<td>$sd(\log R_{t}^{s})$</td>
<td>0.187</td>
<td>0.187</td>
<td>0.190</td>
</tr>
<tr>
<td>$corr(\Delta u_{t}^{1}, \log R_{t}^{s})$</td>
<td>0.207</td>
<td>0.194</td>
<td>0.192</td>
</tr>
<tr>
<td>$corr(\Delta u_{t}^{1}, \Delta u_{t-1}^{1})$</td>
<td>0.386</td>
<td>0.416</td>
<td>0.408</td>
</tr>
<tr>
<td>$corr(\log R_{t}^{s}, \log R_{t-1}^{s})$</td>
<td>0.057</td>
<td>0.047</td>
<td>0.037</td>
</tr>
<tr>
<td>$corr(\Delta u_{t}^{1}, \log R_{t-1}^{s})$</td>
<td>0.387</td>
<td>0.420</td>
<td>0.420</td>
</tr>
<tr>
<td>$corr(\log R_{t}^{s}, \Delta u_{t-1}^{1})$</td>
<td>-0.289</td>
<td>-0.261</td>
<td>-0.277</td>
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Table 4: Parameter Values for the Benchmark Model

<table>
<thead>
<tr>
<th>Category</th>
<th>Symbol</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime</td>
<td>( J, Ret )</td>
<td>((J, Ret) = (69, 40))</td>
</tr>
<tr>
<td>Preferences</td>
<td>( \alpha )</td>
<td>Risk Aversion, ( \alpha \in {4, 6, 8, 10} )</td>
</tr>
<tr>
<td></td>
<td>( 1/\rho )</td>
<td>Intertemporal Substitution, ( 1/\rho = 1.17 )</td>
</tr>
<tr>
<td></td>
<td>( \psi_{j+1} )</td>
<td>Survival Probability, U.S. Life Table</td>
</tr>
<tr>
<td></td>
<td>( \beta )</td>
<td>Discount Factor, see Notes</td>
</tr>
<tr>
<td>Earnings</td>
<td>( e_j(z) = z_1g_j(z_2)(1 - \tau) )</td>
<td>( j &lt; Ret )</td>
</tr>
<tr>
<td></td>
<td>( e_j(z) = z_1b(\alpha) )</td>
<td>( j \geq Ret )</td>
</tr>
<tr>
<td>Returns</td>
<td>( R^s, R^b )</td>
<td>Table 2 - 3</td>
</tr>
<tr>
<td>Leverage</td>
<td>( p )</td>
<td>( p = 1 )</td>
</tr>
</tbody>
</table>
Figure 2: Life-cycle profiles in the benchmark model

Notes:
Figure 3: Human capital values and decomposition

Notes:
Decomposing Values:

\[ v_j = E_j[m_{j,j+1}y_{j+1}] = E_j[m_{j,j+1}(\alpha^b R^b_{j+1} + \alpha^s R^s_{j+1} + \epsilon)] \]

\[ v_j = \alpha^b E_j[m_{j,j+1}R^b_{j+1}] + \alpha^s E_j[m_{j,j+1}R^s_{j+1}] + E_j[m_{j,j+1}\epsilon] \]

\[ 1 = \frac{\alpha^b}{v_j} E_j[m_{j,j+1}R^b_{j+1}] + \frac{\alpha^s}{v_j} E_j[m_{j,j+1}R^s_{j+1}] + \frac{E_j[m_{j,j+1}\epsilon]}{v_j} \]

Upshot: decompose \( v_j \) into three components. Bond and stock shares sum to more than 1 when \( E_j[m_{j,j+1}\epsilon] < 0 \).
Figure 4: Properties of human capital returns

Notes:
Decomposing Returns:

\[ R^h_{j+1} \equiv \frac{v_j + 1 + e_{j+1}}{v_j} = \frac{\alpha^b R^b_{j+1} + \alpha^s R^s_{j+1} + \epsilon}{v_j} \]

\[ E_j[R^h_{j+1}] = \frac{\alpha^b}{v_j} E_j[R^b_{j+1}] + \frac{\alpha^s}{v_j} E_j[R^s_{j+1}] \]

Upshot: weights sum to more than 1 when \( E_j[m_{j,j+1}\epsilon] < 0 \)
Figure 5: Portfolio shares in the benchmark model

Notes:
1. Our work does not offer a direct link. Earnings are exogenous. However, some of our findings (e.g. \( v_j << v_j^{naive} \) and \( E[R^h_j] > E[R^s_j] > E[R^b_j] \)) should also hold when earnings are endogenously determined.

2. Such a model (e.g. Huggett, Ventura and Yaron (2011)) would allow marginal returns and total human capital returns to be analyzed jointly.

3. Huggett, Ventura and Yaron (2011) would also suggest that analyzing human capital accumulation and its financing from age 20 or so onwards is way too late. They find that (averaging across individuals) there is little net skill accumulation after this age.