The Macroeconomics of Microfinance

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Microfinance Revolution

- Small loans, targeted to the poor
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- Small loans, targeted to the poor
  - business loans
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- Small loans, targeted to the poor
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  - consumption smoothing
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- High growth rates, desire to scale up even more...
Microfinance Revolution

- Small loans, targeted to the poor
  - business loans
  - consumption smoothing
  - human capital investment
- Low default rates: 2.06 – 3.54% (median)
- High growth rates, desire to scale up even more...
- ... but no evaluation of general equilibrium effects
This Paper

- Models the microfinance revolution as an innovation that:
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  1. guarantees a minimum (uncollateralized) loan for production
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  2. has no risk of default
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• Models the microfinance revolution as an innovation that:
  1. guarantees a minimum (uncollateralized) loan for production
  2. has no risk of default
  3. and no intermediation costs
This Paper (cont’d)

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   - little net effect on per-capita income
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  3. increases wages, redistributing from “rich” to “poor”
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Important GE effects
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   - Little net effect on per-capita income
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   - Increases welfare/consumption of workers/marginal entrepreneurs

Important GE effects: more redistribution and welfare gains...but smaller impact on aggregate output and consumption... opposite impact on TFP and capital
Road Map

- Benchmark calibrated model
  - Compare with microevaluations
  - Present GE aggregate impacts
  - Present GE distributional impacts
  - Compare GE with PE effects

- Extensions:
  - Small open economy
  - Model w/ market labor shock
  - Add large-scale sector with fixed cost
Benchmark Model

- Heterogeneous agents: entrepreneurial ability and wealth.
Benchmark Model

- Heterogeneous agents: entrepreneurial ability and wealth.
- Occupational choice: Work for wage or operate their own technology.
Benchmark Model

- Heterogeneous agents: entrepreneurial ability and wealth.
- Occupational choice: Work for wage or operate their own technology.
- Financial friction: limited enforcement.
Model: Plant Technology

\[ f(z, k, l) = z k^\alpha l^\theta \]
Model: Plant Technology

\[ f (z, k, l) = zk^\alpha l^\theta \]

- \(z\): entrepreneurial productivity
- 1 unit of entrepreneur’s time
- \(k\): capital input
- \(l\): labor input (workers)
- \(\alpha + \theta < 1\)
Model: Process of Entrepreneurial Talent

\[ z_s = \begin{cases} 
  z_{s-1} & \text{w/ prob. } \gamma \\
  \zeta_s & \text{w/ prob. } 1 - \gamma 
\end{cases} \]

\[ \zeta_s \sim \eta \zeta^{-1}, \zeta \geq 1 \]

- \( \gamma \) measures persistence
Model: Process of Entrepreneurial Talent

\[ z_s = \begin{cases} 
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  \zeta_s & \text{w/ prob. } 1 - \gamma 
\end{cases} \]

\[ \zeta_s \overset{iid}{\sim} \eta \zeta^{-\eta-1}, \quad \zeta \geq 1 \]

- \( \gamma \) measures persistence
- \( -\eta \) measures the thickness of the right tail
Model: Timing

borrow

entrepreneur

(k,l)

produce
repay/default
consume

z

z'~μ(z')

1-γ

γ

z

z'~μ(z')

1-γ

γ

z

z'~μ(z')

t

(a,z)

occupational
choice

worker

produce
consume

t+1
Workers supply 1 unit of labor at $w$
Workers supply 1 unit of labor at $w$

$$v^w(a, z) = \max_{c, a' \geq 0} \ u(c) + \beta \mathbb{E}_z \max \{ v^w(a', z'), v^e(a', z') \}$$

$$c + a' \leq w + (1 + r) a$$
Model: Individual Problem
Workers’ Bellman Equation

Workers supply 1 unit of labor at $w$

$$v^w(a, z) = \max_{c,a' \geq 0} u(c) + \beta \mathbb{E}_z \max \left\{ v^w(a', z'), v^e(a', z') \right\}$$

$$c + a' \leq w + (1 + r) a$$

where entrepreneur’s value $v^e(a', z')$ is given next
Model: Individual Problem
Entrepreneurs’ Bellman Equation

\[ v^e(a, z) = \max_{c, a', k, l} u(c) + \beta \mathbb{E}_z \max \{ v^w(a', z'), v^e(a', z') \} \]

\[ c + a' \leq zk^\alpha l^\theta - (r + \delta) k - wl + (1 + r) a \]

\[ zk^\alpha l^\theta - (r + \delta) k - wl + (1 + r) a \geq (1 - \phi) \left[ zk^\alpha l^\theta - wl + (1 - \delta) k \right] \]

(enforcement constraint, EC)
Model: Individual Problem
Entrepreneurs’ Bellman Equation

\[ v^e(a, z) = \max_{c,a',k,l} u(c) + \beta \mathbb{E}_z \max \{ v^w(a', z'), v^e(a', z') \} \]

\[ c + a' \leq zk^\alpha l^\theta - (r + \delta) k - wl + (1 + r) a \]

\[ k \leq \bar{k}(a, z; \phi) \]

(rental limit)
Rental Limit

\[ k(a, z, \phi)/w \]

- \( z_\infty \)
- \( z_{99} \)
- \( z_{95} \)
- \( z_{90} \)
Occupational Choice

![Graph showing the relationship between log(z) (Entrepreneurial Ability) and log(a) (Wealth). The graph distinguishes between Entrepreneurs and Workers.](image-url)
Occupational Choice (cont’d)
Occupational Choice (cont’d)
Introduce new technology that:

1. guarantees a minimum (uncollateralized) loan for production
2. has no risk of default
3. and no intermediation costs
Modeling Microfinance Revolution

New technology that changes rental limit from:
Modeling Microfinance Revolution

New technology that changes rental limit from:

\[ k \leq \bar{k}(a, z; \phi) \]
Modeling Microfinance Revolution

New technology that changes rental limit from:

\[ k \leq \bar{k}(a, z; \phi) \]

to

\[ k \leq \max\{\bar{k}(a, z; \phi), a + b^{MF}\} \]
Rental Limit

\[ k(a, z, \phi)/w = \begin{cases} z_\infty \\ z_{99} \\ z_{95} \\ z_{90} \end{cases} \]
Rental Limit w/ Microfinance, $b^{MF} = \frac{1}{2}w$
(Partial Equilibrium) Impact on Occupational Choice
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![Graph showing the impact of log(z) (Entrepreneurial Ability) on log(a) (Wealth). The graph includes vertical lines at z_{90}, z_{95}, z_{99}, and z_{∞}.]
Rental Limit w/ Microfinance, $b^{MF} = w$
(Partial Equilibrium) Impact on Occupational Choice

![Graph showing the impact of entrepreneurial ability on wealth. The graph plots log(z) (Entrepreneurial Ability) on the x-axis and log(a) (Wealth) on the y-axis. The line on the graph represents the relationship between entrepreneurial ability and wealth. Vertical lines at z_90, z_95, and z_99 indicate specific points of interest. The graph illustrates how wealth changes with entrepreneurial ability.]
(Partial Equilibrium) Impact on Occupational Choice
Objects for Stationary Competitive Equilibria

- \( o(a, z) \): occupational choice
- \( G(a, z) \): joint distribution of \( a \), \( z \)
- \( \mu(z) = 1 - z^{-\eta} \): stationary distribution of \( z \)
Definition: Stationary Competitive Equilibria

\[ G(a, z), \text{ policies } o(a, z), c(a, z), a'(a, z), k(a, z), l(a, z), \text{ rental limit } \bar{k}(a, z; \phi), \text{ and prices } w \text{ and } r \text{ such that:} \]

- Allocations solve individuals’ problems given prices and rental limit;
- \( \bar{k}(a, z; \phi) \) satisfies EC;
- Labor and credit markets clear;
- \( G(a, z) \) satisfies

\[
G(a, z) = \gamma \int_{\bar{z} < z, a'(<a, \bar{z}) \leq a} G(d\bar{a}, d\bar{z}) \\
+ (1 - \gamma) \mu(z) \int_{a'(<\bar{a}, \bar{z}) \leq a} G(d\bar{a}, d\bar{z}).
\]
Empirical Strategy

- Choose technology \((\alpha, \theta)\) and productivity process \((\eta^{US}, \gamma)\) to match US data on size distribution and dynamics of establishments and income concentration, given \(\phi^{US} = 1\).
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- Choose contract enforcement and distribution of productivity \((\eta^{IND}, \phi^{IND})\) to match Indian data on the size distribution and external finance to GDP.
Empirical Strategy

- Choose technology \((\alpha, \theta)\) and productivity process \((\eta^{US}, \gamma)\) to match US data on size distribution and dynamics of establishments and income concentration, given \(\phi^{US} = 1\)
- Choose contract enforcement and distribution of productivity \((\eta^{IND}, \phi^{IND})\) to match Indian data on the size distribution and external finance to GDP
- Evaluate impact of \(b^{MF}\)
## Empirical Strategy

**Target US Data Model Parameter**

<table>
<thead>
<tr>
<th>Target</th>
<th>US Data</th>
<th>Model</th>
<th>Parameter</th>
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</thead>
<tbody>
<tr>
<td>top 10% employment share</td>
<td>0.69</td>
<td>0.69</td>
<td>$\eta^US = 4.84$</td>
</tr>
<tr>
<td>top 5% income share</td>
<td>0.30</td>
<td>0.30</td>
<td>$\alpha + \theta = 0.79$</td>
</tr>
<tr>
<td>Exit rate</td>
<td>0.10</td>
<td>0.10</td>
<td>$\gamma = 0.89$</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.04</td>
<td>0.04</td>
<td>$\beta = 0.92$</td>
</tr>
</tbody>
</table>

**Target Indian Data Model Parameter**

<table>
<thead>
<tr>
<th>Target</th>
<th>Indian Data</th>
<th>Model</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>top 10% employment share</td>
<td>0.58</td>
<td>0.58</td>
<td>$\eta^IND = 5.56$</td>
</tr>
<tr>
<td>Ext. fin./GDP</td>
<td>0.34</td>
<td>0.34</td>
<td>$\phi^IND = 0.08$</td>
</tr>
</tbody>
</table>
Relation to Microevaluations

- Two recent studies evaluate interventions impact on entrepreneurial households
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  - 1. Urban: India Hyderabad study
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• We simulate similar sized intervention and compare short-run, partial equilibrium impacts
Relation to Microevaluations

- Two recent studies evaluate interventions impact on entrepreneurial households
  1. Urban: India Hyderabad study (Banerjee et al, 2010)
  2. Rural: Thai village funds study (Kaboski and Townsend, forthcoming, 2010)
- We simulate similar sized intervention and compare short-run, partial equilibrium impacts
- Model capture key features (heterogeneity, orders of magnitude) reasonably well
Impacts on Marginal Ability Entrepreneurs

Take-up Rate
MF/Ext. Fin.

Income
Consumption

Ability Percentile

0.00 0.20 0.40 0.60 0.80 1.00

Take-up Rate
MF/Ext. Fin.

Income
Consumption

Ability Percentile

0.60 0.70 0.80 0.90 1.00
<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>India</th>
<th>Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Loan/Exp per Cap</td>
<td>1</td>
<td>1-2</td>
<td>1</td>
</tr>
<tr>
<td>Credit/Exp per Cap</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Microfinance/Total Credit</td>
<td>29%</td>
<td>44%</td>
<td>33%</td>
</tr>
<tr>
<td>Entrepreneurship</td>
<td>+4 pp</td>
<td>+2 pp</td>
<td>+1 pp</td>
</tr>
<tr>
<td>Investment</td>
<td>+46%</td>
<td>+16/128%</td>
<td>+30% (prob).</td>
</tr>
<tr>
<td>Consumption</td>
<td>+1%</td>
<td>+16/0%</td>
<td>+15%</td>
</tr>
</tbody>
</table>
More on Thai Study

- Rural Thailand vs. Urban India and Model
More on Thai Study

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  - Stronger evidence for consumption increase
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  - Rural villages likely to have segmented markets, 7 percent overall wage increase
More on Thai Study

- Rural Thailand vs. Urban India and Model
  - Stronger evidence for consumption increase
  - Weaker evidence for entrepreneurship, investment increase
    - only seen in larger samples
  - Rural villages likely to have segmented markets, 7 percent overall wage increase
    - concentrated in low-skilled labor in the village
Aggregate Implications

![Graph showing the impact of $b^{MF}/w(0)$ on output, capital, and TFP.](image)

- **Output**
- **Capital**
- **TFP**

![Graph showing the impact of $b^{MF}/w(0)$ on wage and interest rate.](image)

- **Wage** (left)
- **Interest Rate** (right)
Aggregate Implications: Short-Run vs. Long-Run

- **Output**
- **Capital**
- **TFP**

Graphs show the relationship between various economic indicators and the wage rate for different values of $b^{MF}/w(0)$.
Aggregate Implications: Role of Occupational Choice

\begin{align*}
&\text{Output} \\ &\text{Capital} \\ &\text{TFP}
\end{align*}

\begin{align*}
&b_{w(0)}^M \\
&0.7 \\
&0.8 \\
&0.9 \\
&1.0 \\
&1.1 \\
&1.2
\end{align*}
Explaining Aggregate Effects

• Why does TFP increase?
Explaining Aggregate Effects

- Why does TFP increase?
  - Microfinance allows entrepreneurs with high marginal product of capital to invest more
Explaining Aggregate Effects

- Why does TFP increase?
  - Microfinance allows entrepreneurs with high marginal product of capital to invest more
- Why does capital fall?
Explaining Aggregate Effects

- Why does TFP increase?
  - Microfinance allows entrepreneurs with high marginal product of capital to invest more

- Why does capital fall?
  - Microfinance redistributes income from talented (high saving) to untalented (low saving) individuals
Understanding TFP

\[ \frac{b^{MF}}{w(0)} \]

- **TFP**
- **Efficient \( k \)**
- **Efficient \( z \)**

[Graphs showing evolution of TFP and its components over different values of \( b^{MF}/w(0) \).]

- **Avg. \( z \) (left)**
- **Entre. frac. (right)**

Details
Understanding Capital Accumulation

Aggregate savings rate, $\frac{S}{Y}$, is an (income) weighted average of individual savings:
Aggregate savings rate, $S/Y$, is an (income) weighted average of individual savings:

$$\frac{S}{Y} = \frac{Y(z_{low})}{Y} \frac{S(z_{low})}{Y(z_{low})} + \frac{Y(z_{high})}{Y} \frac{S(z_{high})}{Y(z_{high})}$$
Understanding Capital Accumulation

- Saving $z_{95}^{100}$ (left)
- Income $z_{95}^{100}$ (right)
- Saving $z_{0}^{95}$ (left)

$$b^{MF}/w(0)$$
Distribution of Welfare Gains
Distribution of Welfare Gains

fraction of permanent consumption
How does GE affect results?

1. More redistribution
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   • bigger welfare gains for low ability, low wealth
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2. Smaller positive aggregate impacts
How does GE affect results?

1. More redistribution
   • bigger welfare gains for low ability, low wealth

2. Smaller positive aggregate impacts
   • lower TFP (less entry, talented guys get less resources)
How does GE affect results?

1. More redistribution
   - bigger welfare gains for low ability, low wealth

2. Smaller positive aggregate impacts
   - lower TFP (less entry, talented guys get less resources)
   - less capital (wages redistribute to low savers)
More Redistribution in GE

![Graphs showing the relationship between ability/wealth percentile and redistribution.](image-url)
Smaller Aggregate Impacts in GE

General Equilibrium

Partial Equilibrium

- Output
- Capital
- TFP
Smaller Aggregate Impacts in GE vs PE short-run

General Equilibrium

Partial Equilibrium, Short-Run
Smaller Aggregate Impacts in GE vs PE short-run

TFP Decomposition

General Equilibrium

Partial Equilibrium, Short-Run

TFP
Efficient k
Efficient z

0 1 2 3 4 5

b^{MF}/w(0)
Extensions

- Small open economy (capturing capital supplied by foreign donors)
Extensions

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  - Capital demand still falls: lower wealth accumulation
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Extensions

- Small open economy (Ext1)
  (capturing capital supplied by foreign donors)
  - Capital demand still falls: lower wealth accumulation
  - Smaller TFP gains with $r$ constant
- Zero labor shock (Ext2)
  (capturing poor, low ability entrepreneurs)
Extensions

- Small open economy (Ext1)
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Extensions

- **Small open economy** (Ext1)
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    - Smaller TFP gains with \( r \) constant
- **Zero labor shock** (Ext2)
  - Capturing poor, low ability entrepreneurs
    - Lower TFP, capital accumulation -> wages fall
    - Self-employed benefit relative to workers
- **Two-sector model with fixed costs** (Ext3)
  - Capturing additional GE effect on relative price
Extensions

- Small open economy (capturing capital supplied by foreign donors)
  - Capital demand still falls: lower wealth accumulation
  - Smaller TFP gains with $r$ constant

- Zero labor shock (capturing poor, low ability entrepreneurs)
  - Lower TFP, capital accumulation $\rightarrow$ wages fall
  - Self-employed benefit relative to workers

- Two-sector model with fixed costs (capturing additional GE effect on relative price)
  - Large impact of large loans
Conclusion

- In GE microfinance is primarily a redistributive policy
- Potential impact on consumption & productivity, but not aggregate output as it discourages capital accumulation.
- GE effects differ from PE
  - smaller effects on output and consumption
  - more redistribution in GE
  - opposite effects on TFP and capital accumulation
Small Open Economy Model

- Fixed interest rate, wage rate still adjusts
- Captures idea that microfinance capital may come from abroad
- Capital still linked to savings decisions through collateral constraints
Closed vs. Small Open Economy

Closed Economy

![Graph showing the relationship between \( k^{MF}/w(0) \) and Output, Capital, and TFP in a closed economy.]

Small Open Economy

![Graph showing the relationship between \( k^{MF}/w(0) \) and Output, Capital, and TFP in a small open economy.]

<table>
<thead>
<tr>
<th></th>
<th>Closed Economy</th>
<th>Small Open Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>
Labor Shock Model

- When idea dies, draw zero labor endowment with probability $\pi$
- Captures idea of poor, potentially undercapitalized, low ability entrepreneur
- Calibrate $\pi$ to match 35 percent self-employed (India)
Aggregate Impacts: Labor Shock

![Graph showing the impact of labor shock on Output, Capital, and TFP. The graph plots $b_{MF}/w(0)$ on the x-axis and different measures on the y-axis.]

- Output
- Capital
- TFP

![Graph showing the impact of labor shock on Wage and Interest Rate. The graph plots $b_{MF}/w(0)$ on the x-axis and different measures on the y-axis.]

Wage (left)
Int. Rate (right)
Poorest, i.e., self-employed, benefit most
Two-Sector Model

- Two sectors: \( p = (p_S, p_M) \), with different fixed costs, \( \kappa_S < \kappa_M \), S: Services M: manuf./investment
- Heterogeneous individuals: entrepreneurial ability, \( z_S \) and \( z_M \), and wealth,
- Choice of occupation and sector: Work for wage or operate their own technology in either sector,
- Financial friction: collateral constraint, limited enforcement.
Aggregate Implications: Two-Sector

![Graph showing the relationship between \( \frac{b_{MF}}{w(0)} \) and output, capital, and TFP.](image)
Aggregate Implications: Two-Sector (Cont’d)

\[
\begin{array}{cccc}
\text{Output} & b^M & \text{Capital} & b^M \\
0.7 & 0.2 & 0.8 & 0.2 \\
0.9 & 0.3 & 1.0 & 0.3 \\
0.9 & 0.4 & 1.1 & 0.4 \\
0.9 & 0.5 & 1.2 & 0.5 \\
\end{array}
\]

Wage

Rel. Price

Interest Rate

\[
\begin{array}{cccc}
b^M/w(0) & 0 & 2 & 4 & 6 \\
\text{Output} & & & & \\
\text{Capital} & & & & \\
\text{TFP} & & & & \\
\end{array}
\]
Understanding TFP: Two-Sector
Understanding Capital Accumulation: Two-Sector

Saving rate top $z_M$

Saving rate bottom $z$

Saving rate top $z_S$

Income share top $z_S$

Income share top $z_M$

$k^{MF}/w(0)$
Model: Endogenous Rental Limits

\[
\max_{c,a',l} u(c) + \beta \mathbb{E}_z v(a', z') \geq v^{def}
\]
Model: Endogenous Rental Limits

\[
\max_{c,a',l} u(c) + \beta \mathbb{E}_z v(a', z') \geq v^{def}
\]

where

\[
v^{def} = \max_{c,a',l} u(c) + \beta \mathbb{E}_z v(a', z')
\]

\[
c + a' \leq (1 - \phi) \left[ z k^\alpha l^\theta - \omega l + (1 - \delta) k \right]
\]
Model: Endogenous Rental Limits

\[
\max u(c) + \beta \mathbb{E}_z v(a', z') \geq v^{def}
\]
Model: Endogenous Rental Limits

\[
\max u(c) + \beta \mathbb{E}_z v(a', z') \geq v^{def} \\
\uparrow \\
z k^\alpha l^\theta - (r + \delta)k - wl + (1 + r)a \\
\geq (1 - \phi) \left[ z k^\alpha l^\theta - wl + (1 - \delta)k \right]
\]
Model: Endogenous Rental Limits

\[
\max u(c) + \beta \mathbb{E}_z v(a', z') \geq v^{def} \\
\updownarrow \\
z k^\alpha l^\theta - (r + \delta) k - wl + (1 + r)a \\
\geq (1 - \phi) \left[ z k^\alpha l^\theta - wl + (1 - \delta) k \right] \\
\updownarrow \\
k \leq \bar{k}(a, z; \phi)
\]
### Table: Summary of Public Small Business Credit Programs

<table>
<thead>
<tr>
<th></th>
<th>India</th>
<th>Indonesia</th>
<th>Philippines</th>
<th>Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Program</td>
<td>NABARD</td>
<td>BRI-KUPEDES</td>
<td>PCFC</td>
<td>MBVF</td>
</tr>
<tr>
<td>Program Size</td>
<td>$2.7 Bn</td>
<td>$21 Bn</td>
<td>$150 M</td>
<td>$1.5 Bn</td>
</tr>
<tr>
<td>Typical/Avg. Loan</td>
<td>$1,200</td>
<td>up to $2,800</td>
<td>up to $3,500</td>
<td>$500</td>
</tr>
<tr>
<td>Loan/Income per-Capita</td>
<td>1.4</td>
<td>up to 1.3</td>
<td>up to 2</td>
<td>0.4</td>
</tr>
<tr>
<td>Country</td>
<td>Borrowers per-capita</td>
<td>MF Loans /GDP</td>
<td>Average Loan Balance</td>
<td>Per-capita Income</td>
</tr>
<tr>
<td>------------</td>
<td>----------------------</td>
<td>----------------</td>
<td>----------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>0.13</td>
<td>0.028</td>
<td>112</td>
<td>547</td>
</tr>
<tr>
<td>Mongolia</td>
<td>0.13</td>
<td>0.129</td>
<td>1393</td>
<td>1410</td>
</tr>
<tr>
<td>Peru</td>
<td>0.11</td>
<td>0.041</td>
<td>1590</td>
<td>4658</td>
</tr>
<tr>
<td>Bolivia</td>
<td>0.09</td>
<td>0.107</td>
<td>1926</td>
<td>1776</td>
</tr>
<tr>
<td>Vietnam</td>
<td>0.09</td>
<td>0.044</td>
<td>510</td>
<td>1024</td>
</tr>
<tr>
<td>Kenya</td>
<td>0.04</td>
<td>0.036</td>
<td>744</td>
<td>803</td>
</tr>
<tr>
<td>India</td>
<td>0.02</td>
<td>0.003</td>
<td>146</td>
<td>1154</td>
</tr>
<tr>
<td>Mean</td>
<td>0.02</td>
<td>0.004</td>
<td>655</td>
<td>3192</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.03</td>
<td>0.020</td>
<td>3192</td>
<td>3071</td>
</tr>
</tbody>
</table>
Two-Sector Model: Plant Technology

Fixed cost $\kappa_S < \kappa_M$ (units of sector output)
Two-Sector Model: Plant Technology

Fixed cost $\kappa_S < \kappa_M$ (units of sector output)

Gross output: $f^i(z_i, k, l) = z_i k^\alpha l^\theta$
Two-Sector Model: Preferences

Households maximize

\[ U(c) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \]

\[ u(c_t) = \frac{1}{1 - \sigma} \left( c_{S,t}^{1-\varepsilon} + c_{M,t}^{1-\varepsilon} \right)^{\frac{1-\sigma}{1-\varepsilon}} \]
Two-Sector Model: Individual Problem

Entrepreneurs’ Bellman Equation, Sector i

\[ v^i (a, z) = \max_{c, a', k, l} \left[ u (c) + \beta \mathbb{E}_z v \left( a', z' \right) \right] \]

\[ pc + a' \leq p_i f (z_s, k, l) - Rk - wl - (1 + r)p_i \kappa_i + (1 + r) a \]

\[ k \leq \bar{k}^i (a, z; \phi) \]
Modeling Microfinance

\[ k \leq \max\{\bar{k}(a, z; \phi), k^{MF} - p_i \kappa_i\} \]
Pareto Distribution of Productivity

\[ z_i \sim \eta z_i^{-(\eta+1)}, \quad z_S \perp z_M \]

- Thick right tail within each sector.
- Exact Cobb-Douglas benchmark.
Perfect Credit Benchmark
Size Distribution of Establishments

• Sector $i$:

\[
\text{Pr} \left[ l_i > l \right] = \left( \frac{l \left( \hat{z}_i \right)}{l} \right)^{\eta(1-\alpha-\theta)}
\]
Perfect Credit Benchmark
Size Distribution of Establishments

- Sector $i$:
  \[
  \Pr \left[ \tilde{l}_i > l \right] = \left( \frac{l(\tilde{z}_i)}{\tilde{l}_i} \right)^{\eta(1-\alpha-\theta)}
  \]

- Average employment per establishment $\bar{l}_i$:
  \[
  \frac{\bar{l}_i}{\bar{l}_i'} = \frac{p_i \kappa_i + w}{p_i' \kappa_i' + w}
  \]
### Empirical Strategy

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>US</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. scale in services</td>
<td>14</td>
<td>14</td>
<td>$\kappa_S = 0.00$</td>
</tr>
<tr>
<td>Avg. scale in manuf.</td>
<td>47</td>
<td>47</td>
<td>$\kappa_S = 1.00$</td>
</tr>
<tr>
<td>Manuf. share of GDP</td>
<td>0.25</td>
<td>0.25</td>
<td>$\psi = 0.91$</td>
</tr>
</tbody>
</table>
“It is worth noting that a fairly low take-up (16% after two years), similar to what was found in other studies, suggest that the effect of the program on poverty reduction and welfare is necessarily going to be relatively limited, even in the longer run. This is not necessarily a failure of this program in particular, or micro-credit in general. It may well be a very effective tool precisely for the minority of households who wants to expand their activity.”

Crepon, Devoto, Duflo and Pariente (2011)
Understanding TFP

\[
Y = \left[ \int_{i:o_i=e} z_i^{1-\theta} \left( \frac{k_i}{K} \right)^{\frac{\alpha}{1-\theta}} \, di \right]^{1-\theta} \frac{N^{1-\alpha-\theta}}{\left( \frac{L}{N} \right)^\theta K^\alpha N^{1-\alpha}}
\]

where \( N = L + E \), \( L = \int_{i:o_i=w} di \) and \( E = \int_{i:o_i=e} di \)
Understanding TFP (cont’d)

\[ TFP^{k-\text{eff}} = \left[ \int_{i:o_i=e} \frac{Z_i^{1-\alpha-\theta}}{E} \, di \right]^{1-\alpha-\theta} \left( \frac{E}{N} \right)^{1-\alpha-\theta} \left( \frac{L}{N} \right)^\theta \]
Understanding TFP (cont’d)

\[ \frac{TFP(b^{MF})}{TFP(0)} = \frac{TFP(0)}{TFP^{k-\text{eff}}(0)} \frac{TFP(b^{MF})}{TFP^{k-\text{eff}}(b^{MF})} \]

\( k\text{-efficiency} \)

\( z\text{-efficiency} \)