Social Interactions V: The Great Gatsby Curve

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Understanding the Great Gatsby Curve

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Overview I: Basic Claim

This paper is designed to provide insights into the relationship between cross-sectional inequality in the United States and the associated level of intergenerational mobility.

Originally identified for the OECD by Miles Corak (dubbed the Great Gatsby Curve by Alan Krueger) as a cross-section relation, we focus on intertemporal curve for the US.
We argue that an intertemporal Gatsby Curve is a salient feature of inequality in the US and that this relationship is causal.

In this analysis, inequality within one generation determines the level of mobility of its children, and so argue that the Gatsby curve phenomenon is an equilibrium feature where mechanisms run from inequality to mobility.

Our analysis proceeds at both theoretical and empirical levels.
Overview II: Basic Ideas: Mechanisms and Implications

Increases in cross-sectional inequality increase the magnitude of the differences in the characteristics of social contexts in which children and adolescents develop.

This is so both because increased cross sectional inequality implies greater differences in the quality of social context experienced by the relatively rich and the relatively poor, conditional on to an initial income distribution, and because the degree of segregation of rich and poor into disparate social contexts is itself an increasing function of the level of cross sectional inequality and so can increase.
We make these ideas concrete in the consensus of neighborhoods, so increased income inequality is linked to greater income segregation of neighborhoods which in turn increases the intergenerational persistence of socioeconomic status.

The model we develop constitutes an example of what Durlauf (1996c, 2006) titled the “memberships theory on inequality”: a perspective that identifies segregation as an essential determinant of inequality within and across generations. Benabou’s work in 1990’s essential predecessor.

While we focus on education, the causal chain between greater cross-sectional inequality, greater segregation, and slower mobility may apply to a host of contexts.
What We Do

1. Theoretical model of neighborhoods, segregation and mobility formalizes the inequality/mobility nexus

2. Stylized facts organized to provide empirical support for general claims.

3. Reduced form intergenerational mobility regressions constructed to explore mobility dynamics

4. Structural model calibrated.
What We Conclude

Evidence, in our judgment, supports perspective, but is not decisive.

Magnitudes of Gatsby effects not large enough to come close to theory of everything.
Theory Background 1: IGE Regression and the Great Gatsby Curve

One way to understand our argument is to start with a linear model relating parental income $Y_{ip}$ and offspring income $Y_{io}$

$$Y_{io} = \alpha + \beta Y_{ip} + \varepsilon_{io}$$  \hspace{1cm} (1)

As a statistical object, (1) can produce a Gatsby curve, but only one where causality runs from mobility to inequality.
In contrast, if the equilibrium model mapping of parent to offspring income is

$$Y_{io} = \alpha + \beta (X_i)Y_{ip} + \varepsilon_{io}$$  \hspace{1cm} (2)

for some set of variables $X_i$, a causal mapping from changes in the variance of income to the measure of mobility $\beta$, i.e. the coefficient produced by estimating (1) when (2) is the correct intergenerational relationship, can exist. If $X_i = Y_{ip}\beta(Y_{ip})Y_{ip} = f(Y_{ip})$, then (2) becomes a nonlinear family investment income transmission model.
Theory Background 2: Our Model

Our theoretical model is based on Durlauf (1996a,b) which developed a social analogue to the class of family investment models of intergenerational mobility developed by Becker and Tomes (1979) and Loury (1981).

By social analogue, we mean a model in which education and human capital are socially determined and thereby mediate the mapping of parental income into offspring economic attainment. Relative to (2), we thus implicitly consider $X_i$ variables that are determined at a community level.
Summary of Environment and Equilibrium Properties

1. Labor market outcomes for adults are determined by the human capital that they accumulate earlier in life.

2. Human capital accumulation is, along important dimensions, socially determined. Local public finance of education creates dependence between the income distribution of a school district and the per capita expenditure on each student in the community. Social interactions, ranging from peer effects to role models to formation of personal identity, create a distinct relationship between the communities in which children develop and the skills they bring to the labor market.
3. In making a choice of a neighborhood, incentives exist for parents to prefer more affluent neighbors. Other incentives exist to prefer larger communities. These incentives interact to determine the extent to which communities are segregated by income in equilibrium. Permanent segregation of descendants of the most and least affluent families is possible even though there are no poverty traps or affluence traps, as conventionally defined.

4. Greater cross-sectional inequality of income increases the degree of segregation of neighborhoods. The greater the segregation the greater are the disparities in human capital between children from more and less affluent families, which creates the Great Gatsby Curve.
Alternative Routes to Gatsby Curve

It is important to recognize that our social determination of education approach is only one route to generating equilibrium mobility dynamics of the form (2).

Mulligan (1999) showed how credit market constraints, by inducing differing degrees in constraints for families of different incomes, could produce (2). In this case, $X_i$ can be thought of as family income.

Becker, Kominers, Murphy, and Spenkuch (2015) show how the Gatsby Curve behavior can emerge in a family investment model in which the productivity of human capital investment in a child is increasing in the level of parental human capital, which is another choice of $X_i$ in (2).
A Formal Model

a. demography

The population possesses a standard overlapping generations structure.

There is a countable population of family types, indexed by $i$, which we refer to as dynasties. Each family type consists of many identical “small” families.

This is a technical “cheat” to avoid adults considering the effect of their presence in a neighborhood on the income distribution. It can be relaxed without affecting any qualitative results.

Each agent lives for two periods.
Agent \( it \) is the adult member of dynasty \( i \) and so is born at time \( t - 1 \).

In period 1 of life, an agent is born and receives human capital investment from the neighborhood in which she grows up. In period 2, adulthood, the agent receives income, becomes a member of a neighborhood, has one child, consumes and pays taxes.
b. preferences

The utility of adult $it$ is determined in adulthood and depends on consumption $C_{it}$ and income of her offspring, $Y_{it+1}$. Offspring income is not known at $t$, so each agent is assumed to maximize expected utility that has a Cobb-Douglas specification.

$$EU_{it} = \pi_1 \log(C_{it}) + \pi_2 E(\log(Y_{it+1})|F_t) \quad (3)$$

where $F_t$ denotes parent’s information set.
c. income and human capital

Adult it’s income is determined by two factors.

First, each adult possesses a level of human capital that is determined in childhood, $H_{it-1}$.

Income is also affected by a shock experienced in adulthood $\xi_{it}$. These shocks may be regarded as the labor market luck, but their interpretation is inessential conditional on whatever is assumed with respect to their dependence on variables known to the parents. We model the shocks as independent of any parental information, independent and identically distributed across individuals and time with finite variance.
We assume a multiplicative functional form for the income generation process.

\[ Y_{it} = \phi H_{it-1} \xi_{it} \quad (4) \]

This functional form matters as it will allow the model to generate endogenous long term growth in dynasty-specific income. Equation (4) is an example of the AK technology studied in the growth literature.

We employ this technology in order to understand inequality dynamics between dynasties in growing economies.
d. family expenditures

A parent’s income decomposes between consumption and taxes.

\[ Y_{it} = C_{it} + T_{it} \]  

(5)
e. educational expenditure and educational investment in children

Taxes are linear in income and are neighborhood- and time-specific

\[ \forall i \in nt, \ T_{it} = \tau_{nt}Y_{it}. \]  \hspace{1cm} (6)

The total expenditure available for education in neighborhood \( n \) at \( t \) is

\[ TE_{nt} = \sum_{j \in nt} T_{jt} \]  \hspace{1cm} (7)

and so constitutes the resources available for educational investment.
We assume that the education process exhibits non-convexities with respect to population size, i.e. there exists a type of returns to scale (with respect to student population size) in the educational process.
Let $p_{nt}$ denotes the population size of $n$ at time $t$. The educational investment provided by the neighborhood to each child, $ED_{nt}$ (equivalent to educational quality), requires total expenditures

$$ED_{nt} = \frac{TE_{nt}}{\nu(p_{nt})}$$

(8)

where $\nu(p_{nt})$ is increasing such that that for some positive parameters $\lambda_1$ and $\lambda_2$, $0 < \lambda_1 < \frac{\nu(p_{nt})}{p_{nt}} < \lambda_2 < 1$
f. human capital

The human capital of a child is determined by two factors: the child’s skill level $s_{it}$ and the educational investment level $ED_{nt}$

$$H_{it} = \theta(s_{it}) ED_{nt},$$ (9)

where $\theta(\cdot)$ is positive and increasing. The term “skills” is used as a catch-all to capture the class of personality traits, preferences, and beliefs that transform a given level of educational investment into human capital.
The linear structure of (9) is extremely important as it will allow dynasty income to grow over time. Together, equations (4), (8), and (9) produce an AK-type growth structure relating educational investment and human capital, which can lead family dynasties to exhibit income growth because of increasing investment over time.
Entry level skills are determined by an interplay of family and neighborhood characteristics

\[ s_{it} = \zeta \left( Y_i, Y_{-i} \right) \quad (10) \]

where \( \zeta \) is increasing and exhibits complementarities. Dependence on \( Y_i \) is a placeholder for the role of families in skill formation. Dependence on \( Y_{-i} \) is readily motivated by a range of social interactions models.
g. neighborhood formation

Neighborhoods reform every period, i.e. there is no housing stock. As such, neighborhoods are like clubs. Neighborhoods are groupings of families, i.e. all families who wish to form a common neighborhood and set a minimum income threshold for membership. This is a strong assumption. That said, we would emphasize that zoning restrictions matter in neighborhood stratification, so the core assumption should not be regarded as obviously inferior to a neighborhood formation rule based on prices.
h. political economy

The equilibrium tax rate in a neighborhood is one such that there does not exist an alternative one preferred by a majority of adults in the neighborhood.

The Cobb-Douglas preference assumption renders existence of a unique majority voting equilibrium trivial because, under these preferences, there is no disagreement on the preferred tax rate. o desired budget share allocation.
i. borrowing constraints

Neither families nor neighborhoods can borrow. This extends the standard borrowing constraints in models of this type. With respect to families, we adopt Loury (1981) idea that parents cannot borrow against future offspring income. Unlike his case, the borrowing constraint matters for neighborhood membership, not because of direct family investment. In addition, in our analysis, communities cannot entail children who grow up as members to pay off debts accrued for their education. Both assumptions follow legal standards, and so are not controversial.
Neighborhood formation and intergenerational income dynamics: model properties

Proposition 1. Equilibrium neighborhood structure

i. At each $t$ for every cross-sectional income distribution, there is at least one equilibrium configuration of families across neighborhoods.

ii. In any equilibrium, neighborhoods are segregated.
Proposition 1 does not establish that income segregation will occur. Clearly it is possible that all families are members of a common neighborhood. If all families have the same income, complete integration into a single neighborhood will occur because of the nonconvexity in the education investment process. Income inequality is needed for segregation. Proposition 2 follows immediately from the form of the education production function nonconvexity we have assumed.
Proposition 2. Segregation and inequality

There exist income levels $\bar{Y}^{\text{high}}$ and $\bar{Y}^{\text{low}}$ such that families with $Y_{it} > \bar{Y}^{\text{high}}$
will not form neighborhoods with families with incomes $\bar{Y}^{\text{low}} > Y_{it}$.

Intuitively, if family incomes are sufficiently different, then more affluent families do not want neighbors whose tax base and social interactions effects are substantially lower than their own. Benefits to agglomeration for the affluent can be reversed when families are sufficiently poorer.
Income dynamics

Along an equilibrium path for neighborhoods, dynasty income dynamics follow the transition process

\[ \Pr(Y_{it+1} | F_t) = \Pr(Y_{it+1} | \bar{Y}_{nt}, p_{nt}) \]  

(11)

This equation illustrates the primary difficulty in analyzing income dynamics in this framework: one has to forecast the neighborhood composition. This leads us to focus on the behavior of families in the tails of the income distribution, in particular the highest and lowest income families at a given point in time.
Proposition 3. Equilibrium income segregation and its effect on the highest and lowest income families

i. Conditional on the income distribution at $t$, the expected offspring income for the highest family in the population is maximized relative to any other configuration of families across neighborhoods.

ii. Conditional on the income distribution at $t$, the expected offspring income of the lowest income family in the population is minimized relative to any other configuration of families across neighborhoods that does not reduce the size of that family’s neighborhood.
Proposition 4. Expected average growth rate for children in higher income neighborhoods than for children in lower income neighborhoods

Let $g_{nt+1}$ denote the average expected income growth between parents and offspring in neighborhood $n,t$. For any two neighborhoods $n$ and $n'$ if $\bar{Y}_{nt} < \bar{Y}_{n't}$, $p_{nt} \geq p_{n't}$, then $g_{nt+1} - g_{n't+1} > 0$. 
Proposition 4 does not speak to the sign of $g_{nt}$. Under the linear assumptions of this model, there exists a formulation of $\Theta(\cdot)$ and $\xi(\cdot,\cdot,\cdot)$ such that neighborhoods exhibit positive expected growth in all time periods, i.e. $\forall nt \ g_{nt} > g_{\text{min}} > 0$. In essence, this will hold when educational investment is sufficiently productive relative to the preference-determined equilibrium tax rates so that investment levels grow (this is the AK growth model requirement as modified by the presence of social interactions). We assume positive growth in what follows.
Proposition 6. Decoupling of upper and lower tails from the rest of the population of family dynasties

i. If $\forall nt \ g_{nt} > 0$, then there exists a set of time $t$ income distributions such that the top $\alpha \%$ of families in the distribution never experience a reduction in the ratios their incomes compared to any dynasty outside this group

ii. If $\forall nt \ g_{nt} > 0$, then there exists a set of time $t$ income distributions such that the bottom $\beta \%$ of families in the distribution never experience an increase in the ratios their incomes compared to any dynasty outside this group
Proposition 7. Intergenerational Great Gatsby curve

There are skill formation technologies such that there exists a set of time $t$ income distributions such that the intergenerational elasticity of parent/offspring income will be increased by a mean preserving increase in the variance of logarithm of initial income.
Underlying the theorem, there are two routes by which Gatsby Curves can be generated.

First, mean-preserving spreads alter the family-specific IGEs, which in this model take the form $\beta(Y_i, \bar{Y}_i)$. Hence once can construct cases where the linear approximation, i.e regression coefficient, increases with a mean-preserving spread.

Second, increased inequality can alter segregation.
Proposition 7 does not logically entail that increases in variance of income increase the intergenerational elasticity of income. The reason is that the model we have set up is nonlinear and effects of changes in parental income inequality into a scalar measure of mobility such as the IGE will typically not be independent of the shape of the income density, conditional on the variance. Put differently, the construction of a Great Gatsby Curve from our model involves two moments of a nonlinear, multidimensional stochastic process of family dynasties, and so the most one can expect is logical compatibility. The subtleties of producing Gatsby-like behavior in nonlinear models of course is not unique to our framework; see discussion in Becker, Kominers, Murphy and Spenkuch (2015).
General Evidence on the Inequality/Mobility Nexus

1. Direct evidence of an intertemporal Gatsby Curve: inequality and mobility are negatively associated.
Figure 1a Aaronson and Mazumder Rising intergenerational elasticities

The 90-10 Wage Gap and the IGE

Source: Aaronson and Mazumder (2008)
Figure 1b: Aaronson and Mazumder Rising intergenerational elasticities

The Income Share of Top 10% and the IGE

Source: Aaronson and Mazumder (2008)
Figure 1c: Aaronson and Mazumder: Rising intergenerational elasticities

The Return to College and the IGE

Source: Aaronson and Mazumder (2008)
Figure 2. Kearney and Levine: 90/10 and other ratios

Source: Kearney and Levine (2016). Notes: The x-axis reflects the year in which income is measured for the 90/50 and 50/10 ratios. For the mobility measure in Chetty, et al. (2014b), year reflects birth cohort. For the mobility measure in Lee and Solon (2009), year reflects the year in which the son's income was recorded.
2. Location/Mobility Nexus
Figure 3. Relationship between inequality and the rate of high school non-completion

Source: Kearney and Levine (2016). Notes: The graduation data is from Stetser and Stillwell (2014). The 50/10 ratios are calculated by the authors. The District of Columbia is omitted from this figure because it is an extreme outlier on the X axis (50/10 ratio = 5.66).
Figure 4. Chetty, Hendren, Kline, and Saez (2014): Spatial heterogeneity in rates of relative mobility

This map shows rates of upward mobility for children born in the 1980s for 741 metro and rural areas ("commuting zones") in the U.S. Upward mobility is measured by the fraction of children who reach the top fifth of the national income distribution, conditional on having parents in the bottom fifth. Lighter colors represent areas with higher levels of upward mobility.
3. Income Segregation is pervasive and growing
Figure 5. Spatial distribution of poverty rates

Source: US Census Bureau
Figure 6. Income segregation in Chicago

Source: US Census Bureau
Figure 7. Trends in family income segregation, by race

Source: Bischoff and Reardon (2013); authors' tabulations of data from U.S. Census (1970-2000) and American Community Survey (2005-2011). Averages include all metropolitan areas with at least 500,000 residents in 2007 and at least 10,000 families of a given race in each year 1970-2009 (or each year 1980-2009 for Hispanics). This includes 116 metropolitan areas for the trends in total and white income segregation, 65 metropolitan areas for the trends in income segregation among black families, and 37 metropolitan areas for the trends in income segregation among Hispanic families. Note: the averages presented here are unweighted. The trends are very similar if metropolitan areas are weighted by the population of the group of interest.
Figure 8. Changes in census tract income averages over time

Note: All income deflated using CPI-U-RS and expressed in logs.
Figure 9. Evolution of state income averages over time

Mean income in state (thous. $, log scale)

Note: All income deflated using CPI-U-RS and expressed in logs.
Figure 10. Evolution of census tract income variances over time

Note: All income deflated using CPI-U-RS and expressed in logs.
Figure 11. Evolution of state income variances over time

Note: All income deflated using CPI-U-RS and expressed in logs.
4. Spatial Heterogeneity in Factors Relevant to Human Capital/Sills Formation
Figure 12. Spatial variation in per capita public school expenditure

Note: 2014 per pupil expenditure, in dollars. Source: NCES.
Figure 13. Spending per student, by school district, Texas

Note: 2014 per pupil expenditure, in dollars. Source: NCES.
Figure 14. Exposure to violent crime

Figure 15. Distribution of homicides in Chicago

Empirical Evidence II: IGE Regressions

Our next collection of empirical evidence focuses on the properties of the IGE and implications for a Gatsby Curve.
Our first exercise considers nonlinearities in the intergenerational mobility process. One explanation of the Gatsby curve linking the variance of income to mobility is that the linear transmission process is misspecified, i.e.

\[ y_{io} = f(y_{ip}) + \varepsilon_{io} \]  \hspace{1cm} (12)

It is obvious that, depending on the shape of \( f(\cdot) \), increases in the variance of \( y_{ip} \) can increase the variance of \( y_{io} \).
To explore this possibility, we first construct a nonparametric estimate of $f()$. Figure 16 presents the nonparametric function. Figure 17 presents two ways of measuring local IGE values: $\frac{f(Y_{ip})}{Y_{ip}}$ and $f'(y_{ip})$ respectively.
Figure 16. Non-parametric estimation of offspring’s income given parental income

The figure shows that expected offspring income is non-linearly dependent on parental income. Offspring income conditional on parental income (red line) was non-parametrically calculated using a kernel density estimator with a normal density weighting function. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30—34. Parental income is individual's family income in adolescence (averaged over ages 13-17). The orange line represents the piece-wise linear prediction of offspring's income given parental income.
Figure 17a. Local IGE estimates for income

The graph displays local IGE estimates - defined as the marginal effect of parental income at each income level - obtained from non-parametric estimation of offspring's income conditional on parental income. The dependent variable is the marginal effect of parental income. Lower and upper bounds represent 1 standard deviation from the local IGE. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30—34. Parental income is individual's family income in adolescence (averaged over ages 13-17).
Figure 17b. Local IGE estimates for income

The graph displays local IGE estimates - defined as the ratio of offspring income to parental income level - obtained from non-parametric estimation of offspring's income conditional on parental income. The dependent variable is the ratio of offspring income to parental income. Lower and upper bounds represent 1 standard deviation from the local IGE. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30—34. Parental income is individual's family income in adolescence (averaged over ages 13-17).
As the point estimates and associated standard errors indicate, there is some evidence of nonlinearity, particularly in the tails of the income distribution. The decreasing \( \frac{f(Y_{ip})}{Y_{ip}} \) values are consistent with Chetty, Hendren, Kline and Saez (2014). The derivatives of the transmission function \( f'(y_{ip}) \), while roughly consistent with the first measure, are too erratic to interpret.

Together, we conclude that there is some, but not extremely strong evidence of nonlinearity in the sense of (2).
We complement these nonparametric results with some simple regressions which allow for differences in the linear IGE coefficients for parents in the tails of the income distribution as opposed to the middle.

Table 1 splits the sample according to whether a family was in the bottom 10%, the middle 80%, or the top 10% of the national income distribution. Table 2 repeats this exercise when income distribution location is calculated at the state level while Table 3 performs the same exercise at the census tract level.
Table 1. IGE regressions for bottom 10%, middle 80% and top 10% relative to nation

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Observations 1,617 1,617
R-squared 0.172 0.996

Robust standard errors in parentheses. All income in logs.
*** p<0.01, ** p<0.05, * p<0.1
Table 2. IGE regressions for bottom 10%, middle 80% and top 10% relative to state

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<td>R-squared</td>
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<td>0.996</td>
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Robust standard errors in parentheses. All income in logs.

*** p<0.01, ** p<0.05, * p<0.1
Table 3. IGE regressions for bottom 10%, middle 80% and top 10% relative to census tract

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Robust standard errors in parentheses. All income in logs.

*** p<0.01, ** p<0.05, * p<0.1
The national, state, and census tract level results are similar. In each case, there is relatively little heterogeneity in the IGE coefficients, while there is heterogeneity in the intercepts, with the bottom and top 10% growing more rapidly than the middle 80%. While the precision of the intercept estimates does not allow for very strong statements, these results are suggestive of decoupling of the upper tail of the type that is consistent with the admittedly extreme case of complete immobility that appears as a theoretical possibility. Note that the relatively higher growth of the lower 10% than the middle 80% is evidence of a convergence mechanism that lies outside the linear structure of (1), but nevertheless can generate the Gatsby Curve like behavior.
neighborhood income and the IGE levels

Our second set of exercises considers how the IGE may depend on the mean and variance of neighborhood income. We focus on parametric models that are variations of

\[
y_{io} = \alpha + \beta y_{ip} + \gamma_1 \bar{y}_{ig(p)} + \gamma_2 y_{ip} \bar{y}_{ig(p)} + \gamma_3 \text{ineq}(y_{ig(p)}) + \gamma_4 y_{ip} \text{ineq}(y_{ig(p)}) + \varepsilon_{io}
\]  

(13)

The parameters \( \gamma_1 \) and \( \gamma_2 \) capture average group income effects while \( \gamma_3 \) and \( \gamma_4 \) capture inequality effects.
Table 4 presents results where parental income is interacted with census tract income. Table 5 conducts the same exercise at the state level. Bloome (2015) estimates analogous models for variance at the state level. Table 6 combines census tract and state variables. We report results using the variance of log income. Models using the Gini coefficient to measure inequality produce extremely similar results.
Table 4. IGE and interactions with census tract income distribution

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family income, ages 13-17</td>
<td>0.471***</td>
<td>0.361***</td>
<td>0.363***</td>
<td>0.363***</td>
<td>0.450***</td>
<td>0.370***</td>
</tr>
<tr>
<td></td>
<td>(0.0294)</td>
<td>(0.0389)</td>
<td>(0.0387)</td>
<td>(0.0390)</td>
<td>(0.0354)</td>
<td>(0.0404)</td>
</tr>
<tr>
<td>Average income in tract</td>
<td>0.330***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.571</td>
</tr>
<tr>
<td></td>
<td>(0.0672)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.968)</td>
</tr>
<tr>
<td>Income variance in tract</td>
<td>0.0438</td>
<td></td>
<td></td>
<td></td>
<td>1.081</td>
<td>1.296</td>
</tr>
<tr>
<td></td>
<td>(0.0950)</td>
<td></td>
<td></td>
<td>(1.176)</td>
<td>(1.504)</td>
<td></td>
</tr>
<tr>
<td>Family income*tract avg.</td>
<td></td>
<td>0.0326***</td>
<td>0.0235</td>
<td>-0.0244</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00658)</td>
<td>(0.0729)</td>
<td>(0.0953)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family income*tract var.</td>
<td></td>
<td></td>
<td>0.00266</td>
<td>-0.134</td>
<td>-0.128</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.00959)</td>
<td>(0.121)</td>
<td>(0.152)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.293)</td>
<td>(0.389)</td>
<td>(0.388)</td>
<td>(0.391)</td>
<td>(0.356)</td>
<td>(0.405)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,617</td>
<td>1,153</td>
<td>1,153</td>
<td>1,153</td>
<td>1,153</td>
<td>1,153</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.170</td>
<td>0.179</td>
<td>0.179</td>
<td>0.179</td>
<td>0.163</td>
<td>0.180</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes for tables 4–6: All income deflated using CPI-U-RS. Tract measures are normalized to have zero mean. The dependent variable in the linear regression results of Tables 4–6 is an individual’s family income averaged over ages 30–34; individual’s family income in adolescence is averaged over ages 13–17.
Table 4, while revealing some fragility in coefficient estimates across specifications, does allow some conclusions to be discerned. There is evidence that census tract income increases expected offspring income additively (column 2) and via interaction with parental income (column 3). Column 4 fails to identify statistically significant effects when both types of average income effects are included, presumably due to collinearity. In contrast, statistically significant evidence is found that census tract inequality affects offspring income. With respect to our model, we expected the coefficient on the interaction of family income and variance log income to be negative. This is consistent with the negative signs on family income \( \times \) log income in columns 5 and 6. But large standard errors make results of these specifications disappointing in terms of corroboration of our ideas.
### Table 5. IGEs and interaction with state income distribution

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family income, ages 13-17</td>
<td>0.471***</td>
<td>0.434***</td>
<td>0.436***</td>
<td>0.426***</td>
<td>0.449***</td>
<td>0.414***</td>
</tr>
<tr>
<td></td>
<td>(0.0294)</td>
<td>(0.0294)</td>
<td>(0.0294)</td>
<td>(0.0287)</td>
<td>(0.0283)</td>
<td>(0.0284)</td>
</tr>
<tr>
<td>Average income in state</td>
<td>0.788***</td>
<td>6.962***</td>
<td>4.871**</td>
<td>4.871**</td>
<td>4.871**</td>
<td>4.871**</td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td>(2.132)</td>
<td>(2.462)</td>
<td>(2.462)</td>
<td>(2.462)</td>
<td>(2.462)</td>
</tr>
<tr>
<td>Income variance in state</td>
<td>0.644***</td>
<td>-9.647***</td>
<td>-5.772</td>
<td>-5.772</td>
<td>-5.772</td>
<td>-5.772</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
<td>(3.189)</td>
<td>(3.625)</td>
<td>(3.625)</td>
<td>(3.625)</td>
<td>(3.625)</td>
</tr>
<tr>
<td>Family income*state avg.</td>
<td>0.0773***</td>
<td>-0.654***</td>
<td>-0.416*</td>
<td>-0.416*</td>
<td>-0.416*</td>
<td>-0.416*</td>
</tr>
<tr>
<td></td>
<td>(0.0146)</td>
<td>(0.215)</td>
<td>(0.248)</td>
<td>(0.248)</td>
<td>(0.248)</td>
<td>(0.248)</td>
</tr>
<tr>
<td>Family income*state var.</td>
<td>0.0675***</td>
<td>1.002***</td>
<td>0.656*</td>
<td>0.656*</td>
<td>0.656*</td>
<td>0.656*</td>
</tr>
<tr>
<td></td>
<td>(0.0177)</td>
<td>(0.320)</td>
<td>(0.364)</td>
<td>(0.364)</td>
<td>(0.364)</td>
<td>(0.364)</td>
</tr>
<tr>
<td>Constant</td>
<td>5.136***</td>
<td>5.502***</td>
<td>5.483***</td>
<td>5.602***</td>
<td>5.363***</td>
<td>5.717***</td>
</tr>
<tr>
<td></td>
<td>(0.293)</td>
<td>(0.292)</td>
<td>(0.293)</td>
<td>(0.285)</td>
<td>(0.282)</td>
<td>(0.282)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,617</td>
<td>1,611</td>
<td>1,611</td>
<td>1,611</td>
<td>1,611</td>
<td>1,611</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.170</td>
<td>0.184</td>
<td>0.183</td>
<td>0.183</td>
<td>0.178</td>
<td>0.193</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses. All income in logs; state measures normalized to have zero mean.  

*** p<0.01, ** p<0.05, * p<0.1
The state level results in Table 5 provide clearer evidence that average state income helps predict offspring income. Again, the results for the variance of log income and the Gini coefficient are very similar. Columns 2, 4, 6 all contain positive and statistically significant estimates of an additive state mean effect. Interactions of family income with average state income, which appear in specifications for columns 3, 4, and 6, are statistically significant but exhibit fragile signs as the coefficient in 2 is positive while negative for the others. Income variance is positive and significant in 5 while negative and insignificant in specification 6. This fragility can be understood as a derivative of collinearity. Finally, income variance, when interacted with family income, affects the IGE positively. This finding is consistent with the logic of our theoretical ideas,
which suggest that states with higher income variance will exhibit greater segregation at lower levels.
Table 6. IGE’s and census tract and state income distributions

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family income, ages 13-17</td>
<td>0.361*** (0.0391)</td>
<td>0.442*** (0.0355)</td>
<td>0.362*** (0.0384)</td>
<td>0.366*** (0.0407)</td>
</tr>
<tr>
<td>Family income*tract average</td>
<td>0.0942 (0.0824)</td>
<td>0.0282*** (0.00604)</td>
<td>0.0334 (0.104)</td>
<td></td>
</tr>
<tr>
<td>Family income*state average</td>
<td>-0.519* (0.270)</td>
<td>0.0492*** (0.0186)</td>
<td>-0.504 (0.313)</td>
<td></td>
</tr>
<tr>
<td>Average income in tract</td>
<td>-0.633 (0.826)</td>
<td>-0.0627 (1.050)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average income in state</td>
<td>5.329** (2.697)</td>
<td>5.507* (3.130)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family income*tract variance</td>
<td>-0.197 (0.129)</td>
<td>-0.116 (0.158)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Family income*state variance</td>
<td>0.493 (0.315)</td>
<td>0.0768*** (0.0198)</td>
<td>0.0664 (0.377)</td>
<td></td>
</tr>
<tr>
<td>Income variance in tract</td>
<td>1.638 (1.264)</td>
<td>1.073 (1.564)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income variance in state</td>
<td>-4.357 (3.155)</td>
<td>0.143 (3.777)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>6.257*** (0.392)</td>
<td>5.455*** (0.358)</td>
<td>6.238*** (0.385)</td>
<td>6.208*** (0.409)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,153</td>
<td>1,153</td>
<td>1,153</td>
<td>1,153</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.183</td>
<td>0.171</td>
<td>0.190</td>
<td>0.193</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses. All income in logs; measures normalized to have zero mean. 
*** p<0.01, ** p<0.05, * p<0.1
We complete this discussion by considering regressions which allow for both census tract and state effects. These appear in Table 6. Column 1, which considers census tract and state income averages, finds relatively stronger evidence that average census tract income matters as compared to state income. Column 2 focuses on census tract and state variances. No variables are statistically significant in isolation and there is a substantial reduction in goodness of fit relative to the model with average incomes. Column 4 focuses on interactions of means and variances with parental income. Here, average census tract and state income interactions are positive and statistically significant as is state variance interaction.
The insignificance of the interactions of census tract variance and income echoes earlier results. When all variables are combined, average state income survives as being statistically significant.
In summary, with respect to the general ideas of our theoretical framework, we would expect census tract and state means to enhance offspring income as well as interact positively with family income.

We would predict census tract income to reduce the family IGE because of increased local integration and state variance to increase the IGE because of the potential for increased segregation, and census tract variance to reduce it. Thus these reduced form findings are qualitatively consistent with our priors, although the lack of robustness to census tract variance/mobility link is disappointing, at least with reference to our theoretical model.
reduced form Great Gatsby Curves

Our final exercises construct some Gatsby Curves from our statistical models. Figure 18 reports the Great Gatsby Curves that are implied by equation (13). To generate them, we construct counterfactual values of $y_{io}$ given changes in the variance of $y_{io}$ as produced by scaling the historical $y_{io}$ values. For each counterfactual parental income series, we calculate the implied value of $\beta$ if (1) is the linear model used to analyze the parent-offspring income relationship.
Figure 18. Great Gatsby Curve implied by nonparametric specification under scaling of parental income

The graph depicts how the IGE—the marginal effect of parental income on offspring’s income—responds to scaling of parental income. The initial parental income distribution corresponds to the parental income in the PSID sample. We, first, non-parametrically estimated offspring’s income given parental income and saved residuals from the estimation. Then for each scaling of log of parental income - that also scaled variance of parental income (horizontal axis) - offspring income is predicted using the non-parametric estimation and residuals from the first step. Afterwards, predicted offspring income is regressed on scaled parental income; the regression coefficients - the implied IGEs - are plotted. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
As indicated by Figure 18, the nonparametric family income model does not generate a relationship between inequality and mobility. This is not consistent with the Gatsby Curve idea: greater variance in parental income is associated with higher mobility. Some insight into the reasons for this may be seen in Figures 17a-b The nonlinearities in our sample suggest high means and lower local IGE coefficients for families in the tails of the income distribution than in the middle. Hence increased spread of parental incomes pushes more families into the lower IGE regions.
Figure 19. Great Gatsby curve implied by parametric specification including parents’ percentile in nation

This graph depicts how the IGE—the marginal effect of parental income on offspring’s income—responds to scaling of parental income. For each scaling of log parental income (from -50% to +100%), offspring incomes are predicted using the estimated coefficients from Table 1, specification 2. Then predicted offspring income is regressed on scaled parental income; the regression coefficients are plotted. The horizontal axis displays the variance of the scaled log parental incomes. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
Figure 19 reports the implied Gatsby Curve associated with our parametric nonlinear model that is reported in Table 1. The unusual shape reflects the fact that spreading income distribution moves families away from the middle linear IGE model towards the models for the upper and lower tails.

For our purposes, there is one important message from Figures 18 and 19: nonlinearities in family income dynamics do not provide good reasons to think an intemporal Great Gatsby Curve exists for the US.
Our second set of reduced form Gatsby Curves is generated by parametric models we constructed that included census tract and state income distribution characteristics. In each case, we scale the log parental income, census tract, and state incomes proportionately. Figures 20-21 present Gatsby Curves for census tract variables, 22-23 for state level variables, while 24 and 25 combine both census tract and state variables. We consider cases where the results are based on means as well as the ones where results are based both on means and variances.
A consistent picture emerges from these calculations. At the census tract level, a Gatsby curve is implied by our parametric regressions, but the slope is small. For state-level variables, a large negative slope occurs. Hence the state level interactions produce the opposite phenomena from the Gatsby Curve property per se. When census tract and state variables are combined, a gently sloped positive relationship between income inequality and mobility reemerges.
Figure 20. Great Gatsby curve implied by parametric specification including tract average, under scaling of parental income

This graph depicts how the IGE—the marginal effect of parental income on offspring's income—responds to scaling of parental income. For each scaling of log parental income (from -50% to +100%), which is also applied to tract averages, offspring incomes are predicted using the estimated coefficients from Table 4a, specification 4. Then predicted offspring income is regressed on scaled parental income; the regression coefficients are plotted. The horizontal axis displays the variance of the scaled log parental incomes. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
Figure 21. Great Gatsby curve implied by parametric specification including tract average and variance, under scaling of parental income

This graph depicts how the IGE—the marginal effect of parental income on offspring’s income—responds to scaling of parental income. For each scaling of log parental income (from -50% to +100%), which is also applied to tract averages and variances, offspring incomes are predicted using the estimated coefficients from Table 4, specification 6. Then predicted offspring income is regressed on scaled parental income; the regression coefficients are plotted. The horizontal axis displays the variance of the scaled log parental incomes. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
This graph depicts how the IGE—the marginal effect of parental income on offspring’s income—responds to scaling of parental income. For each scaling of log parental income (from -50% to +100%), which is also applied to state averages, offspring incomes are predicted using the estimated coefficients from Table 5, specification 4. Then predicted offspring income is regressed on scaled parental income; the regression coefficients are plotted. The horizontal axis displays the variance of the scaled log parental incomes. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
Figure 23. Great Gatsby curve implied by parametric specification including state average and variance, under scaling of parental income

This graph depicts how the IGE—the marginal effect of parental income on offspring's income—responds to scaling of parental income. For each scaling of log parental income (from -50% to +100%), which is also applied to state averages and variances, offspring incomes are predicted using the estimated coefficients from Table 5, specification 6. Then predicted offspring income is regressed on scaled parental income; the regression coefficients are plotted. The horizontal axis displays the variance of the scaled log parental incomes. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
Figure 24. Great Gatsby curve implied by parametric specification including tract and state average, under scaling of parental income

The graph depicts how the IGE—the marginal effect of parental income on offspring's income—responds to scaling of parental income. This figure assumes that offspring income depends linearly on parental income, average tract and state income, and the interaction of parental income with these variables. For each scaling of log parental income (from -50% to +100%), which is also applied to tract and state averages, offspring incomes are predicted using the estimated coefficients from Table 6, specification 1. Then predicted offspring income is regressed on scaled parental income; the regression coefficients are plotted. The horizontal axis displays the variance of the scaled log parental incomes. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
The graph depicts how the IGE—the marginal effect of parental income on offspring's income—responds to scaling of parental income. This figure assumes that offspring income depends linearly on parental income, average and variance of tract and state income, and the interaction of parental income with these variables. For each scaling of log parental income (from -50% to +100%), which is also applied to tract and state averages and variances, offspring incomes are predicted using the estimated coefficients from Table 6, specification 4. Then predicted offspring income is regressed on scaled parental income; the regression coefficients are plotted. The horizontal axis displays the variance of the scaled log parental incomes. All income measures are deflated using CPI-U-RS and expressed in logs. Offspring income is an individual's family income averaged over ages 30–34. Parental income is individual's family income in adolescence (averaged over ages 13–17).
We conclude from these exercises that there is some weak evidence of the Gatsby Curve like phenomena from the parametric IGE regressions with neighborhood effects. Perhaps unsurprisingly, a necessary condition for stronger evidence is a greater attention to the mechanisms underlying the social interactions/Gatsby relationship. And as argued in Section 4, there is evidence to think the mechanisms that underlie our theoretical model matter in ways that create Gatsby-like outcomes. We thus move from these reduced for exercises to see whether a calibrated structural model can provide additional insights.
A Calibrated Model

In this section, we integrate the theoretical ideas of Sections 2 and 3 with the various facts highlighted in Sections 4 and 5 via a model calibration exercise. The model is a version of Kotera and Seshadri (2017) extended to incorporate heterogeneity at the school district level.
Households live for four periods, one as an offspring and three as an adult. The first period is 18 years and the next three periods are 6 years each.

We keep track of each offspring from birth until the age of 36. Each household \( i \) in a school district \( j \) maximizes utility given by

\[
  u(c^i_{j1}) + \theta V(a^i_j, h^i_{j2}, g^i_j)
\]

where \( u(c^i_{j1}) \) is the utility from consumption \( c^i_j \), \( V(a^i_j, h^i_{j2}, g^i_j) \) is the lifetime utility of the offspring at the beginning of the second period, and \( \theta \) is a measure of parental altruism. \( g^i_j \) is a transfer from a parent to his offspring who can use these resources in the second period.
A central feature of the model is the human capital production function – an offspring’s human capital depends on his own ability, public and private inputs, parent’s human capital and the average human capital in the neighborhood.

Thus, the offspring’s human capital varies at the school district level.
Specifically, for household $i$’s offspring in school district $j$, the stock of offspring human capital at the beginning of the second period, $h_{j2}^i$, is given by

$$h_{j2}^i = a_j^i \left( x_{j1}^i + \bar{x}_j \right)^{\alpha_1} \left( h_{j0}^i \right)^{\alpha_2} \left( h_j^i \right)^{\alpha_3}$$

(15)

where $a_j^i$ is the learning ability, $\bar{x}_j$ represents public inputs, $h_{j0}^i$ is parent’s human capital, and $h_j$ is the average parental human capital in a school district, i.e. $h_j = \frac{1}{n} \sum_i h_{j0}^i$. We assume that $\alpha_1 < 1$, $\alpha_2 < 1$ and $\alpha_3 < 1$. Additionally, $\bar{x}_j$ is collected using local tax rates on income, so $\bar{x}_j = \frac{1}{n} \sum_j y_j^i$. We take these rates as given.
An offspring becomes independent at the beginning of the second period.

He makes decisions on human capital accumulation and consumption in the second, third, and fourth periods ($\{c^i_{j2}, c^i_{j3}\}$) to maximize his utility

$$V(a^i_j, h^i_{j2}, g^i_j) = \max_{\{c^i_{j2}, c^i_{j3}, c^i_{j4}, n^i_{j2}, n^i_{j3}, x^i_{j1}\}} u(c^i_{j2}) + \beta u(c^i_{j3}) + \beta^2 u(c^i_{j4})$$

(17)
subject to the budget constraint

\[
c_{j2}^i + \frac{c_{j3}^i}{1+r} + \frac{c_{j4}^i}{(1+r)^2} = wh_{j2}^i (1 - n_{j2}^i) + \frac{wh_{j3}^i (1 - n_{j3}^i)}{1+r} + \frac{wh_{j4}^i}{(1+r)^2} + g_j^i
\]

(18)

and human capital production functions

\[
h_{j3}^i = a_j^i \left( n_{j2}^i h_{j2}^i \right)^{\eta_i} + h_{j2}^i
\]

\[
h_{j4}^i = a_j^i \left( n_{j3}^i h_{j3}^i \right)^{\eta_i} + h_{j3}^i
\]

\[n_{j2}^i\] and \[n_{j3}^i\] are the time spent on human capital accumulation in the second and third periods.
The solution to the model in the last three periods is straightforward. In particular, individuals invest to maximize lifetime income and then allocate consumption across the two periods to maximize discounted utility.
Next, the maximization problem in the first period can be written as

$$\max_{c_{j1}, x_{j1}, g_j} u(c_{j1}^i) + \theta V(a_i^j, h_{j2}^i, g_j^i)$$  \hspace{1cm} (20)$$

subject to (15), the budget constraint

$$c_{j1}' + x_{j1}' + g_j^i = (1-\tau)y_j'$$  \hspace{1cm} (21)$$

and a non-negativity condition $g_j^i \geq 0$.

The first-order conditions for $x_{j1}'$ and $g_j^i$ are given by
\[ \partial V_{h^{j2}} \left( a^{i}, h^{i}_{j2}, g^{i}_{j} \right) a^{i}_{j} \alpha_{1} \left( x^{i}_{j1} + \bar{x}_{j} \right)^{\alpha_{1}-1} \left( h^{i}_{j} \right)^{\alpha_{2}} \left( h_{j} \right)^{\alpha_{3}} = u \left( c^{i}_{ji} \right) \] (22)

and

\[ \partial V_{g^{j}_{i}} \left( a^{i}_{j}, h^{i}_{j2}, g^{i}_{j} \right) \leq u \left( c^{i}_{ji} \right) \] (23)

where \( V_{h^{j2}} \left( a^{i}_{j}, h^{i}_{j2}, g^{i}_{j} \right) \) and \( V_{g^{j}_{i}} \left( a^{i}_{j}, h^{i}_{j2}, g^{i}_{j} \right) \) are the derivatives of \( V \left( a^{i}_{j}, h^{i}_{j2}, g^{i}_{j} \right) \) with respect to \( h^{i}_{j2} \) and \( g^{i}_{j} \), respectively, and \( u' \left( c_{j1}^{i} \right) \) denotes the derivative of \( u \left( c_{j1}^{i} \right) \) with respect to \( c_{j1}^{i} \).
The first condition implies that private investment would equate the marginal benefits for offspring in the last two periods with the marginal costs incurred by parents in the first period.

The second condition holds with equality if $g_j^i > 0$. In this case, the value of a dollar to the parent is the same regardless of whether it's consumed or left to the offspring. Otherwise, if the value of a dollar to the parent is larger when it's consumed even if $g_j^i = 0$, the inequality in the third condition would be strict.
Calibration

fixed parameters

We assume a standard CRRA utility function over consumption, \( u(c) = \frac{c^{1-\alpha}}{1-\alpha} \), so that

\[
V(a^i_{j1}, h^i_{j2}, g^i_j) = \frac{(c^i_{j2})^{1-\alpha}}{1-\alpha} + \beta \frac{(c^i_{j3})^{1-\alpha}}{1-\alpha} + \beta^2 \frac{(c^i_{j4})^{1-\alpha}}{1-\alpha}
\]

We set \( \alpha = 2 \), \( \beta = 0.96 \), and \( r = (1.04)^6 - 1 \), where 6 is the number of years in each of the last three periods of our model.
Table 7: Fixed parameters in the calibration exercise

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRRA coefficient</td>
<td>$\alpha$</td>
<td>2.0</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.96^6</td>
</tr>
<tr>
<td>Return to schooling</td>
<td>$\varphi$</td>
<td>0.1</td>
</tr>
<tr>
<td>Average wage rate in the U.S</td>
<td>$w$</td>
<td>0.1707</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>$r$</td>
<td>$(1+0.04)^6-1$</td>
</tr>
</tbody>
</table>
Estimation Strategy

We estimate the parameters using the Method of Simulated Moments. Let $\Theta_s$ be the set of parameters to be estimated. Using data moments $M_s$, we obtain the estimated

$$\hat{\Theta}_s = \arg\min_{\Theta_s} [M_s(\Theta_s) - M_s]' W_s [M_s(\Theta_s) - M_s]'$$

where $M_s(\Theta_s)$ is the simulated model moments and $W_s$ is a weighting matrix. In practice, we use the variance-covariance matrix of $M_s$ as the weighting matrix $W_s$. 
Table 8: Data moments used in the calibration exercise

<table>
<thead>
<tr>
<th>Moments</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average offspring income between 24 and 28</td>
<td>$18,313</td>
</tr>
<tr>
<td>Average offspring income between 30 and 34</td>
<td>$23,737</td>
</tr>
<tr>
<td>Average offspring income between 24 and 28 in Group 1</td>
<td>$16,418</td>
</tr>
<tr>
<td>Average offspring income between 24 and 28 in Group 2</td>
<td>$18,409</td>
</tr>
<tr>
<td>Average offspring income between 30 and 34 in Group 1</td>
<td>$21,059</td>
</tr>
<tr>
<td>Average offspring income between 30 and 34 in Group 2</td>
<td>$24,248</td>
</tr>
<tr>
<td>Average offspring income between 24 and 28 in Group 11</td>
<td>$17,583</td>
</tr>
<tr>
<td>Average offspring income between 24 and 28 in Group 12</td>
<td>$18,470</td>
</tr>
<tr>
<td>Average school years in college</td>
<td>1.6016</td>
</tr>
</tbody>
</table>
Baseline results

targeted moments

Tables 9 and 10 provide values of the estimated parameters and the targeted moments, respectively. It is worth noting that, with regards to the targeted moments, we do a fine job matching moments for all variables except for average school years in college. One possible reason is that since the level of offspring’s human capital in the second period is small, children need to spend time on accumulating human capital to match moments for their income in the following periods.
Table 9: Estimated parameters for the calibration exercise

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.5657 (0.0172)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.1283 (0.001)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.2642 (0.1202)</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.5314 (0.1302)</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.5263 (0.003)</td>
</tr>
<tr>
<td>$\mu_{a_j}$</td>
<td>0.3634 (0.0048)</td>
</tr>
<tr>
<td>$\sigma_{a_j}$</td>
<td>0.1691 (0.0053)</td>
</tr>
<tr>
<td>$\rho_{h_{j_0}a_j}$</td>
<td>0.2531 (0.1112)</td>
</tr>
<tr>
<td>Moments</td>
<td>Data</td>
</tr>
<tr>
<td>-------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Average child income between 24 and 28</td>
<td>$18,313</td>
</tr>
<tr>
<td>Average child income between 30 and 34</td>
<td>$23,737</td>
</tr>
<tr>
<td>Average child income between 24 and 28 in Group 1</td>
<td>$16,418</td>
</tr>
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<td>$18,470</td>
</tr>
<tr>
<td>Average school years in college</td>
<td>1.6016</td>
</tr>
</tbody>
</table>
First, we turn to the relationship between parental income and offspring income (in logs). For offspring income, we use offspring income in the fourth period, $wh^i_{j4}$. Figure 26 contains our findings.
Figure 26. Relationship between parental income and offspring income in the model
There is a positive correlation between parental income and offspring income. However, the coefficient is smaller than the one implied by the data. In the previous section, the range of the correlation is between 0.36 and 0.44. By contrast, the correlation in the model is 0.23. The reason for this difference might be because the sample size used in the calibration is smaller. In this exercise, we use only 193 individual data due to data limitations. This underestimates the magnitude of the correlation coefficient.
We then turn our attention to the local IGE estimates for income displayed in Figure 27.
Figure 27. Relationship between ratio of offspring income to parental income and offspring income
As in the previous section, the local IGE estimates are defined as the ratio of offspring income to parental income level. Figure 27 points out that the local IGE estimates fall as parental income rises.

Both qualitatively and quantitatively, this exhibits the same tendency as the one observed in the data. Unlike the data, however, the local IGE estimates fall to 0 in the calibration. Again, this gap is, in part, due to our smaller sample size.
Counterfactuals

To improve our understanding of the forces at work in our model that help explain the positive correlation between parental income and offspring income, we use the estimated model to conduct two counterfactual simulations.
The first counterfactual simulation examines what would happen if there was no return to some factors that are important in forming offspring’s human capital in the second period.

In our model, offspring’s human capital contains three elements: inputs including both public and private ones, parent’s human capital, and average human capital in a school district. In this simulation, we study how important each element is in forming offspring’s human capital. The second counterfactual simulation examines the importance of parental income distribution.

This exercise allows us to quantify the contribution of parental income distribution to intergenerational mobility.
Return to elements for offspring’s human capital in the second period

Figure 28 summarizes intergenerational mobility in the following five cases: i) baseline, ii) no return to all elements: \( \alpha_1 = \alpha_2 = \alpha_3 = 0 \), iii) no return to inputs: \( \alpha_1 = 0 \), iv) no return to parent’s human capital: \( \alpha_2 = 0 \), and v) no return to average human capital in a school district: \( \alpha_3 = 0 \). If all three elements were eliminated, the correlation coefficient would fall to 0.16. Thus, these forces play a key role in driving intergenerational mobility. Additionally, when we decompose this effect, we find that the contribution of inputs is the largest. On the other hand, parent’s human capital and average human capital have a smaller effect on intergenerational mobility.
Figure 28. Counterfactual simulation: contribution of various elements to intergenerational mobility
A Structural Gatsby Curve

How much the distribution of parental income affects intergenerational mobility is also an interesting question. Since the distribution of parental income is given in our model, we conduct a counterfactual simulation in which we make this distribution more disperse.

Specifically, we raise its standard deviation by 10% and 20% holding its mean fixed. School district variables are fixed. Figure 29 presents the results,
Figure 29. Counterfactual simulation: effect of changing parental inequality on intergenerational mobility
The counterfactuals illustrate Gatsby like behavior.
But the effect is rather modest — the 20% increase in dispersion of parental income increases the IGE by 2%. Attanasio, Hurst, and Pistaferri (2015) argue that the standard deviation of income increased by about 20% between 1980 and 2008. Consider the difference between an IGE of .3 and .4. If our theory explained 2% of a 33% increase, that would represent over 5% of the overall change. Not a large fraction, but still a meaningful piece of the overall story.
Conclusions and Conjectures

Plausible theoretical conditions and moderate evidence for the proposition that segregation-based phenomena induce Gatsby-like behavior.

Assocational redistribution policies may be appropriate.