1. Introduction

In an earlier paper, Becker and Lewis (1973) explained why the quantity and quality of children (and, by extension, of many other commodities) are more closely related than are any two commodities chosen at random, without assuming that substitution in consumption between quantity and quality is greater than average. It is sufficient to recognize than an increase in the quantity of children raises the cost or shadow price of the quality of children, and vice versa. This was used to explain, among other things, why the observed income elasticity of demand for quality of children is high at the same time that the observed quantity elasticity is low and often even negative.

As part of a more recent paper on social interactions, Becker (1974b) discussed some other determinants of the demand for quality of children. These include the preferences of parents with regard to their own consumption relative to that of their children, public expenditures on schooling, and genetic inheritance. Becker shows that "social interactions" can also explain the high observed income elasticity of demand for quality of children.

This paper brings together and integrates social interactions and the special relation between quantity and quality. We are able to show that the observed quality income elasticity would be relatively high and the quantity elasticity relatively low and sometimes negative, even if the true underlying income elasticities for quantity and quality were equal and of average value. Moreover, the observed quality elasticity would fall and the observed quantity elasticity would rise as parental income rose. These

We are indebted for helpful comments to Dennis De Tray, Zvi Griliches, Reuben Gronau, Lawrence W. Kenny, Jacob Mincer, and George J. Stigler.

[Journal of Political Economy, 1976, vol. 84, no. 4, pt. 2]
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and related results on the relation of observed quantity and quality income elasticities to social mobility and economic growth are discussed in Section 2.

Section 3 drops the assumption made in Section 2 that all children are of equal quality and considers differences in endowment and quality. It analyzes how parental expenditures are related to their children's endowment; in particular, whether better endowments are reinforced or poorer endowments are compensated. It explores the resulting biases in estimates of rates of return on investments in human capital and in estimates of the direct effect of family background on earnings. It also shows why compensatory education programs may appear to "fail" even when the children being "compensated" are as able and well motivated as other children.

2. Interaction between the Quantity and Quality of Children

We assume in this section that each household has a utility function of the following form:

\[ U = U(n, w, y), \]  

(2.1)

where \( n \) is the number of children, \( w \) the quality of each child, and \( y \) the aggregate amount of all other commodities. By saying "the" quality, we have introduced the assumption that the quality of each child is the same. This quality is partly controlled by the household through its expenditures on children and is partly outside its control because inherited ability, \(^2\) public investments in children, "luck," and other variables also affect quality. The aggregate "endowment" of each child is assumed to be the same, \(^3\) so that parental contributions must be the same if total quality is to be the same.

With some assumptions, the effect of household and endowed inputs on child quality can be expressed simply as the additive function

\[ w = e + q, \]  

(2.2)

^1 Our indebtedness to Lewis in this section should be obvious from its reliance on the Becker and Lewis paper (1973). We also discussed with him the developments in this section and had access to some notes that he prepared. He would be a joint author were that seemingly in a festschrift in his honor!

^2 The introduction of endowments into the analysis of the interaction between the quantity and quality of children was first done in the content of genetic inheritance (see Tomes 1974).

^3 This assumption and the assumption that the quality of each child is the same are made only for convenience of exposition. It would be sufficient to assume that the expected quality and endowment of a child is uncorrelated with the birth order of the child, and even that is much too strong. Therefore, the discussion in Sec. 3 of differences in the quality and endowment of each child does not greatly alter the conclusions of this section.

^4 A general production function for child quality can be written as \( w = f(x, t; z) \), where \( x \) and \( t \) are the household's inputs of goods and time and \( z \) is the endowed inputs. If the time input is ignored and if goods had a constant marginal product, this function...
where \( e \) is the endowed contribution and \( q \) is the household contribution to the total quality \( w \). The household’s budget constraint is

\[
p_y y + p_q q = I, \tag{2.3}
\]

where \( I \) is its own income, \( p_y \) is the price of \( y \), \( p_q \) is the average cost of increasing \( q \) by one unit, and \( p_q q \) is its total expenditure on children.\(^5\)

If \( e \) were exogenous and independent of the level of \( q \), a household could take \( e \) as given in determining its optimal \( q \). Then maximizing the utility function (2.1) subject to the budget constraint (2.3) and the quality function (2.2) yields, if \( p_q \) were fixed,

\[
M_y u_y = \lambda p_y = \lambda \pi_y, \quad M_w u_w = \lambda p_q q = \lambda \pi_w, \quad M_n u_n = \lambda p_q q = \lambda \pi_n, \tag{2.4}
\]

where \( \lambda \) is the marginal utility of income, \( \pi_w = np_q \) is the shadow price or cost of increasing quality, and \( \pi_n = q p_q \) is the shadow price of increasing quantity. By substituting these prices and the function (2.2) into the own-income equation (2.3), the equation for commodity consumption is obtained,

\[
\pi_y y + \pi_n n + \pi_w w = I + \pi_w w = S, \tag{2.5}
\]

where \( S \) is the household’s social income.

The important point is that the shadow price of the quality of children is proportional to the number of children and the shadow price of quantity is proportional to the household’s contribution to quality.\(^6\) Quantity and quality interact in this way because an increase in the number of children increases the cost of raising the quality of children, since the

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\(^5\) Becker and Lewis (1973) develop a slightly more general budget constraint by introducing “fixed” as well as “variable” costs of the number and quality of children. This generalization is not pursued here.

\(^6\) The equilibrium conditions in (2.4) are similar to what they would be if there were mutual harmful joint production in the household production functions for quantity and quality, with an increase in quantity lowering the output of quality and an increase in quality lowering quantity (for a general discussion of joint production, see Grossman [1971]). Recently the household production function approach was criticized, partly because commodity shadow prices are dependent on commodity outputs if there is joint production (Pollak and Wachter 1975). This property would be a virtue rather than a vice, however, if it were helpful in understanding behavior (strangely, Pollak and Wachter never discuss the value of the household production function approach in understanding behavior). Indeed, this paper, as well as the one by Becker and Lewis and the even earlier one by Grossman, indicates that the effect of commodity outputs on commodity prices can be used to understand otherwise puzzling empirical findings.
higher quality applies to more children; similarly, an increase in parental contributions to quality raises the cost of an additional child, since higher-quality children would then be more expensive. Therefore, an "exogenous" change in the quantity or quality of children would induce further changes through this interaction. For example, an increase in quality would raise the shadow price of quantity, which would reduce the demand for quantity, but this in turn would lower the shadow price of quality, which would induce an additional increase in quality, and so on. 7

To see the consequences of this interaction for behavior, define the "true" commodity income elasticities \( n_i, i = y, w, \) and \( n \), as the percentage change in commodity consumption per 1 percent change in social income \( S \), with commodity shadow prices \( \pi_i \) held constant. Also define the "observed" income elasticities \( \bar{n}_i \) as the percentage change in consumption per 1 percent change in own income \( I \), with market prices \( p_i \) held constant. Becker and Lewis (1973) ignore social interactions by assuming that the endowment is zero, so that by equation (2.2) child quality would simply be identical with parental contributions. They show that the observed income elasticities of quantity and quality would be equal if their true elasticities were equal and if other commodities were equally substitutable with quantity and quality.

They state, however, that "it is plausible to assume that the true income elasticity with respect to quality \( (n_q) \) is substantially larger than that with respect to quantity \( (n_n) \)." It then follows from the increase in the relative shadow price of quantity induced by the relatively large increase in quality that the observed quantity income elasticity would be lower than its true elasticity; indeed, the observed elasticity could be low and even negative at the same time that the true elasticity was significantly positive. On the other hand, the observed quality elasticity would be even larger than its true elasticity if the induced substitution of quality for quantity dominated other effects. If we drop the Becker and Lewis assumption of no endowment, all of their results can be derived without assuming that the true quality elasticity \( n_w \) exceeds the true quantity elasticity \( n_n \). For if \( e > 0 \) were fixed, \( dw = dq \) and

\[
\frac{dq}{q} = \frac{w \ dw}{w} = R \frac{dw}{w} > \frac{dw}{w},
\]

(2.6)

where \( R = w/q = 1 + e/q; \) then clearly

\[
q_n = Rn_w > n_w = n_n.
\]

(2.7)

7 The elasticity of substitution between quality and quantity has to be less than unity (quality and quantity cannot be close substitutes) if both are to be positive and finite. This effectively rules out (commonly used) utility functions that depend on total child services—the product of quality and quantity—because the elasticity of substitution of quantity with total quality would equal unity and, with parental contributions to quality, would approach unity as these contributions increased relative to the endowment.
If the endowment’s contribution were sizable, \( R \) would be much above unity, and the true own-contribution quality elasticity \( n_q \) would be substantially above the true total quality elasticity \( n_w \). Since the latter is assumed to equal the true quantity elasticity \( n_q \), it has been shown why “the true [own contribution] income elasticity . . . \( n_q \) is substantially larger than . . . \( n_w \)” even when \( n_w = n_q \).

All the results of Becker and Lewis on the relation between observed and true income elasticities would then follow even though the true quantity and total quality elasticities were equal. In particular, the observed would be less than the true quantity elasticity: the former could be small and even negative at the same time that the latter was sizable. The observed elasticity for quantity \( (\hat{n}_q) \) would be smaller and for quality \( (\hat{n}_q) \) larger the more important the endowment’s contribution to total quality.

As own income continued to rise with the endowment fixed, the ratio \( R = w/q \) would fall as long as child quality had a positive observed income elasticity, because the increase in \( w \) would result entirely from an increase in \( q \). By equation (2.7), the fall in \( R \) would reduce the true own-contribution quality elasticity \( n_q \). A reduction in this elasticity would, via the induced interaction between quantity and quality, then raise the observed quantity elasticity \( (\hat{n}_q) \) and lower even further the observed own-contribution quality elasticity \( (\hat{n}_q) \). That is, the observed quantity elasticity, small and perhaps negative at lower income levels, would rise as income rose, while the observed quality elasticity, large at lower income levels, would fall as income rose. Therefore, observed quality and quantity would tend to be more negatively related at lower than at higher income levels (see figs. 1 and 2).

Figure 1 plots the relation between number of children and income that is typically observed. The curves generally decline at the lowest income levels and often turn up at the highest levels. By using the distinction between a household’s and the endowment’s contribution to quality, and the interaction between quantity and quality, we have been able to explain all the important features of this observed relation, including its nonmonotonicity, while assuming that the true quantity elasticity was constant and of average value.

The shape of the typical relation between household expenditures on quality of children and income is not as well known. The same analysis that explains the nonmonotonic relation in figure 1 implies the concave relations in figure 2. Household expenditures on quality would grow rapidly at lower incomes and then at a decreasing rate, even though the true total quality elasticity was constant and of average value. Con-
sequently, the very different patterns for observed quantity and quality graphed in figures 1 and 2 are consistent with true quantity and quality income elasticities that are constant, equal to each other, and equal to the average income elasticity.

The assumption that the endowment is the same at all levels of parent's own income is not realistic. For example, if own income were higher because of greater parental ability, some of that greater ability would be genetically transmitted to children, and the endowment's contribution would thereby increase. Or, since higher-income persons live in wealthier communities, public contributions to their children's schooling would be greater.9

Therefore, instead of assuming that the endowment's contribution to quality is constant, we assume that its size increases by $\delta$ percent for each 1 percent increase in the parent's own income. The coefficient $1 - \delta$ is a

9 We assume that the Tiebout equilibrium among different communities is not perfect; hence, public expenditures would not be fully converted into effectively private ones.
measure of the degree of intrinsic social mobility,\textsuperscript{10} or intrinsic regression to the mean across generations. Typically, $1 - \delta$ is less than 1 and significantly greater than 0, although values out of this range are possible.

An increase only in the endowment would increase social income and, therefore, would increase the demand for children, child quality, and all other superior commodities. The own contribution to child quality must fall, however, because the increased expenditures on children and other commodities would be "financed" by a reduction in own expenditures on child quality. The decline in own contributions must be less than the rise in the endowment if total child quality increases;\textsuperscript{11} the difference is determined by the income and price elasticities of demand for quality.

Therefore, an increase in own income would have a more positive effect

\textsuperscript{10} For the distinction between intrinsic and actual mobility, see Becker (1974b).

\textsuperscript{11} Total child quality could decrease, however, if the rise in the demand for children induced by the decline in own contributions were sufficiently large. The resulting rise in the price of quality could reduce the demand for quality by more than enough to offset the effect of the rise in social income.
on the quantity of children the larger $\delta$ is or the more the endowment increased as own income increased; similarly, the effect on own contributions to quality would be smaller the larger $\delta$ is. Put differently, the observed quantity own-income elasticity would be smaller and the observed quality own-income elasticity would be bigger the greater the degree of intrinsic intergenerational mobility.

Two curves are shown in figures 1 and 2: one when $\delta = 0$, or when the intrinsic mobility is complete, and the other when $\delta = 0.5$, a 50 percent intrinsic regression toward the mean. Since the quantity elasticity is positively related to $\delta$, the curve representing $\delta = 0.5$ in figure 1 falls less rapidly at lower income levels, hits a trough earlier, and rises more substantially than does the curve for $\delta = 0$. Similarly, since the quality elasticity is inversely related to $\delta$, the curve representing $\delta = 0.5$ in figure 2 is below the one for $\delta = 0$.

The Indianapolis survey in 1941 was probably the first major survey of fertility behavior in the United States. One hypothesis investigated was that "the families of socially mobile couples are smaller than those of socially nonmobile couples of comparable status" (Whelpton and Kiser 1951, p. 1355). The evidence tended to support this hypothesis. Our analysis does imply that the number of children and the degree of social mobility are negatively related; it also implies, however, that (own contributions to) the quality of children and mobility are positively related. The evidence from the Indianapolis survey supports the implication about quantity, although our analysis directly links quantity to the anticipated intrinsic mobility of the children, not to the observed mobility of their parents.\(^{12}\)

Our analysis of the effects of social mobility also implies that a general increase in income due, say, to economic growth, has quite different effects on the quantity and quality of children than does an increase in one household's income relative to that of other households. Presumably, persons experiencing an increase in relative income expect greater regression to the mean in their children's endowment than do persons experiencing an increase due to economic growth. Therefore, the quantity income elasticity estimated from differences in relative income—that is, from "cross-sectional" differences in income—would be smaller than the elasticity due to economic growth—that is, from "time-series" differences; similarly, the quality elasticity estimated from differences in relative income would be larger.

An increase in the rate of growth of income over time has additional implications because it increases the endowments of children relative to

\(^{12}\) Indirectly, however, there could be a close link if more mobile parents expect to have more mobile children, because parents' mobility is partly due to luck, ability, and other factors that are very imperfectly transmitted to their children.
the incomes of their parents. Since an increase in child endowments would reduce the investment by parents in children, which in turn would reduce the (shadow) cost of children, the (relative) redistribution of social income to children produced by increased growth would increase the number of children. Therefore, the number of children would be positively related and parental investment per child negatively related to the (autonomous) rate of growth in income.

Our conclusions about the effects of economic growth on the number of children are similar to those reached by Easterlin in his important work on fertility. Although both Easterlin’s analysis and our own analysis are based on changes in the economic position of children relative to that of their parents, we do not make any special assumptions about preferences, while Easterlin appears to rely heavily on such assumptions. Since they are not necessary to reach his conclusions, his emphasis on the way preferences are formed is superfluous; moreover, we believe that it has diverted attention from the important part of his analysis.

The distinction between “true” and “observed” income elasticities in the Becker and Lewis paper has been criticized by persons who argue that only the observed elasticities are needed to analyze behavior. We hope that this serious misapprehension is now put to rest. Observed elasticities would tend to be quite unstable, even when the true ones are constant, because they depend on the level of income, the degree of social mobility, the rate of growth of income over time, and other variables. Presumably this is why there is such a bewildering array of empirical estimates of the relation between income and fertility (see Simon 1974). Moreover, the true elasticities could also be estimated if measures of total quality were developed (see the measures of Kenny [1976]) and if shadow prices were

13 We assume that the increased rate of growth is due to autonomous technological progress or other forces unrelated to parental investment in their children and that the relative market price of children is unaffected. If the growth were induced by parental investment or if the market price changed, the analysis would be different. We are certainly not pretending to give a full analysis of the effects of economic growth on the quantity and quality of children.

14 For a recent statement of his approach with some supporting evidence, see Easterlin (1975b). We are indebted to Gilbert Ghez for suggesting that our approach is related to Easterlin’s.

15 Any redistribution of income and endowments between parents and children, no matter what the source, would affect the quantity and quality of children. For example, an increase in the public debt, with the proceeds used by the current generation, and with head taxes levied on their children (the next generation), would reduce the number of children and increase the total income per child; that is, the debt would have a negative burden on the next generation. The results would be more ambiguous if the debt were financed by income taxes on the children, because the cost to parents of increasing the quantity or quality of their children would be increased by an income tax. We owe this last point to Robert Barro; he has discussed the effects of an increase in the public debt with a model that has overlapping generations and interdependent parental preferences but that excludes any effects on the number of children (see Barro 1974).
held constant, but would also be biased if market prices alone were held constant. The distinction between observed and true elasticities helps create order out of the seemingly random variation in the observed relation between income and fertility.

The relation between true and observed quantity and quality price elasticities is also influenced by the size of the endowment. For example, an increase in $p_q$, the market price of quantity and quality, would reduce quality by more than quantity because a fixed endowment implies that the entire reduction in quality is achieved by a reduction in the household’s contribution, which reduces the shadow price of quantity relative to quality.\(^\text{16}\)

### 3. Compensation and Reinforcement of Differences among Children\(^\text{17}\)

We have been assuming that the total quality and endowment of each child are the same, so that parental contributions to each must also be the same. This section explores some consequences of dropping the assumption of equal endowments and of introducing differences in ability, public support, “luck,” and other factors. We isolate the effect of differences in endowments from differences in preferences toward children by assuming “child-neutral preferences”; that is, the marginal utility to parents from changes in child quality is the same for all children when their qualities are equal. Formally, the utility function

\[
U = U(y, w_1, \ldots, w_n)
\]  

(3.1)

has the following separability property:

\[
\frac{\partial U}{\partial w_i} \left/ \frac{\partial U}{\partial w_j} \right. > 1 \quad \text{if } w_i < w_j
\]

\[
= 1 \quad \text{if } w_i = w_j
\]

\[
< 1 \quad \text{if } w_i > w_j,
\]

(3.2)

where $w_i$ is the quality of the $i$th child.

Let the endowments of two children differ because one inherited greater ability (or for any other reason). If the cost or price of adding to their

\(^{16}\) Becker and Lewis conclude differently: an increase in market price reduces quantity by more than quality because they ignore the endowment and assume that the “fixed” cost of quantity exceeds the “fixed” cost of quality. If we incorporated this assumption about fixed costs into our analysis, an increase in market price no longer necessarily reduces quality by more than quantity, because the endowment and fixed costs have opposite effects.

\(^{17}\) Our discussion in this section benefited from the analysis in Adams (1976).
quality were $p_{q_1}$ and $p_{q_2}$, respectively, parental contributions to the quality of each would be determined from the equilibrium condition

$$\frac{\partial U}{\partial w_1} \mid \frac{\partial U}{\partial w_2} = \frac{p_{q_1}}{p_{q_2}}.$$ \(3.3\)

If the cost of adding to quality were the same, even when children differed in ability or other aspects of their endowment, then equations (3.2) and (3.3) immediately imply that total qualities would also be the same: differences in parental contributions would fully compensate for differences in endowments. In other words, within a family, the amount invested by parents in a child would be perfectly negatively correlated with the endowment of the child.\(^1\)

The family would contribute to equality by redistributing to less endowed children and to parents some of the increased family wealth resulting from better endowments. This conclusion is essentially a special case of a general theorem in social interactions (Becker 1974b), namely, if a family “head” is voluntarily transferring some of his own resources to different members, a redistribution of endowed resources among members would induce the “head” to “tax” the entire gain of those gaining and compensate fully those losing.

The conclusions would be different if the cost of adding to quality were related to endowments. For example, it is often lower for abler children; on the other hand, public programs that compensate for inferior backgrounds or abilities raise the endowments of children with relatively high costs.\(^2\)

If, on balance, the cost of adding to quality were negatively related to the endowment, equations (3.2) and (3.3) would imply that the desired quality of children would be positively related to their endowment. Clearly, less well endowed children no longer are fully compensated by

\(^{18}\) Inequalities replace equalities if some parental contributions are zero.

\(^{19}\) This conclusion would be modified if parents do not want to spend enough on their children to equalize the marginal utilities in eq. (3.3). They might either spend nothing on both children or spend nothing on the better endowed and an amount on the other that raises his total quality to a level below the endowed quality of the abler. They still compensate the less able child, but not fully.

\(^{20}\) According to the production function developed in n. 4 above, $\frac{\partial w}{\partial x} = a(e)$, where $e$ is the endowment and $\frac{\partial w}{\partial x}$ is the marginal productivity of parental expenditures on child quality. The marginal cost of raising quality equals

$$p_{q} = \frac{p_{x}}{\frac{\partial w}{\partial x}} = \frac{p_{x}}{a(e)},$$

where $p_{x}$ is the price of a unit of $x$. Then an increase in the endowment lowers or raises $p_{q}$ as

$$\frac{d}{de} \left(\frac{\partial w}{\partial x}\right) = \frac{da}{de} \geq 0.$$
their parents; indeed, parents could actually reinforce differences in endowments. Two opposing forces are at work; a “wealth” effect that induces parents to compensate less well endowed children and an efficiency or “price” effect that induces them to reinforce better-endowed children.

Although the net outcome of the wealth and price effects may seem to be indeterminant here, as in many other problems, there is actually a strong presumption that the price effect dominates for investments in human capital and the wealth effect for investments in nonhuman capital. That is, parents invest more human and less nonhuman capital in their better-endowed children. This conclusion does not depend on any assumption about the ease of substituting between the qualities of different children in their parents’ utility function.

Assume that the cost to parents of investing in the human capital of their children is negatively related to their endowment, while the cost of investing in their nonhuman capital (via gifts and bequests) is independent of their endowment, or at least much more independent than for human capital.

If parents invested nonhuman as well as human capital in each child, and if the cost of human capital rose with the amount invested in a child, the marginal cost of investing in the human capital of each child would, in equilibrium, equal the given marginal cost of investing in nonhuman capital (otherwise no investment in the latter would be warranted). Since more human capital, perhaps much more, would have to be invested in better-endowed children to equate their marginal cost to the marginal cost of more poorly endowed children, investments in human capital must reinforce differences in endowments. On the other hand, since the marginal costs of all investments are equal in equilibrium, the total quality (based on nonhuman as well as human capital) of all children must be the same. Therefore, investment in nonhuman capital must sufficiently compensate children with poorer endowments to offset exactly the greater investment of human capital in children with better endowments.

Most parents, even poorer ones, usually invest something in the human capital of their children but give them only negligible amounts (sometimes even negative amounts!) of nonhuman capital. They would, however, still tend to invest more human capital in better-endowed children if they anticipated that these children would “care” sufficiently about their

21 Instead of referring to costs, we could equally well say that rates of return are positively related to endowments for human capital and are independent of endowments for nonhuman capital.

22 Modifying n. 20, we let \( \frac{\partial a}{\partial x} = a(e, x) \), with \( \frac{\partial a}{\partial x} < 0 \). Then

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \left[ \frac{p_x}{a(e, x)} \right] > 0.
\]

23 By eqqs. (3.2) and (3.3) when \( p_{q_1} = p_{q_2} \).
siblings to transfer voluntarily enough resources to their siblings, because
the average cost of parental investments would then be lower.24

What may seem surprising is that they would tend to invest more
human capital in better-endowed children even if children were selfish.
This assertion follows from the “rotten-kid” theorem (Becker 1974b),
which says that even selfish children take account of their parents’ desires
if they are receiving transfers from their parents. In particular, better-
endowed children would recognize that their parents would invest more
human capital in them if they transferred enough resources to their
siblings.25 Consequently, they would have a selfish incentive to transfer
resources voluntarily to their siblings; parents then would have an in-
centive to invest more human capital in these better-endowed children.26

Our conclusion is that the price effect dominates the wealth effect, that
more human capital is invested in better-endowed children. Therefore,
parents contribute to the observed inequality in earnings by investing
more human capital in children who would receive higher earnings any-
way because of their greater endowment. However, since parents invest
more nonhuman capital in poorly endowed children, they reduce the
inequality in total income relative to that in earnings.

The “Failure” of Compensatory Education

“Compensatory education has been tried and it apparently has failed. . . .
The chief goal of compensatory education—to remedy the educational
lag of disadvantaged children and thereby narrow the achievement gap
between ‘minority’ and ‘majority’ pupils—has been utterly unrealized in
any of the large compensatory education programs that have been
evaluated so far” (Jensen 1969). So begins Arthur Jensen’s famous and
controversial essay on compensatory education and intelligence. His
assertion about the apparent failure of compensatory education has not
been controversial; indeed, subsequent studies have only buttressed it.
What has been controversial is his linking the apparent failure to the
inferior intelligence of the children, primarily black children, being com-
pensated. Our analysis has nothing directly to add to the controversy
about the relative intelligence of different groups of children, but it is in-

24 The average cost would be minimized if sufficiently more were invested in better-
endowed children to equalize marginal costs.
25 The “rotten-kid” theorem also implies that a child has an incentive to invest in
himself the amount of his own time and energy that is optimal to the family even though
poorly “endowed” children are fully compensated with nonhuman capital.
26 The one difficulty with this application of the “rotten-kid” theorem is that, since
much of the return on human capital is received after the investment period, parents
cannot directly ensure that less endowed children receive the appropriate transfers from
their siblings. This does not mean, however, that parents have no control over the be-
havior of grown children: social and family “pressures” can induce these children to
conform to the terms of implicit contracts with their parents.
directly relevant because it can explain why compensatory education programs would appear to "fail" even when the children compensated are as able as other children.

Public compensatory education programs essentially increase the "endowments" of some children in poorer families. The increase in the wealth of these families produced by the increase in endowments would induce a redistribution of parental time and expenditures away from the children being compensated and toward their other children and themselves. That is, the induced own "compensatory programs" by parents help defeat the intent of public programs. Although family wealth rises by the full extent of the increase in the endowment of a child participating in a compensatory program, the total (parental included) investment in that child may only rise by a small fraction of the increase in his endowment. The fraction depends on the contribution of his endowment to family wealth, the family's income elasticity of demand for child quality, and so forth.

To be sure, compensatory public programs may also have a price or efficiency effect that lowers (it may also raise) the cost of adding to the quality of compensated children. Unlike the wealth effect, such a price effect would reinforce the intent of public programs because lower costs induce greater parental investment in these children.

The important point, however, is that if the effect on costs was relatively unimportant, which is not implausible, the main result of compensatory programs would be to redistribute wealth to families of compensated children, with little increase in the total human capital invested in these children. Since redistributions of wealth to these families could be and is achieved more directly, compensatory programs could legitimately be considered "failures" (although not without any effects on poor families). 27

Note that this conclusion does not require compensated children to be inferior in ability or motivation—that is, to receive low rates of return on investments in their human capital—for they could even be above average. Nor does it require compensated programs to be badly planned or administered; again, these programs could be better run than more successful programs. It requires only that compensatory programs reduce significantly the amount of time and money that parents invest in their "compensated" children. 28

27 Some evidence for higher education suggests that reductions in parental expenditures can nullify most of the effects of a public education program (see Peltzman 1973).
28 This analysis implies that a "total intervention" program—that is, a program whereby the state (Plato's Republic?) or some other authority assumes the entire responsibility for investment in a child—would not appear to "fail" because offsetting parental reactions would not be possible. Similarly, if parents invested little, perhaps because of poverty or neglect, a sizable compensatory program could not "fail" because only a small induced decline in parental investments is possible.
Family Background and Rates of Return on Human Capital

Estimated rates of return on education and other human capital are biased because persons with better ability, motivation, and family background usually accumulate more human capital than other persons do. Several studies have tried to reduce this bias by considering differences between siblings, and many studies recently have adjusted for family background. The argument is that the variation in ability and the social environment is reduced by considering siblings, or even persons with similar backgrounds, and that this might substantially reduce the bias in estimated rates of return.

The difficulty with this argument is that, although ability, motivation, and family background are much less variable between siblings or between persons with similar backgrounds than in the whole population, the covariance between these variables and the amount invested in human capital may not be any less; indeed, it may be greater. In introducing the Gorseline data in his earlier study, Becker said that "some brothers may become relatively well-educated precisely because of unusual ambition and other kinds of ability rather than because of interest, luck, and other factors uncorrelated with earnings" (1964, p. 87). Yet it is the covariation, not the variation itself, that is the source of the bias in estimated rates of return.

The analysis in this section does provide the means for determining the effect on the bias of considering siblings or unrelated persons with similar family backgrounds. Consider the following relation explaining the human capital invested by parents in a child:

\[
S_c = b_0 + b_cE_c + b_yY_p + b_NN + u, \tag{3.4}
\]

where \(E_c\) is the ability, motivation, and other components of his "endowment"; \(Y_p\) is parental education, income, and other dimensions of his family background; \(N\) is the number of children in his family; \(u\) represents other influences on \(S_c\) that are assumed to be independent of \(E_c\) and \(Y_p\); and \(b_0, b_c, b_N, b_y\) are constants.

Since "wealthier" parents invest more in their children, \(b_y > 0\); indeed, our analysis implies that \(b_y\) is "large" because an increase in parental

29 Related issues are discussed in a recent paper by Griliches (1975b). We are indebted to James Heckman for helpful comments that corrected some errors in a previous version.

30 D. Gorseline in the late 1920s was one of the first to collect data on the schooling and incomes of siblings (see Gorseline 1932). Becker (1964) used Gorseline's data in trying to determine the magnitude of the bias in his estimated rates of return on schooling. Recently, Chamberlain and Griliches (1975) developed a sophisticated statistical analysis to reconsider Gorseline's data, as well as later data from the Parnes study. Still more recently, Paul Taubman has been considering the earnings, schooling, and other aspects of twins.

wealth substantially increases their investment if the endowment ($E_c$) is held constant. Similarly, by the interaction between quantity and quality discussed in Section 2, $b_n < 0$. The magnitude and sign of $b_n$ depend on the relative importance of parental compensation or enforcement of the endowment; our analysis suggests that reinforcement dominates for human capital, so that $b_n$ would be $> 0$ (compensation dominates for nonhuman capital).

Empirical estimates of the coefficients in equation (3.4) have been scarce because the same data set has seldom contained information on a child's endowment and his family background. Many studies have found a powerful effect of family background on children's human capital in regressions between these variables. Moreover, using an indirect method of estimation, Chamberlain and Griliches (1975) find that able brothers receive more schooling than less able ones. Both the powerful effect of background and the positive effect of greater child ability on parental investment are consistent with our analysis.

The earnings-generating equation of a person can be written as

$$\log I_c = a_0 + rS_c + a_eE_c + v,$$

(3.5)

where $I_c$ is his potential earnings, $r$ is the rate of return on the human capital invested by his parents $S_c$, $v$ is assumed to be independent of $u$, and $a_e$ is the direct effect of endowment on earnings. If his endowment were omitted from the earnings equation, the bias in the estimated rate of return would depend on $a_e$ and the relation between $S_c$ and $E_c$. That is,

$$r^* = r + a_e\beta_{es},$$

(3.6)

where $\beta_{es}$ is the coefficient in a regression of $E_c$ on $S_c$. Since $a_e > 0$, the direction of the bias is the same as the sign of $\beta_{es}$, and the magnitude of the bias depends on $a_e$ as well as $\beta_{es}$.

If rates of return were estimated from the earnings of siblings or from the earnings of persons with similar family backgrounds, $Y_p$ would be held constant when $S_c$ changed; if the number of siblings were also held constant, the regression coefficient of $E_c$ on $S_c$ would be

$$\beta_{es, y} = \frac{1}{b_e} (1 - d_{us, y}),$$

(3.7)

32 See the references in n. 31 above and the paper by Griliches in this volume.

33 Any interaction between $S_c$ and $E_c$, and direct effects of family background ($Y_p$) on earnings, is ignored (the latter is discussed below). The bias that results from ignoring any covariation between $u$ and $v$ can be treated along the same lines that we use to analyze the bias from omitting the endowment $E_c$.

34 By transposing eq. (3.4), $E_c$ becomes a function of $S_c$ and the other variables:

$$E_c = \beta_0 + \frac{1}{b_e} S_c - \frac{b_y Y_p}{b_e} - \frac{b_y}{b_e} - \frac{1}{b_e} u.$$

Then

$$\beta_{es, y} = \frac{1}{b_e} - \frac{1}{b_e} d_{us, y}.$$
where $d_{us,y}$ is the coefficient in a regression of $u$ on $S$ when $Y$ is held constant. The magnitude of $d_{us,y}$ is bounded by zero and unity and depends on the relative importance of "random" forces and endowments in determining investments in human capital. Hence, the sign of the bias in rates of return estimated from persons with similar family backgrounds would be the same as the sign of $b_e$. This sign is determined by whether parents on balance compensate less endowed or reinforce better-endowed children; if reinforcement dominates, as we argue above, $b_e > 0$ and rates of return would be overestimated; if compensation dominates, $b_e < 0$ and these rates would be underestimated. The bias would be very large if "random" forces were unimportant and if compensation and reinforcement almost offset each other.

There is a somewhat paradoxical relation between family background and endowment. If reinforcement dominated compensation ($b_e > 0$), then children with better backgrounds might well be less endowed than children with poorer backgrounds having the same parental investment in their human capital, in spite of the general presumption that children from better backgrounds are better endowed. From footnote 34 this would occur if

$$\beta_{ey,s} = -\frac{1}{b_e} (b_y + d_{uy,s}) < 0, \text{ or if } b_y > -d_{uy,s}, \quad (3.8)$$

where $d_{uy,s} < 0$ is the coefficient in a regression of $u$ on $Y$ when $S$ is held constant. Since our theory implies that $b_y$ is positive and "large," this inequality very likely would hold.

Fortunately, this paradox is resolved quite readily. Consider the simple case where investment in children is entirely determined by background and endowment. Then an improvement in background increases the investment, as does an improvement in endowment when reinforcement dominates. Hence, if children from different backgrounds had the same

35 That is, the presumption is that (from n. 34)

$$b_{ey} = \frac{1}{b_e} (-b_y - d_{uy,s} + d_{sy} - d_{us,y}d_{sy}) > 0,$$

where $d_{sy}$ is the coefficient in a regression of $S$ on $Y$. Since

$$R_{sy}^2 = (d_{sy})(d_{ys}),$$

where $R_{sy}$ is the simple correlation between $S$ and $Y$, this can be written as

$$b_{ey} = \frac{1}{b_e} [-b_yd_{ys} - d_{uy,s}d_{ys} + R_{sy}^2(1 - d_{us,y})] > 0.$$

36 If persons from different backgrounds had the same investment, those with better backgrounds would tend to be "unluckier"; otherwise they would have greater investment because an increase in background directly raises investment and also indirectly raises it by raising endowment.
investment, those with better backgrounds would have to be less endowed; otherwise, more would be invested in them.37

If reinforcement dominates, and if rates of return were estimated from persons with different backgrounds, the bias is still necessarily positive. However, this bias would tend to be smaller than in the estimates that hold background constant.38 If compensation dominated, the biases could be of opposite signs: necessarily negative when background is held constant and possibly positive when it varies.39 The bias is likely to be smaller when background varies,40 because it introduces considerable variation in the amount invested at any given endowment, which provides more uncontaminated evidence for estimating the true effect of investment on earnings.

One final point remains to be made.41 This analysis is relevant in evaluating the many attempts in recent years to measure the direct effect of family background on earnings by including background variables in earnings-generating equations.42 If the endowment were omitted from or only imperfectly measured in these equations, the estimated direct effect of background would be biased. Its direct effect on earnings would be underestimated if an increase in family background decreased the unobserved endowment when the amount invested in human capital were held constant43 and overestimated if it increased the endowment. Therefore, the weak positive effect observed in these studies could give a misleading impression of the true effect.

37 This conclusion was already reached in Becker (1967) and in Mincer (1970). It would be reversed if compensation dominated, for then \( b_e < 0 \) and \( \beta_{ey,s} > 0 \) if \( b_y < -d_{sy,s} \), where \( d_{sy,s} \) could now be positive, too. Now if children from different backgrounds had the same investment, those with better background would be better endowed, since a better endowment would offset rather than reinforce a better background.

38 It can easily be shown that

\[
\beta_{es} = \beta_{es,y} + \beta_{ey,s} d_{ys} = \frac{1}{b_e} \left[ 1 - d_{ey,s} - d_{ys} (b_y + d_{uy,s}) \right].
\]

Since \( R_{sy} \leq 1 \), n. 35 implies that \( \beta_{es} > 0 \) or that the bias is still positive when background is free to vary. Moreover, since \( d_{ys} > 0 \), then \( \beta_{ey,s} < 0 \) and \( \beta_{es} < \beta_{es,y} \) if \( b_y > -d_{uy,s} > 0 \) (see eq. [3.8]).

39 If \( b_e < 0 \) if compensation dominated, then clearly \( \beta_{ey,s} < 0 \). Moreover, assuming that \( d_{ys} \) still > 0, \( \beta_{es} > \beta_{es,y} \) if \( b_y + d_{uy,s} > 0 \), or the bias is likely to be less negative when background is free to vary. If \( b_y + d_{uy,s} \) and \( d_{ys} \) were sufficiently positive, then \( \beta_{es} > 0 \) or \( \beta_{es,y} \) and \( \beta_{es} \) would have opposite signs.

40 If background directly affects earnings and was directly included in eq. (3.5), an additional source of bias is introduced by variation in background. Although weak direct effects of background on earnings have been found in most empirical studies, the true effects could be seriously understated (see the discussion in the text).

41 This point is more fully discussed in an addendum to the 1975 edition of Becker's Human Capital and is only summarized here.

42 See the references in n. 31.

43 This was assumed to be the case in the addendum cited in n. 41.
4. Conclusions

This paper considers various effects of the existence of an "endowment" to each child of inherited ability, public subsidies, and "luck" on the quantity and "quality" of children. For example, an increase in parental income would lead to a relatively large increase in parental expenditures on children if their endowments were fixed, because all of the desired increase in the quality of children would have to come from an increase in these expenditures. The large increase in expenditures would reduce the demand for children because the cost of each child is directly related to the expenditure on each.

It is further shown that the elastic response of expenditures to an increase in parental income implies that, even if the true income elasticities of demand for quantity and quality of children were equal, constant, and of average size, the observed quantity elasticity would be small and perhaps negative at lower income levels and larger and perhaps positive at higher levels. Moreover, although the observed quality elasticity could be much larger than its true elasticity, the observed quality elasticity would decline as income rose. Both the observed quantity and quality income elasticities depend on the degree of intergenerational mobility in economic position and the rate of growth over time in income: increased mobility would reduce the observed quantity and increase the observed quality elasticity, whereas increased growth would increase the observed quantity and reduce the quality elasticity.

If some children were better endowed than others, parents could either compensate those with poorer endowments by spending more on them or reinforce those with better endowments. We show that parents tend to invest more human capital in better-endowed children and more non-human capital in poorer ones. That is, they reinforce with human capital and compensate with nonhuman capital. Parental responses, therefore, tend to widen the inequality in earnings and narrow that in income relative to earnings.

Public (or private) "compensatory" education programs for certain children would affect the amount of time and money spent on them by their parents. If increased public expenditures induced a decline in parental expenditures of time and money, public "compensatory" programs might have only a small effect on the total investment in "compensated" children, including the investment by parents. "Compensatory" programs would then appear to fail even if "compensated" children were as able and well motivated as other children and even if these programs were efficiently conducted.

Rates of return on the human capital invested by parents in their children are sometimes estimated from comparisons of siblings or from unrelated persons with similar family backgrounds. These estimates would
be biased if endowments were only imperfectly held constant. For example, if, on balance, parents reinforced children with better endowments, the true rates would be overestimated because endowments would also increase as the investment in human capital increased—given the assumption that better endowments are reinforced. Indeed, the bias would tend to be greater than the bias in estimates that permit background to vary.

Furthermore, simply entering family background variables directly into earnings-generating regression equations would result in biased—perhaps quite biased—estimates of the direct effect of family background on earnings. If the investment in human capital were held constant and if endowments were only imperfectly held constant, an increase in background would tend to reduce endowments if parents reinforced children with better endowments (and increase endowments if they compensated poorer endowments). Consequently, the direct effect of background on earnings would be underestimated with reinforcement (and overestimated with compensation).